

MINIMUM DEGREE ENERGY OF GLOBE GRAPH, BISTAR GRAPH AND SQUARE OF BISTAR GRAPH

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Abstract

In this article, our attention is restricted to finite undirected simple graphs. We establish the result that the minimum degree energy of the bistar graph $(B_{n,n})$ is $2\sqrt{n^2 + 6n + 1}$. Additionally, we examine the minimum degree energy of both the square of the bistar graph and the globe graph.

Keywords: Graph Energy, Minimum Degree Energy, Minimum Degree Matrix Minimum Degree Eigen Value.

1. INTRODUCTION

Graph energy, first defined by Gutman, has become an essential branch of spectral graph theory because of its strong relation with linear algebra, combinatorics, and mathematical chemistry. Graph energy is simply the sum of the absolute values of the eigenvalues of the graph adjacency matrix. Among many other branches in spectral graph theory, the study of graph energy has gained much attention because of its interesting features and applications[1]. The term graph energy arose initially in theoretical chemistry; however, today graph energy is studied as an independent area where numerical descriptors of graphs are given using eigenvalues of matrices constructed through graphs. During the last few decades, many different kinds of graph energy have been studied using adjacency, Laplacian, signless Laplacian, distance, and many other matrices of graphs. Such studies have not only helped connect algebra with combinatorics but have also contributed greatly towards distinguishing graph structure and their inherent behaviour [2].

Special families of graphs have always been important in this context as the special nature of these classes allows their spectra to be analytically calculated. Bistar graphs form one such class that has been receiving considerable importance in the recent past. A bistar graph can be defined as a structure formed by connecting two stars at their central nodes resulting in a symmetrical bifurcating graph with a combination of simple and non-trivial spectrum. Due to their hierarchal construction and balanced topologies, bistar graphs offer an ideal medium for the study of eigenvalues of matrices and corresponding energies. While many aspects of bistar graphs have been studied in the literature, their minimum degree energies have received considerably less attention[3], [4], [5], [6].

Another natural extension arises from graph operations such as graph squaring. The square of a graph is obtained by connecting every pair of vertices whose distance in the original graph is at most two. This operation substantially changes adjacency relationships and increases connectivity while preserving the underlying vertex set. As a result, the spectral characteristics of the graph also undergo significant modification. For a bistar graph, taking its square transforms many previously non-adjacent vertices into adjacent ones, producing a richer combinatorial structure. Investigating the minimum degree energy of the square of a bistar graph therefore provides insight into how graph operations influence spectral invariants and reveals patterns that are not visible in the original graph [7].

Another prominent graph family examined in this paper is called a globe graph. Globe graphs have an inherently symmetric structure created by connecting two special vertices using several internally vertex-disjoint paths of equal length[8], [9]. This symmetric structure and repetitive patterns enable the use of the globe graph as a testbed for exact spectral calculations. The symmetric and repetitive properties of the globe graph mean that algebraic methods can be applied successfully, which makes it particularly promising in terms of calculating characteristic polynomials, eigenvalues, and graph energy. The study of minimum degree energy of this graph class, however, has been relatively underexplored until now. Calculating minimum degree energy for these graph classes is not simply a matter of applying standard matrix calculus methods. Exact formulae can help gain insight into the connections between graph topologies and eigenvalues, thus furthering our understanding of spectral graph invariants. Results may also be used to test conjectures and computer algorithms, as well as to compare various definitions of energy. Moreover, solving such problems helps develop techniques that can later be generalized to apply to other graph families[3], [10], [11], [12]

The present study focuses on three closely related problems. First, the minimum degree energy of the bistar graph is determined through an analysis of its minimum degree matrix and corresponding eigenvalues[13]. Secondly, the investigation is extended to the square of the bistar graph, where the altered connectivity introduces new spectral characteristics requiring separate treatment. Finally, the minimum degree energy of the globe graph is established by constructing the associated minimum degree matrix, deriving its characteristic polynomial, and evaluating its eigenvalue spectrum. In each case, emphasis is placed on obtaining exact analytical expressions rather than relying solely on numerical computation[6], [14].

