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EP MATRICES OVER INTERVAL INCLINE MATRICES

P. RAMASAMY

Research Scholar, Department of Mathematics, Rajah Serfoji Government College, Thanjavur, (Affiliated to Bharathidasan University), Tiruchirappalli, Tamil Nadu, India. Email: Proframs76@gmail.com

S. ANBALAGAN

Assistant Professor, Department of Mathematics, Rajah Serfoji Government College, Thanjavur, (Affiliated to Bharathidasan University), Tiruchirappalli, Tamil Nadu, India. Email: sms.anbu18@gmail.com

Abstract

Inclines are additively idempotent semirings in which products are less than or equal to either factor. The characterization of EP elements, Product of EP elements in an incline with involution are obtained as a generalization and development with EP matrices over an Incline and transpose in p^*-regular ring and EP elements in a reflexive Semigroups.

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Keywords: Incline, Semiring, EP Element, EP Matrices.

1. INTRODUCTION

The notion of inclines and their applications are described comprehensively by Cao, Kim and Roush [1]. In [4] Kim and Roush have surveyed and out lined algebraic properties of incline and matrices over incline.

Inclines are generalization of Boolean Algebra, Fuzzy Algebra, distributive lattice and a Special type of semiring. An interval incline matrix is a Structure which has an associative, Commutative addition and distributive multiplication Such that

$$[\alpha_L, \alpha_U]+[\alpha_L, \alpha_U] = [\alpha_L, \alpha_U]$$

$$[\alpha_L, \alpha_U]+[\alpha_L \beta_L, \alpha_U \beta_U] = [\alpha_L, \alpha_U] \text{ and}$$

$[\alpha_L, \alpha_U]+[\alpha_L \beta_L, \alpha_U \beta_U] = [\beta_L, \beta_U]$ for all $[\alpha_L, \alpha_U]$, $[\beta_L, \beta_U] \in L$ has generalization of EP interval elements to P^* regular ring extended this concept to E^kP interval elements for $k \geq 1$.

In [3] we have studied "the structure of EP elements in an incline with involution -T and Characterization of EP elements. In this paper, the concept of EP matrices over an inclines are discussed as a generalization of the results available in the literature [6].

The structure of Interval of EP riatrices over an incline with transpose -T and characterization of EP matrices are obtained. we have determined of EP matrices include in them the wide classes of row matrices and Column matrices.

2. PRELIMINARIES

The basic definitions and results are show in this section.

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Definition 2.1

An interval incline is a nonempty set L with binary operations + and · defined on L x L Such that for all α, β, γ ∈ L. we usually suppress the dot · of α·β and write as αβ.

(i) $[\alpha_L+\beta_L, \alpha_U+\beta_U] = [\beta_L+\alpha_L, \beta_U+\alpha_U]$

(ii) $[\alpha_L, \alpha_U] +([\beta_L, \beta_U]+ [\gamma_L, \gamma_U]) = ([\alpha_L, \alpha_U] +[\beta_L, \beta_U])+ [\gamma_L, \gamma_U]$

(iii) $[\alpha_L, \alpha_U] ([\beta_L, \beta_U]+ [\gamma_L, \gamma_U]) = [\alpha_L, \alpha_U] [\beta_L, \beta_U]+ [\alpha_L, \alpha_U] [\gamma_L, \gamma_U]$

(iv) $([\beta_L, \beta_U]+ [\gamma_L, \gamma_U]) [\alpha_L, \alpha_U] = [\beta_L, \beta_U] [\alpha_L, \alpha_U] + [\gamma_L, \gamma_U] [\alpha_L, \alpha_U]$

(v) $[\alpha_L, \alpha_U] ([\beta_L, \beta_U] \cdot [\gamma_L, \gamma_U]) = ([\alpha_L, \alpha_U] [\beta_L, \beta_U]) \cdot [\gamma_L, \gamma_U]$

(vi) $[\alpha_L, \alpha_U]+[\alpha_L, \alpha_U] = [\alpha_L, \alpha_U]$

(vii) $[\alpha_L, \alpha_U]+[\alpha_L \beta_L, \alpha_U \beta_U] = [\alpha_L, \alpha_U]$

(viii) $[\alpha_L, \alpha_U]+[\alpha_L \beta_L, \alpha_U \beta_U] = [\beta_L, \beta_U]$

An interval incline is said to be Commutative if $[\alpha_L \beta_L, \alpha_U \beta_U] = [\beta_L \alpha_L, \beta_U \alpha_U]$ for all $[\alpha_L, \alpha_U]$, $[\beta_L, \beta_U]\in L$

An incline (α, +, ·) with order relation ≤ defined on L.

$[\alpha_L, \alpha_U] \leq [\beta_L, \beta_U]$ (or) $\alpha_L \leq \beta_L$ and $\alpha_U \leq \beta_U$ if and only if $[\alpha_L, \alpha_U]+[\beta_L, \beta_U] = [\beta_L, \beta_U]$.

