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ON SOME WEAKER REGULAR AND NORMAL FORMS OF ALMOST TOPOLOGICAL SPACE UNDER THE CONTINUOUS MAPPING

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Abstract

In 1971, Čirić introduced the concept of nearly topological spaces, which minimizes topological constraints while keeping key properties, this study examines this generality. We examine why poorer nearly topological spaces affect continuity, convergence, compactness, and connectedness, we want to develop new mathematical frameworks for functional analysis, approximation theory, and fixed-point theory by exploring how axiom and open set definition modifications impact these structures, we found that weaker structures may broaden topological applications beyond mathematics, we also examine how weaker forms impact mappings and continuous transformations. New tools and approaches for interdisciplinary mathematics and related fields are the goal of this program, this study proves that continuous mappings to other spaces need topological space regularity and normalcy, it addresses how virtually topological spaces enhance classical topology and affect dynamical systems, algebraic topology, and functional analysis, studies on these areas enhance topology and related fields, we focus on the effects of weak topological forms in nearly topological spaces, particularly the function \(f \) and its importance in compact regular spaces, this study details reduced regularity and normalcy to understand their relations in essentially topological spaces, the findings enhance topological categorization, provide the framework for future research, and influence mathematical modelling and theoretical computer science by establishing structural stability under continuous mappings, notable contributions, virtual topological space theory advances, understanding the complex relationships between compactness, regularity, and normalcy.

Keywords: Topological Spaces, Weaker Regular Forms, Weaker Natural Forms, Continuous Mappings, Topological Properties.

1. INTRODUCTION AND BACKGROUND

Topology studies spatial features conserved following continuous transformations to explain continuity, convergence, compactness, and connectedness. Several domains have benefitted from classical topological space analysis, recent research has studied weaker topological space definitions to comprehend mathematical structures and applications, and this work maps weaker nearly topological spaces. We study these weaker structures' characteristics, topological links, and applications, Čirić presented the "nearly topological space" notion in 1971, improving on traditional topology, almost topological spaces reduce topology restrictions to define open sets more freely while keeping important properties, this generalization helps functional analysis, approximation theory, and fixed-point theory [1], this study examines weaker nearly topological spaces to better understand mappings and the space. Changing essentially topological space assumptions or limiting open sets and mappings may weaken structures, these modifications widen topological applications and provide new possibilities, we start with a comprehensive examination of nearly topological space rules and features. We emphasize that nearly topological spaces have fewer open sets and continuity constraints than classical ones. It prepares us for lesser forms, next, we examine weaker nearly

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topological spaces, we examine how axiom changes or limits influence weaker structures, we want to thoroughly study variations to identify new mathematical structures and their impacts, the study studies translations over smaller nearly topological domains, topology depends on mappings, therefore studying them in weaker structures may provide new insights and applications, we examine how modifications affect continuity definition and convergence behavior in weaker spaces, this study links convergence to reduced structural continuity, we examine how less nearly topological spaces affect compactness, connectedness, and separation, we study how applying these ideas to weaker structures affects them, this study shows how weaker structures work and their promise in many mathematical domains, this study may affect math fields, the mapping extends virtually topological spaces, allowing us to apply these notions to analysis, algebraic topology, mathematical physics, and more, this work may provide new mathematical tools and methodologies for numerous fields and increase research and application opportunities., to enhance topology, the article finishes by examining weaker forms of essentially topological spaces under mapping, we aim to study these structures to understand about continuous transformations and mappings' space interactions, weaker structures may disclose new mathematical frameworks and applications, expanding topology and promoting multidisciplinary collaboration, topology is mathematics' most creative field, comparable points apply to the group, weak topology is crucial for basic topological spaces over topological vector spaces or linear operator spaces like Hilbert space because of its mathematical importance, similar to a conventional vector space's basic structure, it relates to a topological vector space's continuous duality, weak topology allows researchers to identify portions of a weakly closed space closed (or compressed), functions that are constant about the weak topology called weakly continuous, let us ignore the broad issue and focus on new topology-designated properties, weak structures under f function mapping, the function with continuous inversion, the new continuous mapping concept, is invariant in all new topological spaces. In general topology, we approach continuous weak topology.

Many scientists have built continuous topological space forms using f, they made relevant generalizations, the functions they perform determined their topological families and fundamental descriptions, and later thoughts and concepts developed in this field. Some of the following scholars have extensively studied these families and their effects on weak topological forms under f functions, many functional space topologies are becoming critical, where each structure in space X for all functions put on space H in another space V relies on some idea of functions "nearly" such that $H \in X$ and $V \in Y$. Every topological family in an "almost" topological space has weak topology features, the function f expands "nearly topological space in functions." to give V a weak topology.

2. RESEARCH OBJECTIVES, SIGNIFICANCE, PROBLEMS, AND QUESTIONS

Certain topological space articles provide facts, ideas, generalizations, and definitions. Because it affects the semi-weak topological universe, we will study the f function in Compact Regular Space, Completely Compact Regular Space, Normal Regular Space, Normal Compact Regular Space, Continuous Compact Regular Space, and Mildly

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Normal Compact Regular, this study introduces and studies topological spaces, which generalize topological groups, academics and specialists claim these places have continuous mappings, this paper examines invariance in most topological spaces under f. Using f, this approach describes virtually topological spaces, topological spaces' structure and behavior under continuous mappings depend on regularity and normalcy. Even after significant investigation, tiny modifications of these features in virtual topological spaces remain unknown and uncategorized. Continuous mappings affect compactness and regularity in nearly topological spaces; thus, we must concentrate on them.

The questions addressed by our study:

1) How can a compact subset produce a space approximately compact regular?
2) Explain nearly compact regular regions.
3) What conditions make an extremely compact regular space virtually continuous with continuous mapping?
4) Can we identify normal and nearly normal compact regular spaces under subsets and coverings?
5) What further features make a nearly regular space relatively typical compact regular?