Overall, the findings presented in this paper enrich the theory of graph energies and provide new results concerning the minimum degree matrix of important special graphs. It is expected that these developments will stimulate further investigations into minimum degree energy for other graph operations and network families, thereby strengthening the role of spectral techniques in modern graph theory and its applications.

The minimum degree matrix $M(G)$ of a simple, undirected graph G is a square $n \times n$ matrix (where n is the number of vertices).

It is formally defined as

$$M_{ij} = \begin{cases} \min(d_i, d_j), & \text{if } v_i v_j \in E(G); \\ 0, & \text{otherwise.} \end{cases}$$

The minimum degree eigenvalues are obtained from the matrix $M(G)$. This paper focuses exclusively on finite simple undirected graphs, and the following section recalls several fundamental definitions from graph theory [3].

Definition 1.1:[13] The minimum degree energy, written as $E_{min}(G)$, is defined as the total of the absolute values of the eigenvalues derived from the minimum degree matrix of the graph.

$$E_{min}(G) = \sum_{i=1}^n |\lambda_i|,$$

Where $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ are the eigenvalues of the minimum degree matrix $Min(G)$ associated with the graph.

Definition 1.2: [15] A Graph G is termed bipartite if its vertex set V can be separated into two non-overlapping sets V_1 and V_2 such that $V_1 \cap V_2 = \emptyset$, and every edge joins a vertex in V_1 with a vertex in V_2 .

Definition 1.2: [15] A bipartite graph is called complete when each vertex in one partition is adjacent to all vertices in the other partition. Specifically, if $|V_1| = m$ and $|V_2| = n$, the resulting complete bipartite graph is represented by $K_{m,n}$.

Definition 1.4: [15] The graph $k_{1,n}$ represents a star graph, which is a special instance of a complete bipartite graph.

Definition 1.5:[16] The square of a simple connected graph G , represented by G^2 , is defined as the graph with the identical vertex set as G , where an edge exists between two vertices if their shortest-path distance G in is at most 2.

Definition 1.6: [17] The globe graph $G/(n)$ is defined as the graph obtained by joining two isolated vertices with n internally disjoint paths, where every path has length two.

References to sections (as well as figures, tables, theorems and so on), should be capitalized, as in "In Section 4, we show that..."

2. MAIN RESULT

Theorem 2.1: Let $n \in \mathbb{N}, n \neq 1$. Then $E_{min}(B_{n,n}) = 2\sqrt{n^2 + 6n + 1}$, where $E_{Min}(B_{n,n})$ is the Minimum Degree of Bistar graph $B_{n,n}$.

Proof: Let $V(B_{n,n}) = \{v_1, v_2, \dots, v_n, v, u, u_1, u_2, \dots, u_n\}$. Note that The Bistar graph $(B_{n,n})$ consists of $(2n + 2)$ vertices and $(2n + 1)$ edges. Its structure is illustrated in the figure 1 given below.

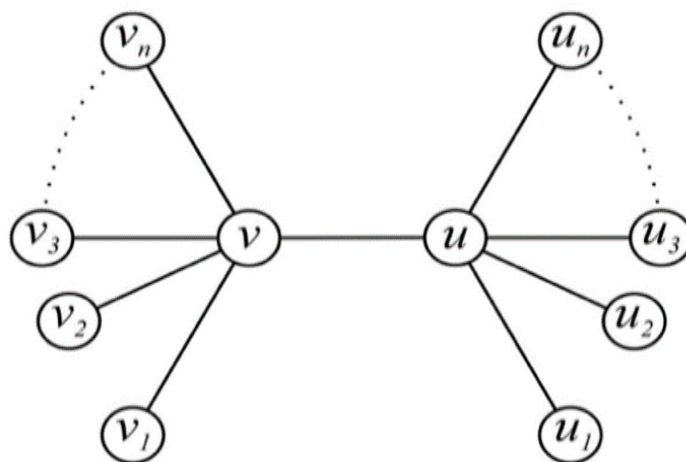


Figure 1: Bistar Graph of $B_{n,n}$

The following matrix defines the minimum degree matrix of the Bistar graph $[B_{n,n}]$