If $[\alpha_L, \alpha_U] \leq [\beta_L, \beta_U]$ then $[\beta_L, \beta_U]$ is said to dominate $[\alpha_L, \alpha_U]$

such that for $[\alpha_L, \alpha_U]$, $[\beta_L, \beta_U]\in L$.

by the incline axioms

$[\alpha_L, \alpha_U]+[\alpha_L \beta_L, \alpha_U \beta_U] = [\alpha_L, \alpha_U]$ and $[\alpha_L, \alpha_U]+[\alpha_L \beta_L, \alpha_U \beta_U] = [\beta_L, \beta_U]$

we get $[\alpha_L \beta_L, \alpha_U \beta_U] \leq [\alpha_L, \alpha_U]$ and $[\alpha_L \beta_L, \alpha_U \beta_U] \leq [\beta_L, \beta_U]$

Inclines are additively idempotent semirings in which products are less than or equal to either factor. The following characterization of this interval incline order Connection are from (P1) and (P2).

Properties:

(P1): $[\alpha_L +\beta_L, \alpha_U +\beta_U] \geq [\alpha_L, \alpha_U]$ and $[\alpha_L + \beta_L, \alpha_U + \beta_U] \geq [\beta_L, \beta_U]$

(P2): $[\alpha_L\beta_L, \alpha_U\beta_U] \leq [\alpha_L, \alpha_U]$ and $[\alpha_L \beta_L, \alpha_U\beta_U] \leq [\beta_L, \beta_U]$ for $[\alpha_L, \alpha_U]$, $[\beta_L, \beta_U]\in L$.

Definition 2.2

An incline (L, +, ·) using the binary operations + and · it satisfy the following conditions.

(i) (L, +) is a semilattice.

(ii) (L, ·) is a Semi group

(iii) α(β+γ) = αβ+αγ for all α,β,γ∈L.

(iv) α+αβ = α and α+αβ = β for all α,β∈L.

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Definition 2.3

Let L denotes an interval incline with (+, ·) operations and order relation ≤ defined by $[\alpha_L, \alpha_U]+[\beta_L, \beta_U] = [\alpha_L, \alpha_U] \Leftrightarrow [\beta_L, \beta_U] \leq [\alpha_L, \alpha_U]$ (or) $\beta_L \leq \alpha_L$ and $\beta_U \leq \alpha_U$. In this definition, EP elements in an interval incline is introduced as a generalization of symmetric clernents and Characterization of EP elements in an incline with involution -T are determined.

Definition 2.4

$p \in L$ is said to be regular if there is an element $r \in L$, Such that prp = p, then r is called a generalized inverse, in short g-inverse (or 1-inverse) of 'p' and is denoted as p^-. let p{1} denotes the set of all 1-inverse of 'p'. L is regular if every element of L is regular.

Definition 2.5

[3] An element *p* in an incline L is said to be EP if $pL = Lp$.

Definition 2.6

An interval incline matrices of order *m*x*n* is defined as $P(=[P_L, P_U]) = [p_{ijL}, p_{ijU}]$ where $p_{ij} = [p_{ijL}, p_{ijU}]$ is ijth element of P interval incline matrix containing all the elements as intervals P_L is the lower matrix of P and P_U is the upper matrix of P.

The following definition deals with the basic operations on IIM.

Definition 2.7

(i) For $[P_L, P_U]$ and $[Q_L, Q_U]$ ™ $L^I mxn$, the Sum is defined as

$[P_L, P_U] + [Q_L, Q_U] = [P_L+Q_L, P_U+Q_U] = [p_{ijL}+q_{ijL}, p_{ijU}+q_{ijU}]$.

(ii) For $[P_L, P_U] = [p_{ijL}, p_{ijU}]_{mxn}$ and $[Q_L, Q_U] = [q_{ijL}, q_{ijU}]_{nxp}$,

i.e., $[R_L, R_U] = [P_LQ_L, P_UQ_U]$ is defined as follows with order *m*x*p*.

$$[R_L, R_U] = [P_LQ_L, P_UQ_U] = \left[\sum_k p_{ikL}q_{jkL}, \sum_k p_{ikU}q_{jkU}\right]$$

$$= [r_{ikL}, r_{ikU}].$$

(iii) P™ L^I_n then the transpose of P is defined as

$P^T = [P^T_L, P^T_U] = [p_{ijL}{}^T, p_{ijU}{}^T] = [p_{ijL}, p_{ijU}]$.

(iv) For P,Q™$L^I mn$, $[P_L, P_U] \leq [Q_L, Q_U]$ iff $P_L \leq Q_L$ and $P_U \leq Q_U]$

iff $p_{ijL} \leq q_{ijL}$ and $p_{ijU} \leq q_{ijU}$, for i = 1, 2 ,. . . ,m and j = 1, 2, . . . ,n.

Remark 2.8

Each entries of P_L and P_U are same then the interval incline matrix coincides with incline matrix.