Define and analyze these lower forms of regular spaces to understand nearly topological spaces' structures and behavior under continuous mappings, this effort will classify and characterize these spaces to improve topology and applications, we consider both weak topologies while transferring the continuous function f to various topologies, show these spaces share continuous function f mapping properties, the thesis, "On Some Weaker Regular and Normal Forms of Almost Topological Space Under the Continuous Mapping," is important in topology, this study's main contributions and relevance:

Nearly Topological Space Advancement: This study defines and explores nearly compact regular, almost entirely compact regular, and almost continuous compact regular spaces, advancing their theory and providing new insights. The findings show complex relationships between regularity, compactness, and normalcy in virtually topological spaces. Understanding these linkages categorizes topological spaces and creates inheritance and expression criteria, the basis for Future Research: Results enable research of almost compact regular spaces under alternative continuous mappings and generalizations of almost entirely compact regular spaces. Topological space analysis may reveal new features or subclasses, mathematics, and Theoretical Applications: This study defines compactness and normality for nearly regular spaces, affecting mathematical modeling and theoretical computer science, these fields may rely on these spaces' structural stability and resilience under continuous mappings, topological Space Theory Contribution: This work discovers topological space hierarchies to the better categorization of regular spaces, particularly normal compact regular spaces, and slightly normal compact regular spaces, more Continuous Mapping Knowledge: This study

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explores virtual topological space regularity and normality using continuous mappings. Continuous mappings to other topological spaces emphasize their relevance and potential applications in dynamical systems, algebraic topology, and functional analysis. This study presents virtual topological spaces and continuous mappings, advancing topology and preparing topological space and application research.

3. TOPOLOGICAL SPACES, TOPOLOGICAL MAPPING SPACE, COMPACT SPACE, AND NORMAL SPACE

This paper will explore weak topological spaces under the continuous function of mapping literature. Start with fundamental observations and suggestions. Show these sites. We map all weak topological spaces under the continuous function f and examine their features. Few essential discoveries and ideas explain everything. We analyze outcomes using continuous function graphs. We study all weak topological spaces' features under function assignment f.

Topological maps are distance-function graphs. We use the typical topological map to tackle similar space problems.

1924 says every separable space contains open-closed subspaces, this trait is adequate for a measured ordinary space, thus, we can only separate the measure of an ordinary space if we can transfer it.

In 1934, J. Leray and J. Schauder defined the degree of mapping of entirely continuous motions in Banach space theory [2]. Schauder proposed a field invariance hypothesis for weakly compressed Banach spaces with a single fully continuous motion [31, 32].

This research exclusively investigates continuous mappings. Many mappings are "pleasant" from various angles: Classify mappings f:X → Y using the properties of single point counter-images, f−1y, y ∈ Y.

A mapping is metrisable if all f−1y spaces are metrisable. Compact metrizable mappings, f−1yare compacta, are examples.

A compact mapping exists if all f−1y are compact. A closed, compact mapping is great. An n-compact mapping has compact counter-image bounds, f−1y. Counter-images of S-mappings, f−1y, are spaces with countable bases, making them fascinating.

A mapping f:X → Y is m-mapping if every point y ∈ Y has a neighborhood y with f−1y in m This idea underpins modern dimension theory [30].

A mapping may have additional features, such as a closed continuous f:X → Y for compact X and Hausdorff Y, theorems of Z. Frolik are essential in this field:

1. A perfectly regular space X is paracompact and complete (in Cech's sense) if and only if it maps perfectly onto a whole metric space. The second Z. Frolik theorem is typeA.
2. Close f:X → Y, complete metric on X. Only Cech's whole space Y is metric [30], the Heine-Borel Theorem's closed and bounded interval enhances compactness [7].

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Definition 2.1: A compact topological space X has a finite subcover for every open cover. If K is a compact subset of X, its subspace topology is compact.

A subset K of X is compact if any open subset covering it has a finite subcover under the subspace topology.

Compact topological spaces under continuous mappings have compact images, a carryover from metric space. Compactness and sequential compactness are only equivalent in topological spaces with added structure, topologically, total boundedness and other compact metric space properties are not comparable [10].

Sets with finite intersections have nonempty intersections in every finite subcollection. Due to De Morgan's Identities, a subset of a topological space X is closed if and only if its counterpart in X is open [10].

Disjoint closed sets A and B in Hausdorff spaces result in open sets U and V with A ⊂ U and B ⊂ V. In conclusion, disjoint closed sets have open neighborhoods. Normal Hausdorff spaces have this. We include the Hausdorff condition in the definition of a normal space to close points, thus, open neighborhoods are discontinuous at different places. Disjoint subsets in R, such $\{\mathbf{0}\}$ and $(\mathbf{0}, \mathbf{1})$, may not have open neighborhoods, requiring closed A and B [60].

For example, the open sets of X are $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}\}$, $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$, and $\{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$, these sets create a topology on X that meets axiom (T0) but not (T1). X's closed sets are ∅, $\{\boldsymbol{e}\}$, $\{\boldsymbol{d}, \boldsymbol{e}\}$, $\{\boldsymbol{a}\}$, $\{\boldsymbol{a}, \boldsymbol{i}, \boldsymbol{e}\}$, $\{\boldsymbol{a}, \boldsymbol{c}\}$, $\{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{e}\}$, $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ Consider A and B disjoint closed, proper subsets of X. One of $\boldsymbol{A}$ and B must include $\boldsymbol{a}$ and the other $\boldsymbol{e}$ as the two closed sets lacking $\boldsymbol{a}$ are $\{\boldsymbol{d}, \boldsymbol{e}\}$ and $\{\boldsymbol{e}\}$. A is $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$, $\{\boldsymbol{a}, \boldsymbol{c}\}$, or $\{\boldsymbol{a}\}$, whereas B is $\{\boldsymbol{d}, \boldsymbol{e}\}$ or {Any option of $\boldsymbol{OA} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ and $\boldsymbol{OB} = \{\boldsymbol{d}, \boldsymbol{e}\}$ meets axi X is not a (T3) space since it is the only open set that includes the closed set $\{\boldsymbol{a}, \boldsymbol{d}, \boldsymbol{e}\}$, therefore, (T0) and (T4) do not imply (T3) [34].

4. REGULAR SPACE AND COMPLETELY NORMAL SPACE

For any closed set A and point $\boldsymbol{b} \notin$ A, a topological space X is a (T3) space if it possesses disjoint open sets $\boldsymbol{OA}$ and $\boldsymbol{Ob}$ such that A ⊆ $\boldsymbol{OA}$ and b ⊆ Two simple examples prove axiom (T3) is independent on T0, T1, and T2: Given X = $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$, its open sets are ∅, $\{\boldsymbol{a}\}$, $\{\boldsymbol{b}, \boldsymbol{c}\}$, and $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ These are X's only closed sets. X meets axiom (T3), but axiom (T0) fails because open sets cannot differentiate $\boldsymbol{b}$ and $\boldsymbol{c}$, thus, axiom (T3) does not imply T0 [34].

