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GRAY CODE HAMMING DISTANCE LABELING ON GRAPHS

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Abstract

Computers store data in the form of bit or binary digits. Gray code is a reflected binary code, which is an ordering of the binary numeral system such that two successive values differ by only one bit (binary digit). The hamming distance is a metric used to compare two binary strings of equal length and is defined as the number of bit positions in which the two binary strings differ. The hamming distance between two binary strings a and b of equal length is denoted by $\mathrm{hd}(a, \mathrm{b})$. It is used for error detection in data transmission and in coding theory. In this paper, we define a new labeling called gray code hamming distance labeling. The existence of gray code hamming distance labeling of M- join of Comb graph, H- graph, and Twig graph were discussed and their gray code hamming distance number were obtained.

Keywords: Gray Code Hamming Distance Labeling , Gray Code Hamming Distance Number, M-join of Comb Graph, M-join of H-graph, M-join of Twig Graph.

1. INTRODUCTION

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for every edge uv a label depending on the vertex label of u and v[4].We introduced hamming distance labeling, odd hamming distance labeling and even hamming distance labeling[5]. It is proved that some path related graphs admit hamming distance and odd hamming distance labeling and some cycle related graphs admit even hamming distance labeling. In this paper, a new labeling called gray code hamming distance labeling is introduced. Also we constructed M-join of H-graph, Comb graph, Twig graph, and proved that these graphs admit gray code hamming distance labeling.

2. GRAY CODE HAMMING DISTANCE LABELING OF M-JOIN OF GRAPHS

In this section, we introduce a new labeling called gray code hamming distance labeling and is defined as follows: In this definition, the notation $[x]_2$ denotes the binary conversion of the number *x*, $GC(x)$ denotes the gray code of the number *x* and $[GC(x)]_d$ denotes the decimal value of $GC(x)$.

Definition 2.1. The function $f: V \rightarrow N \cup \{0\}$ is said to be a gray code hamming distance labeling if it satisfies the following conditions.

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(i) For the function $g: f(v) \to [GC(f(v))]_d$ for every $v \epsilon V$, the adjacent vertices receive distinct labels (as decimals of gray codes)

(ii) There exist an induced function $g^* : E \to \{1,2,3 \dots n\}$ for the function g such that $g^*(uv) = hd([g(f(u))]_2, [g(f(u))]_2)$, for every edge $uv \epsilon E$, the adjacent edges receive distinct labels.

The gray code hamming distance number of a graph G is the least positive integer n such that $2^n - 1 \geq k$, where $k = \max\{f(v)/v \in V\}$ and is denoted by $\eta_{gchd}(\text{G})$.

Structure of M-join of H- graph

The H-graph is obtained by adding an edge between even degree vertices of two path graphs each of length two (P_2 and P_2'). I.e. It is a tree on 6 vertices in which exactly two vertices are of degree 3[3, 7]. The vertex set and edge set of M+1 copies of H –graph is defined as follows:

$V = \left\{ u_i, v_i, u_i^{(j)}, v_i^{(j)} \quad /1 \leq i \leq 3, 1 \leq j \leq M \right\}$

E = $E_1 \cup E_2$, where

$E_1 = \{u_i u_{i+i}, v_i v_{i+i}, u_2 v_2 / 1 \leq i \leq 2\}$,

$E_2 = \{u_i^{(j)} u_{i+1}^{(j)}, v_i^{(j)} v_{i+1}^{(j)}, u_2^{(j)} v_2^{(j)} / 1 \leq i \leq 2, 1 \leq j \leq M\}$,

The M- Join of H –graph is obtained from the above M+1 copies of H-graphs by attaching an edge between consecutive H-graphs. The vertex set V' and edge set E' of this graph is given as follows:

$V' = V$ and $E' = E \cup \{v_3 u_1^{(1)}, v_3^{(j)} u_1^{(j+1)} / 1 \leq j \leq M - 1\}$.

This graph as 6(M+1) vertices and 6M+1 edges.