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Definition 2.9

The row (column) space [$\Re(P_L)$, $\Re(P_U)$], ([$C(P_L)$, $C(P_U)$]) an mxn interval matrix [P_L, P_U] be the subspace of V^n generated it by rows (columns).

that is,

[$\Re(P_L)$, $\Re(P_U)$]= the row space of [P_L, P_U] ={[b_L, b_U]={a_LP_L, a_UP_U}, [$C(P_L)$, $C(P_U)$]= the Column space of [P_L, P_U] ={[b_L, b_U]={$a_LP_L^T$, $a_UP_U^T$} for [a_L, a_U]∈L}

Lemma 2.10 [8]

let L is an interval incline matrices, for [P_L, P_U], [Q_L, Q_U]∈L_{mn}, we have satisfy the following:
(i) [$\Re(PQ)_L$, $\Re(PQ)_U$] ⊆[$\Re(P_L)Q_L$, $\Re(P_L)Q_U$]

⊆[$\Re(Q_L)$, $\Re(Q_U)$]

(ii) [$C(PQ)_L$, $C(PQ)_U$] ⊆[$C(P_L)$, $C(P_U)$] .

3. EP MATRICES OVER AN INTERVAL INCLINE MATRICES

Definition 3.1

An IIM P=[P_L, P_U] is said to be EP matrix if [$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^T)$, $\Re(P_U^T)$]= [$C(P_L)$, $C(P_U)$] means $\Re$ -mean row space & C- Column Space are equal.

Theorem 3.2

Let P=[P_L, P_U]∈L_m be an EP matrix. Then the following are equivalent.

(i) [$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^k)$, $\Re(P_U^k)$], for $k \geq 1$

(ii) [$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^T P_L)$, $\Re(P_U^T P_U)$], for $k \geq 1$

(iii) [$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^T P_L^k)$, $\Re(P_U^T P_U^k)$], for $k > 1$

(iv) [$C(P_L)$, $C(P_U)$] = [$C(P_L^k)^T$, $C(P_U^k)^T$], for $k > 1$

(v) [$C(P_L)$, $C(P_U)$] = [$C(P_L P_L^T)$, $C(P_U P_U^T)$],

(vi) [$C(P_L)$, $C(P_U)$] = [$C(P_L^k)^TP_L$, $C(P_U^k)^TP_U$], for $k > 1$

Proof (i) ⇒ (ii)

Let [$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^k)$, $\Re(P_U^k)$]

[$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^k)\Re(P_L^{k-1})$, $\Re(P_U^k)\Re(P_U^{k-1})$]⊆[$\Re(P_L^{k-2})$, $\Re(P_U^{k-2})$]⊆……

……⊆[$\Re(P_L^2)$, $\Re(P_U^2)$][I_L, I_U] [$\Re(P_L)$, $\Re(P_U)$] (By Lemma 2.10)

⇒[$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^2)$, $\Re(P_U^2)$] = [$\Re(P_L)P_L$, $\Re(P_U)P_U$]

By definition (3.1), we have

Thus [$\Re(P_L)$, $\Re(P_U)$] = [$\Re(P_L^T)P_L$, $\Re(P_U^T)P_U$].

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(ii) ⇒ (iii), Let $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^T P_L), \Re(P_U{}^T P_U)]$.

$= [\Re(P_L{}^T)P_L, \Re(P_U{}^T)P_U]$

$= [\Re(P_L)P_L, \Re(P_U)P_U]$

$= [\Re(P_L{}^T P_L)P_L, \Re(P_U{}^T P_U)P_U]$

$= [\Re(P_L{}^T)P_L{}^2, \Re(P_U{}^T)P_U{}^2]$

Since $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^T)P_L, \Re(P_U{}^T)P_U]$

$= [\Re(P_L)P_L{}^2, \Re(P_U)P_U{}^2]$

$= [\Re(P_L{}^T P_L)P_L{}^2, \Re(P_U{}^T P_U)P_U{}^2]$

$= [\Re(P_L)P_L{}^3, \Re(P_U)P_U{}^3]$

= . . . .

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$= [\Re(P_L{}^T)P_L{}^k, \Re(P_U{}^T)P_U{}^k]$

Thus $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^T)P_L{}^k, \Re(P_U{}^T)P_U{}^k]$.

(iii) ⇒(i)

Let $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^T P_L{}^k), \Re(P_U{}^T P_U{}^k)]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^T P_L{}^k), \Re(P_U{}^T P_U{}^k)]$

$\subseteq [\Re(P_L{}^k), \Re(P_U{}^k)]$

$\subseteq [\Re(P_L{}^2), \Re(P_U{}^2)] \subseteq [\Re(P_L), \Re(P_U)]$ (By Lemma 2.10)

Thus $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^k), \Re(P_U{}^k)]$

Since $[\Re(P_L), \Re(P_U)]=[C(P_L), C(P_U)]=[\Re(P_L{}^k), \Re(P_U{}^k)]$,

The equivalence of (iv),(v) and (vi) can be proved In the similar manner and hence omitted.