We can easily prove that each subspace Y of a (Ti) space X with $\boldsymbol{i}=\mathbf{0}$, $\mathbf{2}$, or $\mathbf{3}$ is likewise a Ti space. All Hausdorff and regular topological space subspaces are regular. Different for (T4) and normal spaces: Some normal topological space subspaces violate the (T4) axiom. Let X = Y ∪ ∞, where ∞ is not in Y. Y consists of pairwise ordered real numbers $(\boldsymbol{x1}, \boldsymbol{x2})$ satisfying $\boldsymbol{x2} \geq \mathbf{0}$. We provide X topology. Please clarify whether ∞ ∈ $\mathbf{0}$ and if $\mathbf{0} - \{\boldsymbol{a}$ is open. An open set $(\mathbf{0})$ has all pairings except a restricted number $(\boldsymbol{x1}, \mathbf{0})$. If ∞ ∉ $\mathbf{0}$, $\mathbf{0}$ is open if it ⊆ Y and is open in the same sense as, this topology makes X a Hausdorff

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space and valid axiom (T4), the subspace Y has the same topology, hence it is not normal [34].

Topological spaces with all subspaces (T4) are interesting. Urysohn first examined them and demonstrated that the (T5) axiom of separation characterizes them: A pair of separated sets A and B makes X a (T5) space, there exist disjoint open sets OA and OB with A and B ⊂ them [34], the T4 axiom (normalcy axiom) is essential to understanding topological spaces' structure and properties. If two disjoint closed subsets have disjointed open sets that include them, Schauder (1929) defines a topological space as T4. Division and management of subsets without overlap is possible since the space is separable. Schauder found that every subspace of T2 and T3 had Hausdorff and regular properties, but not T4. Schauder (1929) proved normal topologies may have non-T4 subspaces, this finding reveals how separation axiom levels affect subspaces in subtle ways. Schauder offered the example of a normal (T4) topological space X with a non-T4 subspace Y, though topological principles should preserve normality, X has this peculiarity. Such examples show the need for careful attention while dealing with T4 space subspaces, which may vary from the main space. Schauder found the T5axiom by researching separation axiom correlations. Schauder defined a T5 space as one where a disjoint open set may include any pair of separated sets (A and B). Since this axiom implies T4, all T5 spaces are T4. Not all T4 spaces are T5, indicating a hierarchical separation axiom structure that extends topological space categorization [34].

Finally, Schauder's 1929 functional space area invariance work highlighted T4 spaces and subspaces. His work on T4 features in subspaces and the T5 axiom advanced topological separation axioms and paved the way for future research. A (T1) + (T5) space is normal and Hausdorff since (T5) implies (T4), topological spaces with (T1) and (T5) axioms are normal [34].

5. COMPLETELY REGULAR SPACES, WEAK TOPOLOGICAL SPACES, AND ALMOST TOPOLOGICAL SPACES

In his topology metric research, Urysohn created the axiom (T). Tychonoff recast it and stressed its usefulness for compactifications. Axiom (T) became relevant when Weil proved that a topological space is uniformizable if and only if it holds [34].

A fully regular topological space X is defined by continuous f : X → $[\mathbf{0};\ \mathbf{1}]$ with f(x) = $\mathbf{0}$ and f (F) = $\{\mathbf{1}\}$, where F is a nonempty closed set and x ∈ X Not all regular spaces are Hausdorff, the trivial topology, with just and X as closed sets, is vacuously totally regular but not Hausdorff if X has more than one point. Tychonoff spaces are Hausdorff and fully regular [35].

Topological set theory employs essentially to signify all but a few elements in infinite groups.

Singal and Arya recommended almost regular spacing in 1969. Read publications [6, 19] for context, the topology of topological groups enables continuous operations and inversion mapping ($x \rightarrow x\mathbf{-1}$). Many researchers and mathematicians have been

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interested in this idea since its debut. Mathematician contributions to topology are many. A.D. Alexandroff, N. Bourbaki, M.I. Graev, S. Kakutani, E. van Kampen, A.N. Kolmogorov, A.A. Markov, Pontryagin, and others gave early topological space theory contributions A.V. Arhangel'skii, M.M. Choban, W.W. Comfort, D. Dikranjan, E. van Douwen, V.I. Malykhin, J. van Mill, B.A. Pasynkov, D. Shakhmatov, M. Tkachenko contributed widely to Recent math literature that offers similar topological group notions and generalizations. S-topological groups [8, 10], semi-topological [4, 5, 33]. S-, quasi-S-, irresolute, and para-topological groups are well-known.

In 1970, N. Levine [16] introduced generalized closed sets in a topological space to expand the family of closed sets. Since generalized closed sets are natural generalizations of closed sets, many mathematicians have investigated them (see [16, 17, 18]), the separation axiom is a classic issue in general topology and many other fields. Different mathematicians have studied separation axioms in literature, the 1973 Singal et al. proposal included very regular, virtually normal, and somewhat normal areas. Ekici, Malghan, Navalagi, Noiri, and Park [22, 23, 24, 25, 26, 27] researched weaker separation axioms, whereas [28] studied continuity, this work aims to unify spaces using Á. Császár's generalized topology concept.

Many papers have addressed these topics during the previous 70 years. Ferri, Hernández, and Wu [29] developed a Baire metrizable group topology with reduced left and right translation requirements.

Some academics called Frolík's poor continuity "almost continuity" (see definition below), the second issue, topological games, influences topological dynamics and Banach space theory [20, 22]. Arhangel'skii and Reznichenko [3] investigated whether para-topological groups are topological, these studies include more continuous functions [3]. No surprise, that poor continuity plays a role. It began in early separate vs. cooperative continuity works [27,28]. All of these factors drive us to explore poor continuity in group activities.

In [36], scientists developed new notions in practically weak topological spaces that allowed for almost compact space characterization and filter convergence-based product theorems. A is nearly compact if each open cover has a restricted subfamily of closures (or closure interiors).

In 1968, Freiwald [53] introduced almost continuous functions to weak topological groups. Recently. Mashhour et al. [38] examined a-continuous functions. Hildebrand and Gene Crossley explored these functions [39]. According to Maheshwari et al. [41], functions were hardly continuous. Levire proposed weakly continuous topological functions in [42]. Husain discussed nearly continuous functions [43]. Mashhour et al. call essentially continuity pre-continuity [44]. Jankovic [45] presented nearly weakly continuous functions recently. Independent continuity and weak continuity indicate nearly terrible continuity.

Later characterizations of basically weakly continuous functions enhanced Mashhour et al. [44]'s results, including specific constraints for almost continuous work. using "almost weakly continuous" instead of "very continuous" in several [44] and [46] conclusions.