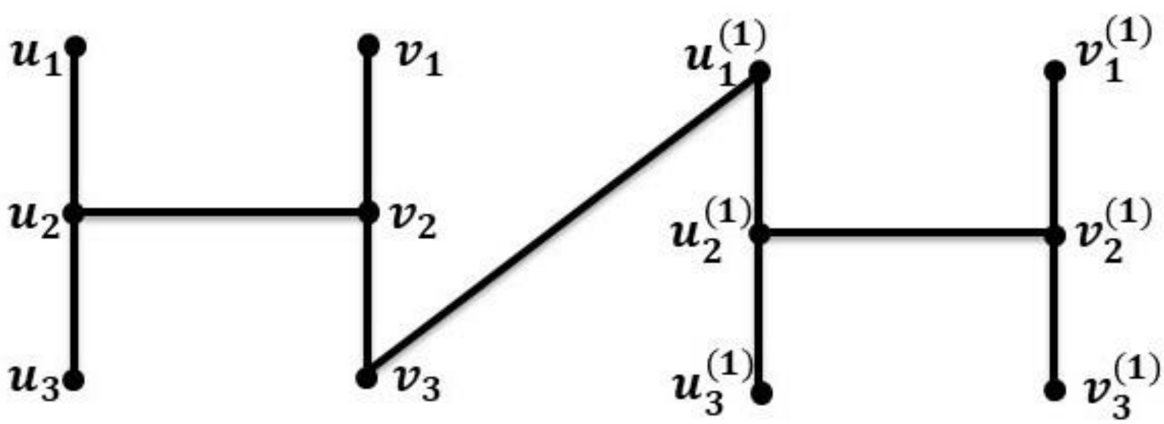

Figure 1: Structure of M-Join of H-graph

Theorem 2.1. The M- join of H-graph admits gray code hamming distance labeled graph and the gray code hamming distance number is $\eta_{gchd}(\text{G})$ =4.

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Proof: consider the M-join of H-graph whose vertex set and edge set are given in the above structure. Define a function $f: V \to N \cup \{0\}$ to label the vertices of the graph in such a way that that $f(u) \neq f(v)$ for any two adjacent vertices u, v and the procedure for labeling the vertices is given in the following algorithm.

Procedure: Vertex labeling of M- join of H-graph, $M \geq 1$.

Input: M- join of H-graph

$V \leftarrow \{u_i, v_i, u_i^{(j)}, v_i^{(j)} / 1 \leq i \leq 3, 1 \leq j \leq M\}$

$u_1 \leftarrow 5; u_2 \leftarrow 0; u_3 \leftarrow 14;$

$v_1 \leftarrow 4; v_2 \leftarrow 1; v_3 \leftarrow 15;$

for $i = 1\ to\ 3$ do

for $j = 1\ to\ M$ do

$$u_i^{(j)} \leftarrow \begin{cases} 14\ if\ i = 1 \\ 0\ if\ i = 2 \\ 5\ if\ i = 3 \end{cases},\ v_i^{(j)} \leftarrow \begin{cases} 4\ if\ i = 1 \\ 1\ if\ i = 2 \\ 15\ if\ i = 3 \end{cases}$$

end for

end for

end procedure

Output: The labeled vertices of M- join of H-graph.

Define a function $g: f(V) \to [GC(f(v))]_d$ where $g(f(u)) = [GC(f(u))]_d$ for every $u \in V$

($GC(x)$ denotes the gray code of the number x) such that $g(f(u_1)) \neq g(f(u_2))$ for every adjacent vertices $u_1, u_2 \in V$.

Now $g(f(u_1)) = 7;\ g(f(u_2)) = 0;\ g(f(u_3)) = 9;$

$g(f(v_1)) = 6;\ g(f(v_2)) = 1;\ g(f(v_3)) = 8;$

For $1 \leq j \leq M$

$$g(f(u_i^{(j)})) \leftarrow \begin{cases} 9\ if\ i = 1 \\ 0\ if\ i = 2; \\ 7\ if\ i = 3 \end{cases}\quad g(f(v_i^{(j)})) \leftarrow \begin{cases} 6\ if\ i = 1 \\ 1\ if\ i = 2 \\ 8\ if\ i = 3 \end{cases}$$

Clearly all the adjacent vertices receive distance gray code as labels. Now The induced edge labels are calculated as follows:

$g^*(u_1u_2) = hd([g(f(u))]_2, [g(f(v))]_2) = hd([7]_2, [0]_2) = 3.$

$g^*(u_2u_3) = hd([0]_2, [9]_2) = 2;\ g^*(v_1v_2) = hd([6]_2, [1]_2) = 3;$

$g^*(v_2v_3) = hd([1]_2, [8]_2) = 2;\ g^*(u_2v_2) = hd([0]_2, [1]_2) = 1;$

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$g^*(v_3u_1^{(1)}) = hd([8]_2, [9]_2) = 1$;