$$\text{Min}(B_{n,n}) = \begin{matrix} & v_1 & v_2 & \dots & v_n & c_1 & c_2 & u_1 & \dots & u_n \\ \begin{matrix} v_1 \\ v_2 \\ \vdots \\ v_n \\ c_1 \\ c_2 \\ u_1 \\ \vdots \\ u_n \end{matrix} & \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & \dots & 1 & 0 & n+1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & n+1 & 0 & 1 & \dots & 1 \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 & 1 & 0 & \ddots & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 \end{bmatrix} \end{matrix}$$

Note that the characteristic polynomial of matrix $\text{Min}(B_{n,n})$ is $\lambda^{2(n+1)} - (n^2 + 4n + 1)\lambda^{2n} + 2\lambda^{2(n-1)}$.

So, eigen value of $\text{Min}(B_{n,n})$ are $0, 0, \dots, 0$ ($2(2n-1)$ times.), $\frac{\sqrt{n^2+6n+1}+n}{2}$, $\frac{-\sqrt{n^2+6n+1}+n}{2}$, $\frac{\sqrt{n^2+6n+1}-n}{2}$, and $\frac{-\sqrt{n^2+6n+1}-n}{2}$.

Hence, Minimum Degree energy of $(B_{n,n}) = 0 + \dots + 0$ (8 times) + $\left| \frac{2\sqrt{n^2+6n+1}+n}{2} \right| + \left| \frac{2\sqrt{n^2+6n+1}-n}{2} \right|$.

$$\begin{aligned} &= \sqrt{n^2 + 6n + 1} + n + \sqrt{n^2 + 6n + 1} - n \\ &= 2\sqrt{n^2 + 6n + 1} \end{aligned}$$

Example 2.2: Minimum Degree energy of Bistar Graph $B_{5,5} = 4\sqrt{14}$

Proof:

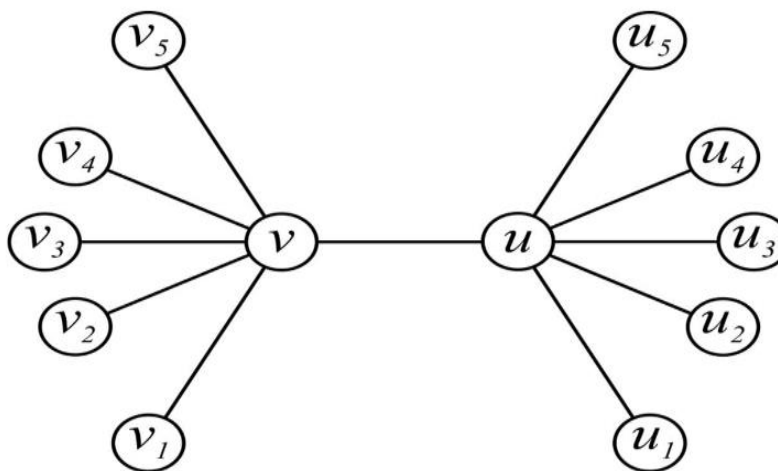


Figure 2: Bistar Graph of $B_{5,5}$

The Minimum Degree matrix of $B_{5,5}$ is given by

$$\text{Min}(B_{5,5}) = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v & u & u_1 & u_2 & u_3 & u_4 & u_5 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v \\ u \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Observe that the characteristic polynomial of matrix $\text{Min}(B_{5,5})$ is

$$\lambda^{12} - 46\lambda^{10} + 25\lambda^8$$

So, Minimum Degree eigen values of $B_{5,5}$ are $0,0,0,\dots,0$ (8 times), $\pm(\sqrt{14} + 3)$ and, $\pm(\sqrt{14} - 3)$

Or Minimum Degree eigen values of $B_{5,5}$ are $0,0,0,\dots,0$ (8 times),

Hence, Minimum Degree energy of $B_{5,5} = 0+0+\dots+0$ (8 times) + $|\sqrt{14} + 3| + |\sqrt{14} - 3| + |\sqrt{14} + 3| + |\sqrt{14} - 3|$.