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6. PRELIMINARIES AND THEORY OF SETS

Our weak topological space literature reviews will follow function f. To demonstrate that function f provides the following characteristics for all weak topological spaces: Continuous Compact Regular Space, Mildly Normal Compact Regular Space, and Normal Regular Space, the spaces are topological.

Sets and Elements [47]: Brain activity depends on grouping. Mental gathering, not physical. Forming and naming a group allows it to discuss and join others. A complex collection of ideas organizes and manipulates math groups. Naïve set theory is a language, not a theory.

We write $\boldsymbol{x} \in$ A to imply that A contains $\boldsymbol{x}$, the sign Δ is a variant of the Greek letter epsilon, which is the first letter of the Latin word element. For more flexibility, put $\boldsymbol{x} \in$ A as A$\ni$ $\boldsymbol{x}$, this notation emphasizes its similarities to inequality symbols < and >, ignoring its provenance. Write $\boldsymbol{x} \notin$ A or A$\not\ni$ $\boldsymbol{x}$ to signify $\boldsymbol{x}$ is not in A.

Membership In naive set theory, a set is any collection of mathematical objects, their constituents. Uppercase letters (e.g., A or B) indicate sets, whereas lowercase letters (e.g., $\boldsymbol{x}$ or $\boldsymbol{y}$) represent things. If A is a set, write "is an element of A" for $\boldsymbol{x}$. If $\boldsymbol{x}$ is not in A, write $\boldsymbol{x} \notin$ A, the "∈" symbol signifies collection membership [48].

Equality of Sets [47]: Elements define sets, the set has just its parts. Most obviously, two sets are equal if and only if they share components. The set is pejorative when used here. Calling something a set, even mistakenly, implies disorganization. Assuming lines are points, two lines coincide if and only if they share points. We will handle distance, order, and other relations between points on a line separately from the idea of a line. Sets make isolating components straightforward. Lightness reduces a box to its contents. It represents our desire to perceive this collection as a whole, not just its components. As elements, sets function like atoms, forgetting their initial nature. Most current math literature uses set and element. Too much usage. Sometimes avoid them. The use of the word element instead of other appropriate terminology is harmful. Calling something an element implies its set. Except for nonmathematical terms like chemical elements or occasional exceptions from mainstream mathematical nomenclature (ancient books name lines, planes, and other geometric image elements). Euclid's Elements is a geometry classic.

Definition 3.1 [47]: Elements must have sets. Sometimes a set is empty, but the set exists, this set is unique because components decide it. ∅ is the empty set.

Definition 3.2 [49]: A finite set E is either empty or equal to $\{\mathbf{1},..., \boldsymbol{n}\}$ for a normal number $\boldsymbol{n}$. E is countably infinite if it equals N normal numbers. A countable set is finite or infinite. Non-countable sets are uncountable.

Definition 3.3 [52]: Disjoint sets (A ∩ B = ∅) have no shared objects.

Definition 3.4 [52] calls it a "union" or "cup".

A ∪ B: = $\{\boldsymbol{x}\in$ (A∪ B) : $\boldsymbol{x}\in$A $\boldsymbol{or}$ $\boldsymbol{x}\in$B$\}$ unites sets A and B.

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(2) A⋃P: = {$\boldsymbol{x}$∈ (A⋃ P) ∶ ∃ A∈P or$\boldsymbol{x}$ ∈$\boldsymbol{A}$} defines the set union. At least one A element has $\boldsymbol{x}$.

See Definition 3.5 [52] for "intersection" and "cap" symbols.

(1) The intersection of sets A and B is {$\boldsymbol{x}$ ∈ (A⋂ B): $\boldsymbol{x}$∈A and $\boldsymbol{x}$∈B}.

(2) A⋂P = {$\boldsymbol{x}$∈ (A⋂P) ∶∀A∈P or $\boldsymbol{x}$∈A} is the intersection of a set A≠ ∅. Every element in A has $\boldsymbol{x}$.

A − B = {$\boldsymbol{x}$ ∈ A ∶ $\boldsymbol{x}$ ∉ B} (Definition 3.6 [53]). If the set A is well-defined, we may call A − B as Bc, the complement of B.

Definition 3.7 [52]: Disjoint sets have no shared elements (A ∩ B = ∅), theor 3.1 [52]. Commutativity, associativity, distributivity (1) Commutative and associative union and intersection:

(i) A^B = B^A; (A^B) ^C = A^(B^C).

(ii) A ∩ B = B ∩ A; (A ∩B) ∩C = A ∩ (B ∩ C).

(2) Union and intersection distribute:

(i) A⋃ (B⋂C) = (A⋂B) ∩ (A⋂C) A ∩ (B ∩ C) = A ∩ B

Theor 3.2 [52]. De Morgan Laws

(1) (A ∪ B) = (C \ A) ∩ (C \ B), (A ∩ B) = (C \ A). ∪ (C \ B) (2) C (A ∪ P) = ∩ {C \ A ∶ A ∈ P}, C \ ∩ P).

Definition 3.8 [52]: A is a subset of Band if every element of A is also in B. B includes or supersets A.

7. A TOPOLOGICAL SPACE

Definition 3.9 [9]. The following components are consistent with X's structure if and only if the following axioms hold. Kindly verify the availability of X.

The set τ includes both X and ∅.

Here we have the second inquiry. The result of combining any two sets τ is the set τ.

This brings us to our third inquiry, also included in τ are any two sets that contact each other.

The equation 3.10 [50]: 1 states that X is a topological space and τ is an open set, while the basic topology is rather straightforward to grasp, X and ∅ denote the coarse topology on X, the broken topology of X's strong set includes all of its subsets.

Make X nonempty, thus, τ$\boldsymbol{i}$ = {{∅}, X} and τ$\boldsymbol{d}$ = P(X) are indiscrete and discrete topologies, respectively. In every alternative topology on X, τ$\boldsymbol{i}$ ⊂ τ.

Let τ$\mathbf{1}$ and τ$\mathbf{2}$ be X topologies. $\mathbf{1}$ is weaker if it ⊆ τ$\mathbf{2}$, this means τ$\mathbf{2}$ is stronger than τ$\mathbf{1}$. Weak topology contains fewer open sets than strong.

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Make A a topological space X subset. A point x∈X is a Limit Point of A if its neighborhood U is either another point of A or the closure of A-$\{x\}$. Refer to A–$\{x\}$=A if $x \notin$ A. A′ is A's limit points in X via derivation.

See Definition 3.12 [54] for a topological space (X, τ). An open subset of X must be in τ.