For $M \geq 2$,$1 \leq j \leq M-1$, $g^*(v_3^{(j)}u_1^{(j+1)}) = hd([8]_2, [9]_2) = 1$;

For $1 \leq j \leq M$, $1 \leq i \leq 2$

$g^*(u_i^{(j)}u_{i+1}^{(j)}) \leftarrow \begin{cases} 2 \; if \; i = 1 \\ 3 \; if \; i = 2 \end{cases}; g^*(v_i^{(j)}v_{i+1}^{(j)}) \leftarrow \begin{cases} 3 \; if \; i = 1 \\ 2 \; if \; i = 2 \end{cases}; g^*\left(u_2^{(j)}v_2^{(j)}\right) = 1.$

From all the above cases, for all the adjacent edges$e_1, e_2 \in E, g^*(e_1) \neq g^*(e_2)$. Hence the M- Join of H-graph admits gray code hamming distance labeling and the gray code hamming distance number is $\eta_{gchd}(G) = 4$.

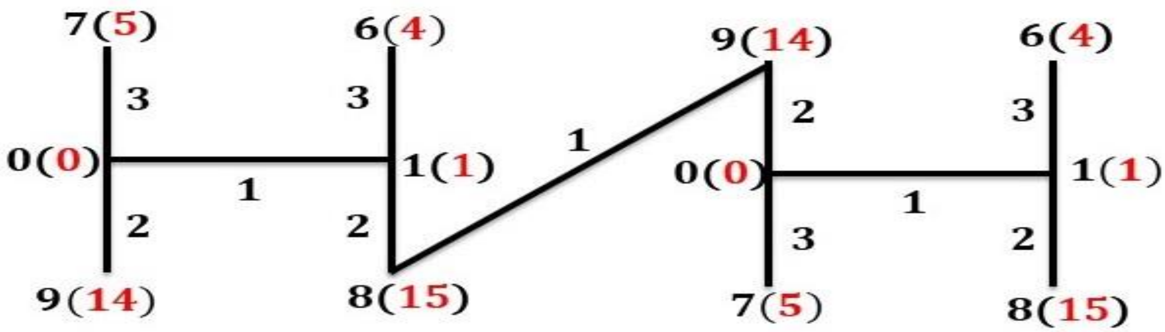

Figure 2: Gray code labeling of 1-Join of H-graph

Structure of M-Join of Comb graph

Comb graph is obtained by joining a single pendant edge to each vertex of a path P_n, $n \geq 2$ and it is denoted by P_n^+[4,6].

The vertex set and edge set of M+1 copies of Comb graph is defined as follows:

$V = \left\{ u_i, v_i, u_i^{(j)}, v_i^{(j)} \;/0 \leq i \leq n, 1 \leq j \leq M \right\}$

E = $E_1 \cup E_2$, where

$E_1 = \{v_iu_i, v_i^{(j)}\,u_i^{(j)}/0 \leq i \leq n, 1 \leq j \leq M, n \geq 2\,\}$.

$E_2 = \{v_iv_{i+1}, v_i^{(j)}v_{i+1}^{(j)},/0 \leq i \leq n-1, 1 \leq j \leq M, n \geq 2\}$.

The M-join of Comb graph is obtained from the above M+1 copies of comb graph by attaching an edge between consecutive Comb graphs. The vertex set $V'$and edge set $E'$of this graph is given as follows:

$V' = V$ and $E' = E \cup \{u_nv_0^{(1)}, u_n^{(j)}v_0^{(j+1)}/1 \leq j \leq M-1, M \geq 2\}$.