Therefore, Minimum Degree energy of $B_{5,5} = E_{\text{Min}} = 4\sqrt{14}$

Theorem 2.3: Let $n \in N, n \neq 1$. Them $E_{min}(B_{n,n}^2) = (2n + 1) + \sqrt{4n^2 + 68n + 1}$, where $E_{Min}(B_{n,n}^2)$ is the Minimum Degree of Bistar graph $(B_{n,n}^2)$.

Proof: Let $V(B_{n,n}^2) = \{v_1, v_2, \dots, v_n, v, u_1, u_2, \dots, u_n\}$. Note that $B_{n,n}^2$ is the graph with $2n + 2$ and $4n + 1$ edges as shown in the following Figure 3

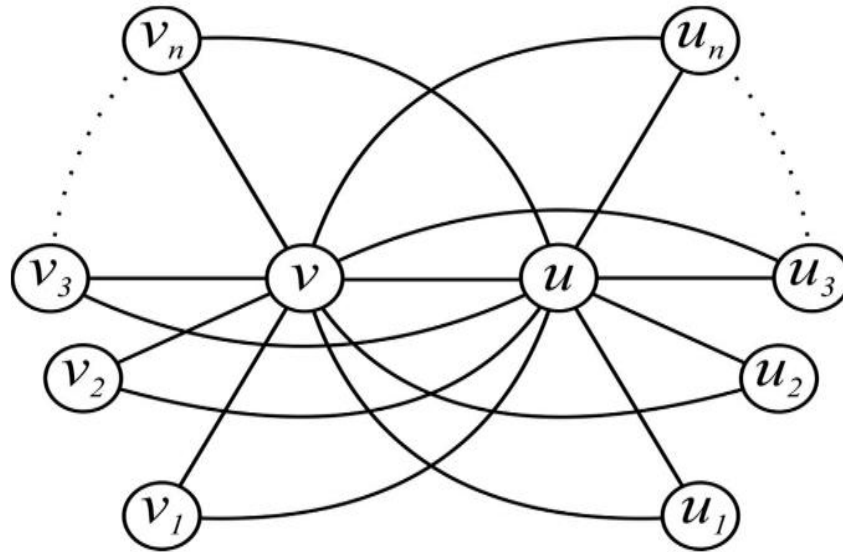


Figure 3: Square of Bistar Graph $B_{n,n}^2$

The following matrix defines the minimum degree matrix of the Square of Bistar graph $[B_{n,n}^2]$

$$Min(B_{n,n}^2) = \begin{bmatrix} 0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \\ 2 & \cdots & 2 & 0 & 2n+1 & 2 & \cdots & 2 \\ 2 & \cdots & 2 & 2n+1 & 0 & 2 & \cdots & 2 \\ 0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \end{bmatrix}$$

The characteristic polynomial derived from the $Min(B_{n,n}^2)$ is given by $\lambda^{2(n+1)} - (2n + 1)\lambda^{2n+1} - (4n + 1)\lambda^{2n} + 4(2n + 1)2\lambda^{2n-1}$

So, Minimum degree eigenvalue are $0, 0, \dots, 0$ ($(2n-1)$ times), $-(2n+1)$, $\frac{(2n+1)+\sqrt{(2n+1)^2+64n}}{2}$, $\frac{(2n+1)-\sqrt{(2n+1)^2+64n}}{2}$