Lemma 3.1 [50]. U is open if and only if Vx ⊆ X for every x ∈ U.

Definition 3.13 [54]: Let X be a topological space with topology τ and A be a non-empty subset, A may have several topologies without {but we want to give it a particular τ-derived topology, hence A is a subspace, we define X subset A in 3.14 [65]. Open set Int(A) is defined as: ∪ {G ⊆ X ∶ G is open and G ⊆ A}, representing the interior of A in X, topological space X and A ⊆ X are defined in 3.15 [66], the closure of A is the closed set (A) = ∩ {K ⊆ X ∶ K is closed and A⊆ K}, theor 3.3 [60], if H = G ∩ A in a subspace A of X, where G is o H pen in X, then H is open.

2) H fits in A if H = K ∩ A, where K is in X.

3) A ∩ X: clA (H).

4) A ∩ X (H) ⊇ A.

Suppose A and B are subsets of X (Theorem 3.4 [60]), then,

1) int (A) ⊆ A.

2) A ⊆ B implies (A) ⊆ (B).

3) int (X) = X.

4) int (int (A)) = int (A).

5) int (A ∩ B) = int (A) ∩ int (B).

6) int (A) ∪ I int nt (B) ⊆ int (A ∪ B).

7) A is open if int (A) = A.

Proposition 3.4 [54]: Let A be a non-empty subset of X with a certain topology. Define the collection of subsets of A as τA: A has a topology of τA = {A ∩ u ∶ u ∈ τ}.

Definition 3.16 [54]: If τ1 and τ2 are topologies on X and τ1 ⊆ τ2, then τ1 is coarser. Or, τ2 is finer or stronger than τ1.

Definition 3.17 [54]: Two X topologies are finer if none are coarser.

According to definition 3.18 [53], a topological property P exists when a space X has it and Y has it, and vice versa.

In topological space X, closed sets are open sets' complements.

Definition 3.20 [64]: (X, τ) is a topological space with P as an element and N as a subset. If P ∈ G ⊆ N, then N is a neighborhood of P.

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X topological spaces share the same properties:

A simple neighborhood system is the closed neighborhoods of all X points.

(ii) Neighborhoods of x and F do not cross for any closed subset F of X and any point x ∉ F.

Lemma 3.2 [57]. Let U ⊆ X be a topological space. U is open in X if and only if it has a neighborhood of x for each x ∈ U.

According to definition 3.21 [63], the smallest closed set is the closure of subset F ⊂ X. Member closure Isolated points are closures without limits.

Definition 3.22 [54]. (Inverse function) f$-\mathbf{1}$: Y → X is the inverse of f.

Definition 3.23 [9]: A topological space X is $\mathbf{T1}$-space if it fits the following axiom:

∈X points may be in an open set without each other; therefore, open sets G and H exist with ∈G, ∉ G and ∈H, ∉ H. Not all open sets G and H are disjoint, according to Definition 3.24 [9], a topological space X is a Hausdorff or $\mathbf{T}$2-space if it has disjointed open sets for each pair of different points (∈X), thus, open sets G and H exist with ∈G, ∈H, and G∩ H= ∅.

Note that Hausdorff spaces are always $\mathbf{T1}$-spaces.

According to Theorem 3.5 [60], a topological space is Hausdorff if and only if the intersection of all closed neighborhoods of a point $\boldsymbol{a}$ is the set $\{\boldsymbol{a}\}$, theorem 3.6 [68] closes all Hausdorff space X finite point sets.

In Hausdorff space, all finite subsets $\boldsymbol{A}$ ⊂ X are closed (Theorem 3.7 [48]).

Claim 3.1 [69]. Every subspace is Hausdorff.

Definition 3.25 [58]: Topological space coverings are X-combining sets. Open sets open the covering, this collection has space-covering subsets.

For any open covering in a compact topological space X, there is a finite subcover. Also known as Heine-Borel property, the Finite Intersection property of a set collection is present if any finite subcollection intersects nonempty, theorem 3.8 [53] applies to all topological spaces (X, τ).

1) Compact X.

2) ∩ $\boldsymbol{\mathcal{F}}$ = ∅ for any closed set families $\boldsymbol{\mathcal{F}}$ in X with finite intersection property.

Consider A a subset of Hausdorff space X. A's limit point is x ∈ X if each neighborhood U encounters A infinitely many times.

A with subspace topology is a Hausdorff space if A ⊂ X, under Theorem 3.10 [72]. ∏i∈I X$\boldsymbol{i}$ is Hausdorff if {X$\boldsymbol{i}$: $\boldsymbol{i}$ ∈ I} is a Hausdorff Family.

Hausdorff spaces are topological spaces that fulfill axioms ($\mathbf{T0}$) and ($\mathbf{T3}$) (Theorem 3.11 [60]).

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X is topological, then these statements are equivalent:

Disjoint the neighborhood for any two distinct X positions. (Hausdorff).

A point of X is the lone member of its closed neighborhood, the diagonal of the product space X × X is closed.

A closed diagonal of the product space Y= XI exists for any set I.

Y is a subspace of X, Lemma 3.4 [68]. Every compact coverage of Y by open sets must have a finite subcollection.

Compact sets' closed subsets are compact.

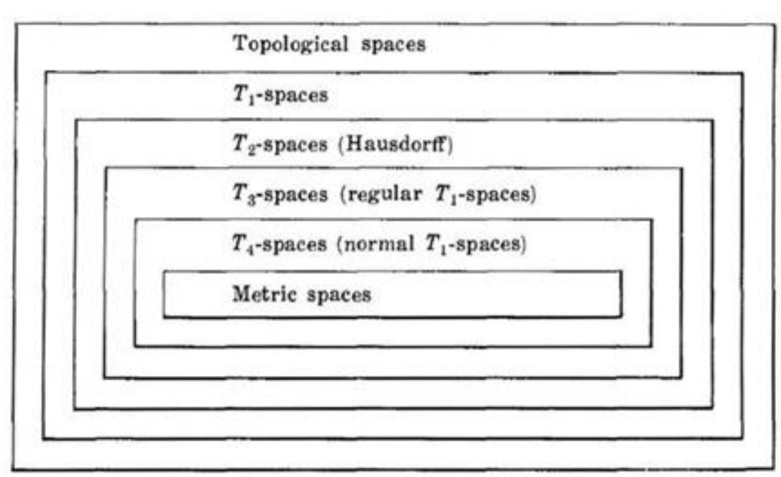
All compact Hausdorff space subspaces are closed, theorem 3.13 [68].