Theorem 3.3

Let $P=[P_L, P_U] \in L_m$ be an EP matrix. Then the following are equivalent.

(i) $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^2), \Re(P_U{}^2)]$, $[C(P_L), C(P_U)] = [C(P_L{}^2), C(P_U{}^2)]$

(ii) $[\Re(P_L{}^T P_L), \Re(P_U{}^T P_U)]= [\Re(P_L), \Re(P_U)] =[\Re(P_L P_L{}^T), \Re(P_U P_U{}^T)]$

(iii) $[\Re(P_L{}^T P_L{}^2), \Re(P_U{}^T P_U{}^2)] =[\Re(P_L), \Re(P_U)] =[\Re(P_L P_L{}^T), \Re(P_U P_U{}^T)]$

(iv) $[\Re(P_L), \Re(P_U)] = [\Re(P_L{}^2), \Re(P_U{}^2)]= [\Re(P_L{}^2)^T, \Re(P_U{}^2)^T]$

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Proof (i) ⇒ (ii)

Let $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)]$

$[C(P_L), C(P_U)] = [C(P_L^2), C(P_U^2)]$

Since, $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)]$

$\Rightarrow[\Re(P_L^2)^T, \Re(P_U^2)^T] = [\Re(P_L)^T, \Re(P_U)^T] = [C(P_L^2), C(P_U^2)]$

Now, $[C(P_L^2), C(P_U^2)]= [C(P_L), C(P_U)]$

$=[C(P_L^T), C(P_U^T)]$

$=[C(P_L^2)^T, C(P_U^2)^T]$

$[\Re(P_L^2), \Re(P_U^2)]= [C(P_L), C(P_U)]$

$=[\Re(P_L), \Re(P_U)]$

$=[C(P_L^2)^T, C(P_U^2)^T]$

$\Rightarrow[\Re(P_L)P_L, \Re(P_U)P_U] =[\Re(P_L), \Re(P_U)]$

$=[C(P_L^2)^T, C(P_U^2)^T]$

$\Rightarrow[\Re(P_L^T)P_L, \Re(P_U^T)P_U]= [\Re(P_L), \Re(P_U)] = [C(P_L^T)P_L^T, C(P_U^T)P_U^T]$

$= [\Re(P_LP_L^T), \Re(P_UP_U^T)]$

Thus (ii) holds.

(ii) ⇒ (iii),

Let $[\Re(P_L^TP_L), \Re(P_U^TP_U)] = [\Re(P_L), \Re(P_U)]$

$= [\Re(P_LP_L^T), \Re(P_UP_U^T)]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_LP_L^T), \Re(P_UP_U^T)]$

$= [\Re(P_L^T)P_L, \Re(P_U^T)P_U]$

$= [\Re(P_L)P_L, \Re(P_U)P_U]$

$= [\Re(P_L^TP_L)P_L, \Re(P_U^TP_U)P_U]$

$= [\Re(P_L^T)P_L^2, \Re(P_U^T)P_U^2]$

$= [\Re(P_L)P_L^2, \Re(P_U)P_U^2]$

$= [\Re(P_LP_L^2), \Re(P_UP_U^2)]$

$= [\Re(P_L^TP_L^2), \Re(P_U^TP_U^2)]$

$= [\Re(P_L), \Re(P_U)]$

$= [\Re(P_LP_L^T), \Re(P_UP_U^T)]$

Thus (iii) holds.

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(iii) ⇒(iv)

Let $[\Re(P_L^T P_L^2), \Re(P_U^T P_U^2)] = [\Re(P_L), \Re(P_U)]$

$= [\Re(P_L P_L^T), \Re(P_U P_U^T)]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^T P_L^2), \Re(P_U^T P_U^2)]$

$\subseteq [\Re(P_L^2), \Re(P_U^2)]$

$\subseteq [\Re(P_L), \Re(P_U)]$

Thus $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)]$

Since $[\Re(P_L), \Re(P_U)] = [\Re(P_L P_L^T), \Re(P_U P_U^T)]$

$= [\Re(P_L^T)P_L^T, \Re(P_U^T)P_U^T]$

$= [\Re(P_L^T)^2, \Re(P_U^T)^2]$

$= [\Re(P_L^2)^T, \Re(P_U^2)^T]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^T)^2, \Re(P_U^T)^2] = [\Re(P_L^2)^T, \Re(P_U^2)^T]$

Thus (iv) holds.

(iv) ⇒(i)

Let $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)] = [\Re(P_L^2)^T, \Re(P_U^2)^T]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^2)^T, \Re(P_U^2)^T]$

$[\Re(P_L)^T, \Re(P_U)^T] = [C(P_L^2), C(P_U^2)]$

$[C(P_L), C(P_U)] = [C(P_L^2), C(P_U^2)]$ $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)]$

Thus (i) holds.

Corollary 3.4

Let $P=[P_L, P_U] \in L_m$ be the EP matrix. Then the following are equivalent.