Therefore, Minimum degree energy $B_{n,n}^2 = 0 + |-(2n+1)| + \left| \frac{(2n+1)+\sqrt{(2n+1)^2+64n}}{2} \right| + \left| \frac{(2n+1)-\sqrt{(2n+1)^2+64n}}{2} \right|$

$$\begin{aligned} &= (2n+1) + \frac{(2n+1) + \sqrt{(2n+1)^2 + 64n}}{2} + \frac{\sqrt{(2n+1)^2 + 64n} - (2n+1)}{2} \\ &= (2n+1) + \sqrt{(2n+1)^2 + 64n} \\ &= (2n+1) + \sqrt{4n^2 + 68n + 1} \end{aligned}$$

Examples 2.4: Minimum Degree energy of square of Bister graph $(B_{5,5}^2) = 32$

Proof:

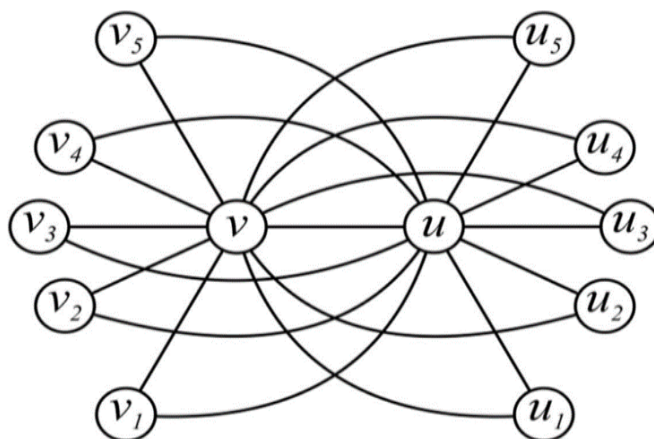


Figure 4: Square of Bister Graph of $B_{5,5}^2$

The Minimum Degree matrix of $B_{5,5}^2$ is given by

$$\text{Min}(B_{5,5}) = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v & u & u_1 & u_2 & u_3 & u_4 & u_5 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v \\ u \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 0 & 11 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 11 & 0 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Note that the characteristic polynomial of $Min_{\delta}(B_{5,5}^2)$ is $\lambda^{12} - 352\lambda^{10} - 880\lambda^9$

So, the minimum degree eigenvalues of $Min_{\delta}(B_{5,5}^2)$ are 0, 0,.....,0 (9 times), 16, -5, -11

Hence, Minimum Degree energy of $B_{5,5}^2 = E_{min}(B_{5,5}^2) = 0 + |16| + |-5| + |-11| = 32$.

Theorem 2.5: Let $n \in \mathbb{N}, n \neq 1$. Then $E_{min}(Gl(n)) = 4\sqrt{2n}$, where $E_{min}(Gl(n))$ is the Minimum Degree of Globe graph $Gl(n)$.

Proof: Let the vertex set of $Gl(n)$ be $V(Gl(n)) = \{v_1, v_2, \dots, v_n, u\}$.

Then $Gl(n)$ is a graph comprising $n + 2$ vertices and $2n$ edges, as illustrated in Figure 5.

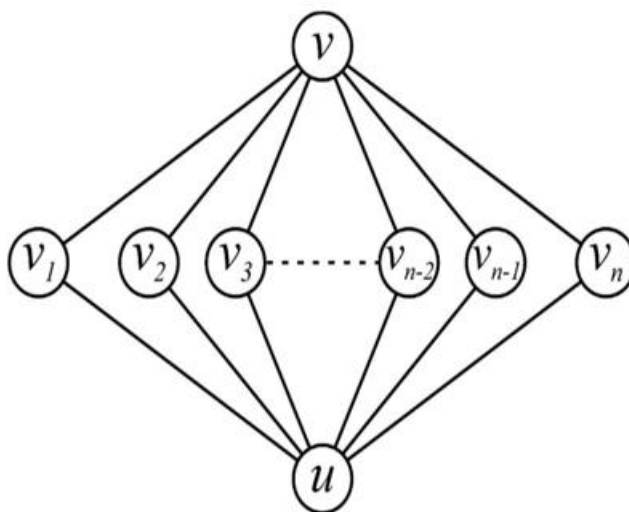


Figure 5: Globe Graph of $Gl(n)$

The following matrix defines the minimum degree matrix of the Globe graph $[Gl(n)]$.