This graph as (2n+2)(M+1) vertices and [(2n+2)(M+1)]-1 edges

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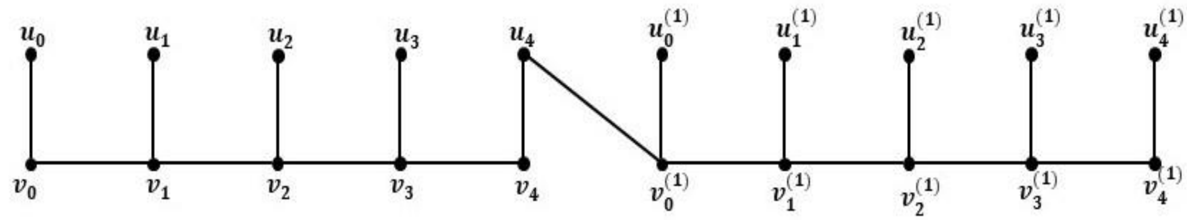

Figure 3: Structure of 1-Join of Comb Graph

Theorem 2.2. The M- join of comb graph P_n^+ is a gray code hamming distance labeled graph for $n \equiv 0(mod4)$ and the gray code hamming distance number is$\eta_{gchd}(G) = 4$.

Proof: Let us consider the M- join of comb graph P_n^+ whose vertex set and edge set are given in the above structure. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of the graph in such a way that that $f(u) \neq f(v)$ for any two adjacent vertices u, v and the procedure for labeling the vertices is explained in the following algorithm.

Procedure: Vertex labeling of M- join of comb graph, $M \geq 1$.

Input: Vertices of M- join of comb graph,

$V = \{u_i, v_i, u_i^{(j)}, v_i^{(j)} / 0 \leq i \leq n, 1 \leq j \leq M\}$

$u_0 \leftarrow 5; u_1 \leftarrow 15; u_n \leftarrow 1;$

$v_0 \leftarrow 0; v_1 \leftarrow 1; v_n \leftarrow 3;$

for $i = 2\ to\ n - 1$ do

$$u_i \leftarrow \begin{cases} 8\ if\ i \equiv 0(mod4) \\ 6\ if\ i \equiv 1(mod4) \\ 3\ if\ i \equiv 2(mod4) \\ 11\ if\ i \equiv 3(mod4) \end{cases}; \quad v_i \leftarrow \begin{cases} 3\ if\ i \equiv 0(mod4) \\ 1\ if\ i \equiv 1(mod4) \\ 4\ if\ i \equiv 2(mod4) \\ 0\ if\ i \equiv 3(mod4) \end{cases}$$

end for

for $j = 1\ to\ M$ do

if $j \equiv 1(mod2)\ do$

$u_0^{(j)} \leftarrow 9; v_0^{(j)} \leftarrow 2; u_n^{(j)} \leftarrow 0; v_n^{(j)} \leftarrow 4;$

for $i = 1\ to\ n - 1$ do

$$u_i^{(j)} \leftarrow \begin{cases} 3\ if\ i \equiv 0(mod4) \\ 11\ if\ i \equiv 1(mod4) \\ 8\ if\ i \equiv 2(mod4) \\ 6\ if\ i \equiv 3(mod4) \end{cases}; \quad v_i^{(j)} \leftarrow \begin{cases} 4\ if\ i \equiv 0(mod4) \\ 0\ if\ i \equiv 1(mod4) \\ 3\ if\ i \equiv 2(mod4) \\ 1\ if\ i \equiv 3(mod4) \end{cases}$$

end for

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else

$u_n^{(j)} \leftarrow 1; v_n^{(j)} \leftarrow 3;$

for $i = 0 \; to \; n-1$ do

$$u_i^{(j)} \leftarrow \begin{cases} 8 \; if \; i \equiv 0(mod4) \\ 6 \; if \; i \equiv 1(mod4) \\ 3 \; if \; i \equiv 2(mod4) \\ 11 \; if \; i \equiv 3(mod4) \end{cases}; \quad v_i^{(j)} \leftarrow \begin{cases} 3 \; if \; i \equiv 0(mod4) \\ 1 \; if \; i \equiv 1(mod4) \\ 4 \; if \; i \equiv 2(mod4) \\ 0 \; if \; i \equiv 3(mod4) \end{cases}$$

end for

end if

end for

end procedure

Output: The labeled vertices of M- join of comb graph

Define a function $g: f(V) \rightarrow [GC(f(v))]_d$ where $g(f(u)) = [GC(f(u))]_d$ for every $u \in V$ such that $g(f(u_1)) \neq g(f(u_2))$ for every adjacent vertices$u_1, u_2 \in V$.