(i) $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)] = [\Re(P_L^T), \Re(P_U^T)] = [\Re(P_L^T)^2, \Re(P_U^T)^2]$

(ii) $[\Re(P_L^k), \Re(P_U^k)]$ is EP;

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^k), \Re(P_U^k)] = [\Re(P_L^T), \Re(P_U^T)]$

$= [\Re(P_L^T)^k, \Re(P_U^T)^k] = [\Re(P_L^T)^{k+1}, \Re(P_U^T)^{k+1}]$

$= [\Re(P_L^{k+1}), \Re(P_U^{k+1})]$, $k \geq 1$

Proof (i) ⇒(ii)

Let $P=[P_L, P_U]$ is EP;

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^T), \Re(P_U^T)]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)] = [\Re(P_L)P_L, \Re(P_U)P_U]$

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$= [\Re(P_L^2)P_L, \Re(P_U^2)P_U]$

$= [\Re(P_LP_L^2), \Re(P_UP_U^2)]$

$= [\Re(P_L^k)P_L, \Re(P_U^k)P_U]$

$=[\Re(P_L^{k+1}), \Re(P_U^{k+1})]$, $k \geq 1$

$[\Re(P_L^T), \Re(P_U^T)] = [\Re(P_L^T)^2, \Re(P_U^T)^2]$

$= [\Re(P_L^T)P_L^T, \Re(P_U^T)P_U^T]$

$= [\Re(P_L^T)^k, \Re(P_U^T)^k]$

$= [\Re(P_L^T)^{k+1}, \Re(P_U^T)^{k+1}]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^k), \Re(P_U^k)]$

$= [\Re(P_L^T), \Re(P_U^T)]$

$= [\Re(P_L^T)^k, \Re(P_U^T)^k]$

$= [\Re(P_L^T)^{k+1}, \Re(P_U^T)^{k+1}]$

$= [\Re(P_L)^{k+1}, \Re(P_U)^{k+1}]$, $k \geq 1$

Thus p^k is EP, $k \geq 1$.

(ii) ⇒ (i) This equivalence holds for k=1.

Theorem 3.5

For $P=[P_L, P_U] \in L_m$; if $[\Re(P_L), \Re(P_U)] = [\Re(P_L^T), \Re(P_U^T)] = [\Re(P_L^2), \Re(P_U^2)]$ then

$[\Re(P_L P_L^TP_L), \Re(P_UP_U^TP_U)] = [\Re(P_L^T P_L^2), \Re(P_U^T P_U^2)]$

$= [\Re(P_L^k P_L^TP_L), \Re(P_U^kP_U^TP_U)]$

$= [\Re(P_L) P_L^k P_L^TP_L, \Re(P_U) P_U^kP_U^TP_U]$

Proof

Let $[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)] = [\Re(P_L^T), \Re(P_U^T)]$

$[\Re(P_L P_L^TP_L), \Re(P_UP_U^TP_U)] = [\Re(P_L)P_L^TP_L, \Re(P_U)P_U^TP_U]$

$=[\Re(P_L^2)P_L^TP_L, \Re(P_U^2)P_U^TP_U]$

$= [\Re(P_L)P_LP_L^TP_L, \Re(P_U)P_UP_U^TP_U]$

$= [\Re(P_L^T)P_LP_L^TP_L, \Re(P_U^T)P_UP_U^TP_U]$

$[\Re(P_L P_L^TP_L), \Re(P_UP_U^TP_U)]= [\Re(P_L^T P_L)^2, \Re(P_U^T P_U)^2]$

$=[\Re(P_L^TP_L P_L^TP_L), \Re(P_U^TP_UP_U^TP_U)]$

$=[\Re(P_L^2)P_L P_L^TP_L, \Re(P_U^2)P_UP_U^TP_U]$

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$=[\Re(P_L)P_L^2\ P_L^TP_L,\ \Re(P_U)P_U^2P_U^TP_U]$

$=[\Re(P_L^2)P_L^2\ P_L^TP_L,\ \Re(P_U^2)P_U^2P_U^TP_U]$

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$[\Re(P_L\ P_L^TP_L),\ \Re(P_UP_U^TP_U)] =[\Re(P_L^kP_L^TP_L),\ \Re(P_U^kP_U^TP_U)]$