Every compact space's closed subspace is compact Theorem 3.14 [68].

Disjoint open sets U and V of Hausdorff space X will include xo and Y if Y is a compact subspace and xo is not in Y (Lemma 3.5 [68]), theorem 3.15 [58] gives the compact unit interval I=$[\mathbf{0}, \mathbf{1}]$.

A regular topological space X does not contradict the following axiom:

When F is a closed subset of X and ∈X is not in F, disjoint open sets G and H exist with F ⊂G and p ∈H.



A topological space is normal if every subspace of X is normal [74]. A topological space is normal if and only if disjoint open subsets U ⊇ A and V ⊇ B exist for all subsets A and B in X with A ∩ B = ∅.

Mildly normal ($\boldsymbol{k}$ − normal) topological spaces are those with two disjoint regularly closed subsets A and B, and two open disjoint subsets U and V such that A ⊆ U and B ⊆ V [65].

Definition 3.36 [76]: (X, τ) is nearly regular if it has disjoint τs (open sets U and V) such that A ⊂ U and x ∈ V for each τ-Regular closed subset A of X and each point $x \notin$ A

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Theorem 3.19 [65]: For X, they are equivalent:

X is almost typical.

(2) Every closed set B and regularly open set A containing B have an open set U that is a subset of B and $\bar{U}$, theorem 3.20 [76] states that (X, τ) is virtually regular if (X, τs) is regular.

According to Theorem 3.21 [76], (X, τ) is virtually regular, meaning that for each x ∈ X and each regularly open neighborhood U of x, there exists a regularly open set V such that x ∈ V ⊂ cl

Lemma 3.7 [76] states that if A and B are disjoint open sets in (X, τ), then τa (A) and τa (B) are also disjoint open sets in (X, τs) containing them

Definition 3.37 [65]: A topological space X is virtually normal if for every two disjoint closed subsets A and B, one of which is regularly closed, there exist two open disjoint subsets U and V such that A ⊆ U and B ⊆ V.

An almost regular compact space is almost compact.

Corollary 3.2 [75]: A tiny, fairly regular space is slightly normal (κ-normal).

Compact ⇒ almost compact ⇒ almost compact Normal ⇒ almost normal ⇒ slightly normal

8. A MAPPINGS FUNCTIONS

Definition 3.38 [54]. Consider two non-empty sets (X and Y may be equal). A function or single-valued mapping from X to Y assigns each X element a unique Y element, the element y of Y assigned to x under the rule f is the image of x or the value of f at x. Image of x: y = f(x).

Definition 3.39 [9]: A function f: R → R is continuous at a point ∈ R if f [Up] ⊂ Vf () for any open set containing f ().

If everywhere, f is continuous.

Venn diagrams may help visualize this idea.

Remark 3.1 [52]. Let X, Y be sets and f: X → Y be a function. Relation definitions define D(f) as f's domain and R(f) as {f(x) :x ∈ D(f)} as its range. If X ⊂Z, then D(f) ⊂ Z, where Y is the function's target.

Definition 3.40 [54]. Let f: X → Y be a topological function. Let $x0$ ∈ X. To make f continuous at $x0$, there must be an open set u in X such that x ∈ u ⊆ f-1(V) for any open set V containing f($x0$).

Definition 3.41 [54]: A one-one or injective function f: X → Y is one-one or injective if distinct elements of X have distinct pictures in Y, i.e., f($x1$) = f($x2$) for $x1$, $x2$

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A function f: X → Y is onto or surjective if its range is Y and its components are the image of X elements, as defined in Definition 3.42 [54], the definition of a bijective function is when it is both injective and onto (definition 3.43 [54]).

Definition 3.44 [54]: Function Composition Let f:X → Y and g:Y → Z be any two functions, the function (gof) from X to Z is defined by (gof) (x) = g(f(x)), theorem 3.22 [55]. Let f: A → B be a function. Bijectivity requires f-1 to be a function from B to A.

Definition 3.45 [55]: Functions f ∶ A →B and g ∶ B → C have a codomain, the function g°f, which combines f and g, is defined as: Assuming x ∈ A, g°f (x) = g (f (x)), so, g°f maps A to C, resulting in A → C.

Definition 3.46 [55]: The identity function on a set A is iA :A → A, where iA (x) = x for every x ∈ A.

Definition 3.47 [52]: A one-to-one map f: ℕ→A exists for infinite sets.

9. A CONTINUITY AND HOMEOMORPHISM IN TOPOLOGICAL SPACES

Topological Spaces and Functions

Definition 3.48 [50]:

- X, T) and (Y, F) are topological spaces.
- A function f: X→ Y is open if, for any open set U ⊆ X, the image f (U) is open in Y, theorem 3.23 [9]:
- Assume (X, τ) is a topological space and F is a family of continuous functions from X to the topological space (X, τ a).
- The resulting weak topology of F is weaker than τ, theorem 3.24 [65]:
- If f: (X, τ) → (Y, τ′) is continuous, then f is continuous.
- If $\mathcal{O}$ ∈ τ′ then f-1($\mathcal{O}$) ∈ τ (the inverse image of an open set is open).
- If F is closed in Y, then f-1(F) is closed in X.
- For every A ⊂ X ∶ f [clX (A)] ⊂ clY(f[A])), theorem 3.49 [48]:
- f is continuous at x if, for each neighborhood V of f(x), there is a neighborhood U of x with f (U) ⊂ V.
- f: X → Y is continuous if and only if it is continuous at each x ∈ X, theorem 3.25 [48]:
- If f ∶ X → Y and g ∶ Y → Z are two continuous functions, then (gof) ∶ X → Z is a continuous function, theorem 3.50 [54]:
- If A is a subset of X, its image set f(A) is a subset of Y defined by f(A) = {f(x): x ∈ A}.
- If B is a subset of Y, its inverse image f-1(B) is the subset of X defined by f-1(B), theorem 3.29 [62]:

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- The property of a space being Hausdorff is preserved by homeomorphism, theorem 2.13 [9]:
- The unit interval [0,1] is non-denumerable.
- A set X is said to have the power of the continuum or has cardinality if it is equivalent to the unit interval [0,1].

Corollary 3.3 [59]:

- If f is a continuous mapping of a topological space X into a Hausdorff space Y, then the graph of f is closed in X × Y, theoretical Definitions of Topological Spaces
- Theorem 3.31: If X is compact and F is closed in X, then F is compact.
- According to Theorem 3.32, a compact Hausdorff space X lacks a perfect subset if f continuously maps Q onto it.
- The continuous image of a compact space is compact.
- Continuous f: X → Y implies compact f (X) (Theorem 3.34).
- Theorem 3.35: A homeomorphism exists if f: X → Y is a continuous bijection and X and Y are compact and Hausdorff.