Now $g(f(u_0)) = 7; \; g(f(u_1)) = 8; \; g(f(u_n)) = 1;$

$$g(f(v_0)) = 0; \; g(f(v_1)) = 1; \; g(f(v_n)) = 2;$$

for $2 \leq i \leq n-1$

$$g(f(u_i)) \leftarrow \begin{cases} 12 \; if \; i \equiv 0(mod4) \\ 5 \; if \; i \equiv 1(mod4) \\ 2 \; if \; i \equiv 2(mod4) \\ 14 \; if \; i \equiv 3(mod4) \end{cases}; \quad g(f(v_i)) \leftarrow \begin{cases} 2 \; if \; i \equiv 0(mod4) \\ 1 \; if \; i \equiv 1(mod4) \\ 6 \; if \; i \equiv 2(mod4) \\ 0 \; if \; i \equiv 3(mod4) \end{cases}$$

for $1 \leq j \leq M$and if $j \equiv 1(mod2)$

$$g\left(f\left(u_0^{(j)}\right)\right) = 13; \; g\left(f\left(u_n^{(j)}\right)\right) = 0; \; g\left(f\left(v_0^{(j)}\right)\right) = 3; \; g\left(f\left(v_n^{(j)}\right)\right) = 6;$$

for $1 \leq j \leq M, 1 \leq i \leq n-1$ and if $j \equiv 1(mod2)$

$$g\left(f\left(u_i^{(j)}\right)\right) \leftarrow \begin{cases} 2 \; if \; i \equiv 0(mod4) \\ 14 \; if \; i \equiv 1(mod4) \\ 12 \; if \; i \equiv 2(mod4) \\ 5 \; if \; i \equiv 3(mod4) \end{cases}; \quad g\left(f\left(v_i^{(j)}\right)\right) \leftarrow \begin{cases} 6 \; if \; i \equiv 0(mod4) \\ 0 \; if \; i \equiv 1(mod4) \\ 2 \; if \; i \equiv 2(mod4) \\ 1 \; if \; i \equiv 3(mod4) \end{cases}$$

for $1 \leq j \leq M, 0 \leq i \leq n-1$ and if $j \equiv 0(mod2)$

$$g\left(f\left(u_n^{(j)}\right)\right) = 1; \quad g\left(f\left(v_n^{(j)}\right)\right) = 2;$$

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$$g\left(f\left(u_i^{(j)}\right)\right) \leftarrow \begin{cases} 12\ if\ i\ \equiv 0(mod4) \\ 5\ if\ i\ \equiv 1(mod4) \\ 2\ if\ i \equiv 2(mod4) \\ 14\ if\ i \equiv 3(mod4) \end{cases}; \quad g\left(f\left(v_i^{(j)}\right)\right) \leftarrow \begin{cases} 2\ if\ i\ \equiv 0(mod4) \\ 1\ if\ i\ \equiv 1(mod4) \\ 6\ if\ i \equiv 2(mod4) \\ 0\ if\ i \equiv 3(mod4) \end{cases}$$

Clearly all the adjacent vertices receive distance gray code as labels. Now The induced edge labels are calculated as follows:

$g^*(v_0u_0) = hd([g(f(v_0))]_2, [g(f(u_0))]_2) = hd([0]_2, [7]_2) = 3$.

$g^*(v_1u_1) = hd([1]_2, [8]_2) = 2$; $g^*(v_0v_1) = hd([0]_2, [1]_2) = 1$;

$g^*(v_nu_n) = hd([2]_2, [1]_2) = 2$; $g^*(v_{n-1}v_n) = hd([0]_2, [2]_2) = 1$;

$g^*(u_nv_0^{(1)}) = hd([1]_2, [3]_2) = 1$.

For $1 \le j \le M-1, M \ge 2$, $g^*(u_n^{(j)}v_0^{(j+1)}) = 1$

For $2 \le i \le n-1$, $g^*(v_iu_i) \leftarrow \begin{cases} 1\ if\ i\ \equiv 1,2(mod4) \\ 3\ if\ i\ \equiv 0,3(mod4) \end{cases}$

For $1 \le i \le n-2$, $g^*(v_iv_{i+1}) \leftarrow \begin{cases} 2\ if\ i\ \equiv 0,2(mod4) \\ 3\ if\ i\ \equiv 1(mod4) \\ 1\ if\ i\ \equiv 3(mod4) \end{cases}$