$=[\Re(P_L)P_L^{k-1}\ P_L^TP_L,\ \Re(P_U)P_U^{k-1}P_U^TP_U]$

$=[\Re(P_L^2)P_L^{k-1}\ P_L^TP_L,\ \Re(P_U^2)P_U^{k-1}P_U^TP_U]$

$[\Re(P_L\ P_L^TP_L),\ \Re(P_UP_U^TP_U)] =[\Re(P_L)P_L^kP_L^TP_L,\ \Re(P_U)P_U^kP_U^TP_U]$

$=[\Re(P_L^T)P_L^kP_L^TP_L,\ \Re(P_U^T)P_U^kP_U^TP_U]$

$=[\Re(P_L)P_L^kP_L^TP_L,\ \Re(P_U)P_U^kP_U^TP_U]$

$=[\Re(P_L^2)P_L^{k-1}\ P_L^TP_L,\ \Re(P_U^2)P_U^{k-1}P_U^TP_U]$

$=[\Re(P_L)P_L^{k-1}\ P_L^TP_L,\ \Re(P_U)P_U^{k-1}P_U^TP_U]$

$=[\Re(P_L^2)P_L^{k-2}P_L^TP_L,\ \Re(P_U^2)P_U^{k-2}P_U^TP_U]$

$=[\Re(P_L)P_L^{k-2}P_L^TP_L,\ \Re(P_U)P_U^{k-2}P_U^TP_U]$

$=[\Re(P_L\ P_L^TP_L),\ \Re(P_UP_U^TP_U)]$

Thus $[\Re(P_L\ P_L^TP_L),\ \Re(P_UP_U^TP_U)]= [\Re(P_L^T\ P_L)^2,\ \Re(P_U^T\ P_U)^2]$

$=[\Re(P_L^kP_L^TP_L),\ \Re(P_U^kP_U^TP_U)]$

$=[\Re(P_L)P_L^kP_L^TP_L,\ \Re(P_U)P_U^kP_U^TP_U]$

Theorem 3.6

if $[P_L,\ P_U]$, $[P_L^TP_L\ P_L^TP_L,\ P_U^TP_UP_U^TP_U]$ are the EP matrices over an incline, then the following hold:

(i) $[\Re(P_L^T)^2\ P_L^2,\ \Re(P_U^T)^2\ P_U^2]= [\Re(P_L^T\ P_L)^2,\ \Re(P_U^T\ P_U)^2]$

(ii) $[\Re(P_LP_L^T)P_L^2,\ \Re(P_UP_U^T)\ P_U^2]= [\Re(P_L^2\ P_L^TP_L),\ \Re(P_U^2P_U^TP_U)]$

Proof

(i) Let $[\Re(P_L\ P_L^TP_L),\ \Re(P_UP_U^TP_U)] = [\Re(P_L^TP_L\ P_L^T),\ \Re(P_U^TP_UP_U^T)]$

$[\Re(P_L\ P_L^TP_L)\ P_L,\ \Re(P_UP_U^TP_U)\ P_U] = [\Re(P_L^TP_L\ P_L^T)\ P_L,\ \Re(P_U^TP_UP_U^T)\ P_U]$

$[\Re(P_L\ P_L^TP_L^2),\ \Re(P_UP_U^TP_U^2)] = [\Re(P_L^TP_L)(P_L^TP_L),\ \Re(P_U^TP_U)(P_U^TP_U)]$

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$[\mathfrak{R}(P_L)P_L{}^TP_L{}^2, \mathfrak{R}(P_U)P_U{}^TP_U{}^2] = [\mathfrak{R}(P_L{}^TP_L)^2, \mathfrak{R}(P_U{}^TP_U)^2]$

$[\mathfrak{R}(P_L{}^T)P_L{}^TP_L{}^2, \mathfrak{R}(P_U{}^T)P_U{}^TP_U{}^2] = [\mathfrak{R}(P_L{}^TP_L)^2, \mathfrak{R}(P_U{}^TP_U)^2]$

$[\mathfrak{R}(P_L{}^T)^2P_L{}^2, \mathfrak{R}(P_U{}^T)^2P_U{}^2] = [\mathfrak{R}(P_L{}^TP_L)^2, \mathfrak{R}(P_U{}^TP_U)^2]$

Thus (i) hold.

(ii) $[\mathfrak{R}(P_L P_L{}^TP_L), \mathfrak{R}(P_UP_U{}^TP_U)] = [\mathfrak{R}(P_L{}^TP_L P_L{}^T), \mathfrak{R}(P_U{}^TP_UP_U{}^T)]$

$[\mathfrak{R}(P_L P_L{}^TP_L) P_L, \mathfrak{R}(P_UP_U{}^TP_U) P_U] = [\mathfrak{R}(P_L{}^TP_L P_L{}^T) P_L, \mathfrak{R}(P_U{}^TP_UP_U{}^T) P_U]$

$[\mathfrak{R}(P_L P_L{}^T) P_L{}^2, \mathfrak{R}(P_UP_U{}^T) P_U{}^2] = [\mathfrak{R}(P_L{}^T)P_LP_L{}^TP_L, \mathfrak{R}(P_U{}^T)P_UP_U{}^TP_U]$

$[\mathfrak{R}(P_LP_L{}^TP_L{}^2), \mathfrak{R}(P_UP_U{}^TP_U{}^2)] = [\mathfrak{R}(P_L)P_LP_L{}^TP_L, \mathfrak{R}(P_U)P_UP_U{}^TP_U]$

$[\mathfrak{R}(P_LP_L{}^T) P_L{}^2, \mathfrak{R}(P_UP_U{}^T) P_U{}^2] = [\mathfrak{R}(P_L{}^2P_L{}^TP_L), \mathfrak{R}(P_U{}^2P_U{}^TP_U)]$

Thus (ii) holds.