$$Min(Gl(n)) = \begin{matrix} & v & v_1 & \dots & v_n & u \\ \begin{matrix} v \\ v_1 \\ \vdots \\ v_n \\ u \end{matrix} & \begin{bmatrix} 0 & 2 & \dots & 2 & 0 \\ 2 & 0 & \ddots & 0 & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 0 & \ddots & 0 & 2 \\ 0 & 2 & \dots & 2 & 0 \end{bmatrix} \end{matrix}$$

The characteristic polynomial derived from the matrix $min(Gl(n))$ is given by $\lambda^{n+2} - 8n\lambda^n$.

So, Minimum Degree eigenvalues of $Gl(n)$ are 0, 0,, 0 (n times), $2\sqrt{2n}$ and $-2\sqrt{2n}$

Hence, Minimum Degree energy of $Gl(n) = E_{min}(Gl(n)) = 0 + |2\sqrt{2n}| + |-2\sqrt{2n}|$

$$= 2\sqrt{2n} + 2\sqrt{2n} = 4\sqrt{2n}$$

Example 2.6: Minimum Degree energy of Globe graph $Gl(5) = 4\sqrt{10}$

Proof:

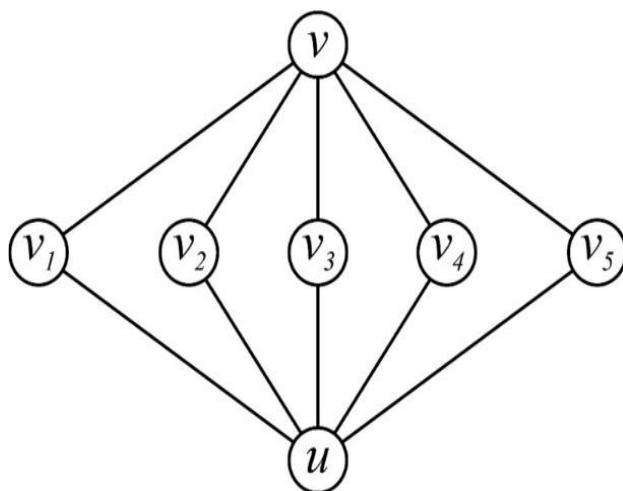


Figure 6: Globe Graph $G(5)$

The Minimum Degree matrix of $Gl(5)$ of $Gl(n)$ is given by

$$\text{Min}(Gl(5)) = \begin{matrix} & \begin{matrix} v & v_1 & v_2 & v_3 & v_4 & v_5 & u \end{matrix} \\ \begin{matrix} v \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ u \end{matrix} & \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \end{matrix}$$

The characteristic polynomial derived from the matrix $\text{min}(Gl(5))$ is given by $\lambda^7 - 40\lambda^5$.

So, Minimum Degree eigenvalues of $Gl(5)$ are $0, 0, 0, 0, 0, 2\sqrt{10}, -2\sqrt{10}$

Hence, Minimum Degree energy of $Gl(5) = 0 + |2\sqrt{10}| + |-2\sqrt{10}|$

Therefore, Minimum Degree energy of $Gl(5) = E_{\min}(Gl(5)) = 4\sqrt{10}$

3. CONCLUSION

This paper investigates the minimum degree energy associated with several important graph families by employing spectral techniques based on the minimum degree matrix. Exact constructions of the corresponding matrices, characteristic polynomials, and eigenvalue distributions have been obtained, leading to explicit expressions for the minimum degree energies of the Bistar graph, the square of the Bistar graph, and the Globe graph.

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