For $1 \le j \le M$ and $if\ j \equiv 1(mod\ 2)$

$g^*(v_0^{(j)}u_0^{(j)}) = hd([3]_2, [13]_2) = 3$; $g^*(v_0^{(j)}v_1^{(j)}) = hd([3]_2, [0]_2) = 2$;

$g^*(v_{n-1}^{(j)}v_n^{(j)}) = hd([1]_2, [6]_2) = 3$; $g^*(v_n^{(j)}u_n^{(j)}) = hd([6]_2, [0]_2) = 2$;

For $1 \le j \le M, 1 \le i \le n-1$ and $if\ j \equiv 1(mod\ 2)$

$g^*(v_i^{(j)}u_i^{(j)}) \leftarrow \begin{cases} 3\ if\ i\ \equiv 1,2(mod4) \\ 1\ if\ i\ \equiv 0,3(mod4) \end{cases}$

For $1 \le j \le M, 1 \le i \le n-2$ and $if\ j \equiv 1(mod\ 2)$

$g^*(v_i^{(j)}v_{i+1}^{(j)}) \leftarrow \begin{cases} 2\ if\ i\ \equiv 0,2(mod4) \\ 1\ if\ i\ \equiv 1(mod4) \\ 3\ if\ i\ \equiv 3(mod4) \end{cases}$

For $2 \le j \le M$ and $if\ j \equiv 0(mod\ 2)$

$g^*(v_{n-1}^{(j)}v_n^{(j)}) = hd([0]_2, [2]_2) = 1$; $g^*(v_n^{(j)}u_n^{(j)}) = hd([2]_2, [1]_2) = 2$;

For $2 \le j \le M, 0 \le i \le n-1$ and $if\ j \equiv 0(mod\ 2)$

$g^*(v_i^{(j)}u_i^{(j)}) \leftarrow \begin{cases} 1\ if\ i\ \equiv 1,2(mod4) \\ 3\ if\ i\ \equiv 0,3(mod4) \end{cases}$

For $2 \le j \le M, 0 \le i \le n-2$ and $if\ j \equiv 0(mod\ 2)$

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$$g^*(v_i^{(j)} v_{i+1}^{(j)}) \leftarrow \begin{cases} 2\ if\ i\ \equiv 0,2(mod4) \\ 3\ if\ i\ \equiv 1(mod4) \\ 1\ if\ i\ \equiv 3(mod4) \end{cases}$$

From all the above cases, for all the adjacent edges$e_1, e_2 \epsilon E, g^*(e_1) \neq g^*(e_2)$. Hence the M- join of comb graph admits gray code hamming distance labeling and the gray code hamming distance number is $\eta_{gchd}(\mathrm{G}) = 4$.

Theorem 2.3. The M- join of comb graph P_n^+ is a gray code hamming distance labeled graph for $n \equiv 1(mod4)$and the gray code hamming distance number is$\eta_{gchd}(\mathrm{G}) = 4$.

Theorem 2.4. The M- join of comb graph P_n^+ is a gray code hamming distance labeled graph for $n \equiv 2(mod4)$and the gray code hamming distance number is$\eta_{gchd}(\mathrm{G}) = 4$.

Theorem 2.5. The M- join of comb graph P_n^+ is a gray code hamming distance labeled graph for $n \equiv 3(mod4)$an {d the gray code hamming distance number is$\eta_{gchd}(\mathrm{G}) = 4$.

Structure of M-Join of Twig graph

A Twig graph is obtained from a path by attaching exactly two pendant edges to each internal vertices of a pathP_n, $n \geq 3$ and it is denoted by $TW(P_n)$[4].

The vertex set and edge set of M+1 copies of Twig graph is defined as follows:

$V = \left\{v_i, v_i^{(j)}\ /0 \leq i \leq n, 1 \leq j \leq M\right\} \cup \left\{u_i, w_i, u_i^{(j)}\ , w_i^{(j)}\ /1 \leq i \leq n-1, 1 \leq j \leq M\right\}$

E = $E_1 \cup E_2\ \cup E_3$, where

$E_1 = \left\{v_i v_{i+1}, v_i^{(j)} v_{i+1}^{(j)}/0 \leq i \leq n-1, 1 \leq j \leq M,\right\}. E_2 = \{v_i u_i, v_i w_i/1 \leq i \leq n-1,$

$E_3 = \{v_i^{(j)} u_i^{(j)}, v_i^{(j)} w_i^{(j)}/1 \leq i \leq n-1, 1 \leq j \leq M\}.$

The M-join of Twig graph is obtained from the above M+1 copies of Twig graph by attaching an edge between consecutive Twig graphs. The vertex set $V'$and edge set $E'$of this graph is given as follows:$V' = V$ and $E' = E \cup \left\{v_n v_0^{(1)}\right\} \cup \{v_n^{(j)} v_0^{(j+1)}/1 \leq j \leq M-1\}$.