Lemma 3.7

Let $[P_L, P_U]$, $[Q_L, Q_U] \in L_m$. if $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L{}^T), \mathfrak{R}(P_U{}^T)] = [\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]$

Then the following are equivalent.

(i) $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L{}^T P_L), \mathfrak{R}(P_U{}^T P_U)]$

(ii) $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_L P_L{}^2), \mathfrak{R}(Q_UP_U{}^2)]$

(iii) $[C(P_L), C(P_U)] = [C(Q_L P_L)^T, C(Q_UP_U)^T]$

(iv) $[C(Q_L{}^T), C(Q_U{}^T)] = [C(P_L P_L{}^T), C(P_U P_U{}^T)]$

Proof (i) ⇒(ii)

Let $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L{}^T P_L), \mathfrak{R}(P_U{}^T P_U)]$

Now, $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L{}^T P_L), \mathfrak{R}(P_U{}^T P_U)]$

$= [\mathfrak{R}(P_L{}^T)P_L, \mathfrak{R}(P_U{}^T)P_U]$

$= [\mathfrak{R}(P_L)P_L, \mathfrak{R}(P_U)P_U]$

$= [\mathfrak{R}(P_L{}^T P_L)P_L, \mathfrak{R}(P_U{}^T P_U)P_U]$

$= [\mathfrak{R}(P_L{}^T)P_L{}^2, \mathfrak{R}(P_U{}^T)P_U{}^2]$

$= [\mathfrak{R}(Q_L)P_L{}^2, \mathfrak{R}(Q_U)P_U{}^2]$

$[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_L)P_L{}^2, \mathfrak{R}(Q_U)P_U{}^2]$

Thus (ii) holds.

(ii) ⇒(iii)

Let $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_L P_L{}^2), \mathfrak{R}(Q_UP_U{}^2)]$

$[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_L P_L{}^2), \mathfrak{R}(Q_U,P_U{}^2)] \subseteq [\mathfrak{R}(P_L{}^2), \mathfrak{R}(P_U{}^2)]$

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$\subseteq[\Re(P_L), \Re(P_U)]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^2), \Re(P_U^2)] = [\Re(P_L)P_L, \Re(P_U)P_U]$

$[\Re(P_L^T), \Re(P_U^T)] = [\Re(Q_L P_L), \Re(Q_U P_U)]$

$[C(P_L), C(P_U)] = [C(Q_LP_L)^T, C(Q_UP_U)^T]$

Thus (iii) holds.

(iii) $\Rightarrow$(iv)

Let $[C(P_L), C(P_U)] = [C(Q_LP_L)^T, C(Q_UP_U)^T]$

$[\Re(Q_L), \Re(Q_U)] = [\Re(P_L^T), \Re(P_U^T)]$

$=[C(P_L), C(P_U)]$

$= [C(Q_LP_L)^T, C(Q_UP_U)^T]$

$= [\Re(Q_L P_L), \Re(Q_UP_U)]$

Therefore, $[\Re(Q_L), \Re(Q_U)] = [\Re(Q_LP_L), \Re(Q_UP_U)]$

$=[\Re(Q_L)P_L, \Re(Q_U)P_U]$

$[C(Q_L^T), C(Q_U^T)] = [\Re(P_L^T) P_L, \Re(P_U^T)P_U]$

$=[C(P_L^TP_L)^T, C(P_U^TP_U)^T]$

$[C(Q_L^T), C(Q_U^T)] = [C(P_L P_L^T), C(P_U P_U^T)]$

Thus (iv) holds.

(iv) $\Rightarrow$(i)

Let $[C(Q_L^T), C(Q_U^T)] = [C(P_L P_L^T), C(P_U P_U^T)]$

$=[C(P_LP_L^T), C(P_UP_U^T)]$

$=[\Re(Q_L), \Re(Q_U)]$

$=[\Re(P_L^TP_L), \Re(P_U^TP_U)]$

$[\Re(P_L), \Re(P_U)] = [\Re(P_L^T P_L), \Re(P_U^T P_U)]$

Thus (i) holds.

Theorem 3.8

Let $[P_L, P_U]$, $[Q_L, Q_U]$ are EP matrices, Then the following are equivalent."