A one-to-one, onto, and continuous function from a compact space X to a Hausdorff space Y is a homeomorphism.

Fully or functionally Hausdorff Spaces

- A topological space X is considered completely or functionally Hausdorff if a continuous function f : X → $[\mathbf{0}, \mathbf{1}]$ such that f $(\boldsymbol{x}) = \mathbf{0}$ and f $(\boldsymbol{y}) = \mathbf{1}$.
- A completely regular $\boldsymbol{T1}$ space is called a Tychonoff space.

Compact Hausdorff Spaces

- A compact Hausdorff space is normal.
- Every closed subspace of a normal space is normal.
- The closed continuous image of a normal space is normal.

Urysohn's Lemma and Tietze's Lemma

- X is normal if and only if there is a continuous function f : X → $[\mathbf{0}, \mathbf{1}]$ such that f (A) = $\mathbf{0}$ and f (B) = $\mathbf{1}$.
- Every T4 space is Tychonoff.

Understanding the Concept of Almost Continuous and Closed Mappings

Definition and Definitions of Almost Continuous and Closed Mappings

- Definition 3.59 [51]: A function f: X → Y is almost continuous if the inverse images of regularly open sets of Y are open in X.

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- Definition 3.60 [77]: The identity f: X → Y is approximately open if the image of every regularly open subset of X is an open subset of Y.

Definition 3.61 [77]: The identity f: X → Y is approximately closed if the image of every regularly closed subset of X is a closed subset of Y.

Theorems and Consequences on the Almost Continuous Match

- Theorem 3.11 [40]: For the identity f: X → Y, the following are equivalent: f is approximately continuous, the inverse image of every regular open subset of Y is a subset of X, and the inverse image of every regular closed subset of Y is a closed subset of X.
- Definition 3.62 [66]: The function f: X → Y is approximately continuous if for every $x \in$ X and for every regular open set V containing f(x), there exists an open set U containing x such that f (U) ⊆ f.
- Definition 3.63 [71]: The function f: X → Y is almost continuous at $x0 \in$ X if and only if for each open V ⊂ Y containing f(xo), cl(f-1(V)) is a neighborhood of $x0$.
- Definition 3.64 [71]: The function f: X → Y is almost continuous if and only if cl(f-1(V)) = f-1(cl (V)) for each open subset V of Y.

Remarks and Theorems on Almost Continuous Mappings

- Theorem 3.38 [71]: If f: X → Y is continuous and U is an open subset of X, then f | U is a. continuous.
- Theorem 3.39 [77]: If f is an open continuous mapping of X onto Y and if g is a mapping of Y into Z, then gof is almost-continuous if fog is almost-continuous.
- Theorem 3.40 [77]: If there exists a neighborhood N of x such that the restriction of f to N is almost continuous at x, then f is almost continuous at x.
- Theorem 3.41 [77]: If f is a mapping of X into Y and X = X1 U X2, where X1 and X2 are closed and f\X$1$ and f\X$2$ are almost continuous, then f is almost continuous at x.
- Theorem 3.42 [77]: If f is an almost-continuous, closed mapping of regular space X and space Y, then Y is almost-regular.

10. WEAKER REGULAR SPACE UNDER THE CONTINUOUS MAPPING

Weak regular spaces are topological spaces with certain regular space features. In topology, weak regular spaces are important because they reveal the variations and limits of regularity and help mathematicians understand the structure and relationships between different types of spaces, a weak regular space under a given mapping has a weak topology induced by that mapping, in which the open sets are generated by the preimages of open sets in the target space under the mapping, this framework retains the separation properties of regular spaces while allowing for a weaker notion of convergence, making it useful for studying convergence behavior and continuity with respect to specific mappings, it is useful for investigating convergence and continuity in situations where the

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original topology is too restrictive, the interaction between the regularity properties and the mapping's weak topology allows for a nuanced understanding of the underlying space and its relationship to the target space, a weakly regular space is a topological space that meets two conditions: for any point x and closed set F not containing x, it exists disjoint open sets U and V with x in U and F in V. For any point x and open set U containing x, there exists an open set V with x in Vand, the closure of Vis in U, like a compact space, an almost compact space has a limited subcover for every open cover. This feature is weaker than compactness, continuous functions on nearly compact regular spaces maintain topological structure, guaranteeing smooth transitions and consistent behavior.

In summary, weak regular spaces provide valuable insights into the structure and relationships between different types of spaces, offering insights into the variations and limits of regularity, the theorem states that if a space is almost compact regular space in a topological space Y, then f(A) is almost compact regular space in Y, this is because if X is a compact regular space, then f(A) is almost compact in Y, this is because every open cover of the space has a finite subcover, which ensures that f(A) has similar compactness properties as X, regularity ensures that f(A) inherits some of the separation properties from X. It also ensures that the preimage of any open set in Y is an open set in X, this property guarantees that the image f(A) preserves the continuity of the mapping. An almost compact space possesses numerous compactness-like features. It signifies that f(A) meets most compactness qualities but may not meet technical constraints, weakly regular and nearly compact spaces are topological ideas that explain space features.

Although similar, they are not the same, topology requires understanding these ideas because they give intriguing instances and insights into topological spaces' structure and behavior, the continuity of functions in these spaces preserves topological structure under mappings, enhancing the study of these spaces and their features, almost totally compact regular spaces are topological spaces with compactness and regularity, the idea of a finite subcover and continuous functions on such spaces allow topology to analyze well-behaved spaces and mappings between them, in the proof, assume f: X → A is a continuous mapping into Y with f/F: F → f(F). By Definition 3.32 and Theorem 3.28, (A, τF) ≅ (f(F), τf(F)) such that τ = {F$\boldsymbol{i}$| F$\boldsymbol{i}$ ⊂ X}; F$\boldsymbol{i}$ ∈ τ ⇔ F$\boldsymbol{i}$ is closed, theorem 4.2 presents a new definition of an almost completely compact regular space. If X is a regular compact space, and f is a continuous mapping, then f(F) is almost completely compact regular space in Y, theorem 4.2 implies that the image of any closed set F under a continuous mapping f from a regular compact space X into the complement of a closed set A in a topological space Y will be an almost completely compact regular space in Y.