This graph as (3n-1) (M+1) vertices and [(3n-1)(M+1)]-1 edges.

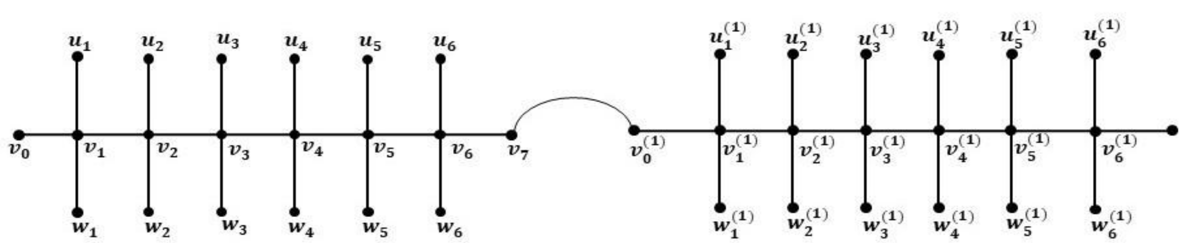

Figure 4: Structure of 1-Join of Twig Graph

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Theorem 2.6. The M- join of twig graph $TW(P_n)$ is a gray code hamming distance labeled graph for $n \equiv 3(mod4)$and the gray code hamming distance number is$\eta_{gchd}(\mathrm{G}) = 4$.

Proof: Let us consider the M- join of twig graph $TW(P_n)$ whose vertex set and edge set are given in the above structure. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of the graph in such a way that $f(u) \neq f(v)$ for any two adjacent vertices u, v and the procedure for labeling the vertices is explained in the following algorithm.

Procedure: Vertex labeling of M- join of twig graph, $M \geq 1$.

Input: Vertices of M- join of twig graph,

$V = \left\{\{v_i, v_i^{(j)} / 0 \leq i \leq n, 1 \leq j \leq M\} \cup \{u_i, w_i, u_i^{(j)}, w_i^{(j)} / 1 \leq i \leq n-1, 1 \leq j \leq M\}\right\}$

$v_0 \leftarrow 0$;

for $i = 1\ to\ n$ do

for j = 1 to M do

$v_0^{(j)} \leftarrow 0$

$$v_i = v_i^{(j)} \leftarrow \begin{cases} 0\ if\ i \equiv 0(mod4) \\ 1\ if\ i \equiv 1(mod4) \\ 15\ if\ i \equiv 2(mod4) \\ 14\ if\ i \equiv 3(mod4) \end{cases}$$

end for

end for

for $i = 1\ to\ n-1$ do

for j = 1 to M do

$$u_i = u_i^{(j)} \leftarrow \begin{cases} 23\ if\ i \equiv 0(mod4) \\ 22\ if\ i \equiv 1(mod4) \\ 18\ if\ i \equiv 2(mod4) \\ 19\ if\ i \equiv 3(mod4) \end{cases};\quad w_i = w_i^{(j)} \leftarrow \begin{cases} 10\ if\ i \equiv 0(mod4) \\ 11\ if\ i \equiv 1(mod4) \\ 5\ if\ i \equiv 2(mod4) \\ 4\ if\ i \equiv 3(mod4) \end{cases}$$

end for

end for

end procedure

Output: The labeled vertices of M- join of twig graph

Define a function $g: f(V) \rightarrow [GC(f(v))]_d$ where $g(f(u)) = [GC(f(u))]_d$ for every $u \in V$ such that $g(f(u_1)) \neq g(f(u_2))$ for every adjacent vertices $u_1, u_2 \in V$.