(i) $[\Re(P_L), \Re(P_U)] = [\Re(Q_L P_L), \Re(Q_UP_U)]$ and

$[\Re(Q_L), \Re(Q_U)] = [\Re(P_LQ_L), \Re(P_UQ_U)]$

(ii) $[\Re(P_L), \Re(P_U)] = [\Re(Q_L^TP_L), \Re(Q_U^TP_U)]$ and

$[\Re(Q_L), \Re(Q_U)] = [\Re(P_L^TQ_L), \Re(P_U^TQ_U)]$

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(iii) $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^TQ_L P_L), \mathfrak{R}(P_U^TQ_UP_U)]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(Q_L^TP_LQ_L), \mathfrak{R}(Q_U^TP_UQ_U)]$

(iv) $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^T)(Q_LP_L)^k, \mathfrak{R}(P_U^T)(Q_UP_U)^k]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(Q_L^T)(P_LQ_L)^k, \mathfrak{R}(Q_U^T)(P_UQ_U)^k]$ for $k \geq 1$

(v) $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^T(Q_LP_L)^k), \mathfrak{R}(P_U^T(Q_UP_U)^k)]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(Q_L^T(P_LQ_L)^k), \mathfrak{R}(Q_U^T(P_UQ_U)^k)]$ for $k \geq 1$

(vi) $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_LP_L)^k, \mathfrak{R}(Q_UP_U)^k]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(P_LQ_L)^k, \mathfrak{R}(P_UQ_U)^k]$ for $k \geq 1$

Proof

Since $[P_L, P_U]$ $[Q_L, Q_U]$ are EP matrices

Let $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^T), \mathfrak{R}(P_U^T)]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)] = [\mathfrak{R}(Q_L^T), \mathfrak{R}(Q_U^T)]$

(i) ⇒(ii), this equivalence automatically holds.

(ii) ⇒(iii) Since, $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_L^TP_L), \mathfrak{R}(Q_U^TP_U)]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(P_L^TQ_L), \mathfrak{R}(P_U^TQ_U)]$

$[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_L^TP_L), \mathfrak{R}(Q_U^TP_U)]$

$= [\mathfrak{R}(Q_LP_L), \mathfrak{R}(Q_UP_U)]$

$= [\mathfrak{R}(P_L^TQ_L)P_L, \mathfrak{R}(P_U^TQ_U)P_U]$

and $[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(P_L^TQ_L), \mathfrak{R}(P_U^TQ_U)]$

$=[\mathfrak{R}(P_LQ_L), \mathfrak{R}(P_UQ_U)]$

$= [\mathfrak{R}(Q_L^TP_L)Q_L, \mathfrak{R}(Q_U^TP_U)Q_U]$

Thus (iii) holds.

(iii)⇒(iv)

Let $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^TQ_L P_L), \mathfrak{R}(P_U^TQ_UP_U)]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)]= [\mathfrak{R}(Q_L^TP_LQ_L), \mathfrak{R}(Q_U^TP_UQ_U)]$

$[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^T)Q_LP_L, \mathfrak{R}(P_U^T)Q_UP_U]$

$= [\mathfrak{R}(P_L)Q_LP_L, \mathfrak{R}(P_U)Q_UP_U]$

$= [\mathfrak{R}(P_L^T Q_LP_L)Q_LP_L, \mathfrak{R}(P_U^T Q_UP_U)Q_UP_U]$

$= [\mathfrak{R}(P_L^T)(Q_LP_L)^2, \mathfrak{R}(P_U^T) (Q_UP_U)^2]$

= . . . .

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$= [\mathfrak{R}(P_L^T)(Q_LP_L)^k, \mathfrak{R}(P_U^T)(Q_UP_U)^k]$

Similarly, $[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)] = [\mathfrak{R}(Q_L^T)(P_LQ_L)^k, \mathfrak{R}(Q_U^T)(P_UQ_U)^k]$

Thus (iv) holds.

(iv)⇒(v)

Let $[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^T)(Q_LP_L)^k, \mathfrak{R}(P_U^T)(Q_UP_U)^k]$ and

$[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)] = [\mathfrak{R}(Q_L^T)(P_LQ_L)^k, \mathfrak{R}(Q_U^T)(P_UQ_U)^k]$ for $k \geq 1$

$[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(P_L^T)(Q_LP_L)^k, \mathfrak{R}(P_U^T)(Q_UP_U)^k]$

$= [\mathfrak{R}(P_L)(Q_LP_L)^k, \mathfrak{R}(P_U)(Q_UP_U)^k]$

$\subseteq [\mathfrak{R}(Q_LP_L)^k, \mathfrak{R}(Q_UP_U)^k]$

$\subseteq [\mathfrak{R}(Q_LP_L), \mathfrak{R}(Q_UP_U)]$

$\subseteq [\mathfrak{R}(P_L), \mathfrak{R}(P_U)]$

$[\mathfrak{R}(P_L), \mathfrak{R}(P_U)] = [\mathfrak{R}(Q_LP_L)^k, \mathfrak{R}(Q_UP_U)^k]$

Similarly, $[\mathfrak{R}(Q_L), \mathfrak{R}(Q_U)] = [\mathfrak{R}(P_LQ_L)^k, \mathfrak{R}(P_UQ_U)^k]$

Thus (v) holds.

(v)⇒(i) this equivalence directly holds for k =1 in (v).

4. CONCLUSION

The main results in the present paper are the generalization of the available results in the [2], [3], [4] for the elemets in *p**- regular ring and for elements in a reflexive semigroup [4]. We have obtaine conditions under which the product of EP elements to be EP matrices which include the characterization of interval EP matrices in row space and column space.

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