Weak regularity and almost complete compactness are two distinct properties of topological spaces that are related in certain ways. A nearly fully compact regular space is a topological space known for its close compactness, covering everything but a finite subset of space, and consisting of three components: a regular space, a compact space, and an almost fully compact space, regular spaces satisfy certain separation axioms, while compact spaces ensure that every open cover has a finite subcover. Almost fully

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compact spaces exhibit most of the properties of a compact space but may lack some specific aspects, usually related to the way infinite sets behave, the continuous function of a nearly fully compact regular space preserves the topology of the space, this concept combines continuity, compactness, and regularity while relaxing the requirement of complete continuity. This means that the initial images of open sets under continuous functions must be only "almost open", rather than completely open, compactness ensures that any open cover of a space has a finite subcover, providing a sense of boundary, and regularity allows points to be separated from closed sets using separate open sets, providing a richer framework for studying topological structures, understanding the properties and behavior of functions within these spaces opens this up, the book opens up prospects for exploring approximation theory, functional analysis, and other areas of mathematics, the relationship between approximately continuous compact regular spaces and continuous functions lies in their ability to approximate approximately continuous functions with continuous functions.

11. WEAK NORMAL SPACE UNDER THE CONTINUOUS MAPPING

Disconnected open sets may separate closed sets from points. A topological space is a "nearly fully compact regular space" if it is compact except for a finite subset. While not requiring total continuity, topology requires continuous functions. Compact, nearly continuous regular spaces are regular, continuous, and compact. Open sets under continuous functions should have "nearly open," not entirely open, starting pictures. In this form of space, compactness gives every open cover a finite subcover, giving it a feeling of finiteness. In topology, weak normal spaces study properties that stay intact after continuous transformations. Regularity allows discontinuous open sets to distinguish points and closed sets, making topological structure investigation more complete. Finiteness requires the separation of disjoint closed sets by disjoint open sets, whereas weak normal spaces separate closed sets and points by open sets. Dividing closed sets by open sets is OK as long as one open set's closure does not cross another. Weak normal spaces are essential in topology and related mathematics because they allow the study of natural spaces with partial borders, bridging the gap between natural and non-natural spaces. In algebraic topology, functional spaces, and general topology, and by studying the unique properties of weak natural sp., knowing how weak naturalism interacts with other topological ideas like a countable contraction, which deals with certain covers of space, is crucial to understanding the complex structure of topological spaces and their behavior, weakly regular and regular spaces are important topological concepts that represent continuity and convergence. Regular spaces separate closed disjoint subsets from open disjoint sets, allowing fundamental topological constructions and theorems. According to the T3 axiom, disjoint open sets may divide a closed set from a point outside it. Regularity is weaker but more ubiquitous than normalcy. Continuous functions retain the topological properties of a regular space because they allow comparison and analysis, identify continuous mapping structures, and establish connections across topological spaces. Topology requires continuous functions. Regular spaces are topological spaces with separation features that allow disjoint open sets to

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divide closed disjoint sets. Compactness, regularity, and normalcy form a topological space regular compact regular. The presence of a finite subcover for each open cover is a feature of Compactness, which indicates that a space is "small" in the finite sense, there are open, disconnected sets that separate any point from a closed set that does not include it, and regularity ensures, the most stringent requirement for regularity is that it must be possible to separate any two closed, disconnected sets by open, disconnected sets, in topology, a compact, semi-normal regular space is a specialized concept that combines regularity, nearly normality, and compactness, because of the large degree of structure and separation that this combination ensures, such spaces are useful in mathematics for constructing complex spaces and evaluating their properties, compactness contributes to the good character of the space and its usefulness in geometry and analysis by ensuring that every open cover has a finite sub-cover, by using disconnected open sets to separate points and closed sets, regularity makes it possible to examine continuity and convergence, there are open, disconnected sets that include any two closed, disconnected sets, according to the relaxation form of naturalness known as "semi-normality", compactness, regularity, and near-normality are three desirable properties of a topological space that are satisfied by a regular space Naturally compact, Close to natural compact Many branches of mathematics, such as functional analysis, topological algebra, and dimension theory, use regular spaces because they allow smooth and consistent modification of spaces over continuous functions. If X is a compact regular space and there are two disjoint closed subsets, one of which is regularly closed, then f(N) is an approximately natural compact regular space in Y, according to Theorem 5.3. Open sets with disjoint closed subsets A and B and their closures are likewise disjoint making the topological space nearly natural. The theory ensures "an approximately natural compact regular space" for the image f(N). Regular spaces are topological spaces with regularity, compactness, and normalcy, but they must be finite and have a finite subcover for every open cover. Regularity separates points from closed sets using open sets, although normalcy is weaker. Continuous functions between spaces retain topological structure, therefore input space changes have minimal influence on output space.

12. CONCLUSIONS AND FUTURE STUDY

This concept provides a complete foundation for understanding compactness, regularity, and normalcy in roughly topological future research has great potential, including studying the properties of approximately compact regular spaces under different continuous mappings, generalizations to almost perfectly compact regular spaces, analysis of regularity-normality interactions in almost perfectly compact topological spaces, and the behavior of perfectly compact regular spaces under continuous mappings, this study aims to provide a comprehensive understanding of weaker regular and normal forms in approximately topological spaces, which improves theoretical knowledge and opens up new practical applications in mathematical modelling, theoretical computer science, dynamical systems, algebraic topology, and functional analysis, the authors think this integrated method will promote topological space study and discoveries, the main results

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of this study are the definitions of approximately compact, approximately perfectly compact, approximately continuous, normal, and somewhat normal compact regular spaces, these findings help explain weaker regular and normal forms in roughly topological spaces and their behavior under continuous mappings, revealing the complicated relationship between regularity, compactness, and normalcy, this study presents a basic knowledge of the features and behavior of roughly topological spaces under continuous mappings More topological space research will open fresh discoveries by investigating regular and weaker normal forms, this study, “On Some Regular and Weakest Normal Forms of Approximately Regular Topological Spaces under Continuous Mapping,” provides many research avenues. These include studying approximately compact regular spaces under different continuous mappings, almost fully compact regular spaces, the interaction between regularity and normality in approximately topological spaces, and the specific conditions under which regular compact spaces behave under different continuous mappings, these studies explore theoretical and practical advances in dynamical systems, algebraic topology, and functional analysis, this work has opened several theoretical and practical options for approximately topological space exploration by exploring regular and least normal forms under continuous mappings, this study opens several opportunities for future research on approximately topological spaces, revealing new theoretical insights and practical applications. Studies of continuous layouts' regular and lesser normal forms might reveal topological spaces' properties.

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