For $0 \leq i \leq n, 1 \leq j \leq M$

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$$g(f(v_i)) = g\left(f\left(v_i^{(j)}\right)\right) \leftarrow \begin{cases} 0 \ if \ i \equiv 0(mod4) \\ 1 \ if \ i \equiv 1(mod4) \\ 8 \ if \ i \equiv 2(mod4) \\ 9 \ if \ i \equiv 3(mod4) \end{cases};$$

For $1 \le i \le n-1, 1 \le j \le M$

$$g(f(u_i)) = g\left(f\left(u_i^{(j)}\right)\right) \leftarrow \begin{cases} 28 \ if \ i \equiv 0(mod4) \\ 29 \ if \ i \equiv 1(mod4) \\ 27 \ if \ i \equiv 2(mod4) \\ 26 \ if \ i \equiv 3(mod4) \end{cases}$$

$$g(f(w_i)) = g\left(f\left(w_i^{(j)}\right)\right) \leftarrow \begin{cases} 15 \ if \ i \equiv 0(mod4) \\ 14 \ if \ i \equiv 1(mod4) \\ 7 \ if \ i \equiv 2(mod4) \\ 6 \ if \ i \equiv 3(mod4) \end{cases}$$

Clearly all the adjacent vertices receive distance gray code as labels. Now The induced edge labels are calculated as follows:

$g^*\left(v_n v_0^{(1)}\right) = 2;$

For $1 \le j \le M-1$ $g^*\left(v_n^{(j)} v_0^{(j+1)}\right) = 2$

For $0 \le i \le n-1, 1 \le j \le M$

$$g^*(v_i v_{i+1}) \leftarrow \begin{cases} 1 \ if \ i \equiv 0(mod\ 2) \\ 2 \ if \ i \equiv 1(mod\ 2) \end{cases}; \ g^*(v_i^{(j)} v_{i+1}^{(j)}) \leftarrow \begin{cases} 1 \ if \ i \equiv 0(mod\ 2) \\ 2 \ if \ i \equiv 1(mod\ 2) \end{cases}$$

For $1 \le i \le n-1$

$g^*(v_i u_i) = 3;\ g^*(v_i w_i) = 4;$

For $1 \le i \le n-1, 1 \le j \le M$

$g^*\left(v_i^{(j)} u_i^{(j)}\right) = 3;\ g^*\left(v_i^{(j)} w_i^{(j)}\right) = 4;$

From all the above cases, for all the adjacent edges $e_1, e_2 \epsilon E, g^*(e_1) \neq g^*(e_2)$. Hence the M- join of twig graph admits gray code hamming distance labeling and the gray code hamming distance number is $\eta_{gchd}(G) = 4$.

Theorem 2.7. The M- join of twig graph $TW(P_n)$ is a gray code hamming distance labeled graph for $n \equiv 0(mod4)$and the gray code hamming distance number is $\eta_{gchd}(G) = 4$.

Theorem 2.8. The M- join of twig graph $TW(P_n)$is a gray code hamming distance labeled graph for $n \equiv 1(mod4)$and the gray code hamming distance number is $\eta_{gchd}(G) = 4$.

Theorem 2.9. The M- join of twig graph $TW(P_n)$is a gray code hamming distance labeled graph for $n \equiv 2(mod4)$an{d the gray code hamming distance number is $\eta_{gchd}(G) = 4$.

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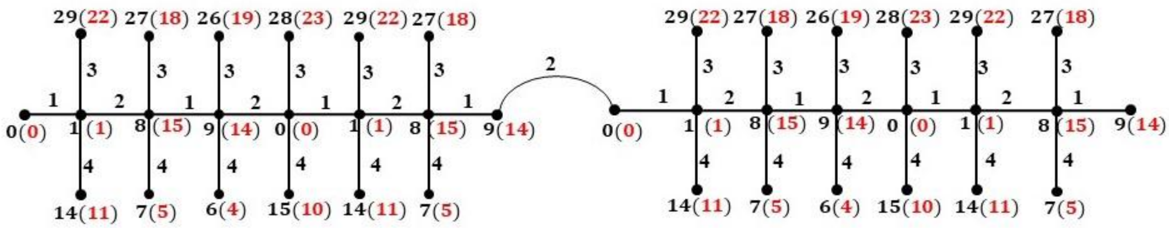

Figure 5: Gray Code labeling of 1-join of Twig Graph

3. CONCLUSION

In this paper, we have introduced gray code hamming distance labeling and proved the existence of gray code hamming distance labeling of M- join of H-graph, comb graph and twig graph and their gray code hamming distance numbers were obtained as 4.

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