

MATHEMATICAL APPROACH THROUGH MODELING ON MULTI-OBJECTIVE TRANSPORTATION PROBLEM

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Abstract

Simple, approachable modeling provides the best compromise approach for solving the linear multi-objective transportation problem (MOTP). Finding solutions that are close to the ideal solution may be done quickly and efficiently by using our technique. This proposed approach provides a distinct and useful solution, also this strategy immediately leading to obtaining the kind of efficient extreme point. With less work and time, the majority of decision makers favor this compromise solution technique. Our methodology is an easy and fast method to identify solutions close to the ideal solution. The implementation of the proposed strategy includes the numerical examples that have been described in the literature, and these examples are then solved by using the proposed technique. This paper concluded with a conclusion and a discussion of the study's future direction.

Keywords: Extreme Point, Ideal Solution, Multi-Objective Linear Programming, Multi-Objective Transportation Problem (MOTP), Transportation Problem (TP).

1. INTRODUCTION

A branch of mathematics applied in multiple objective decision-making, the multi-objective optimization problem, deals with optimization issues where two or more objective functions must be optimised (maximize or minimize) simultaneously. The transportation problem is a specific kind of LPP (linear programming problem) in operation research, the goal of research is to reduce the cost of supplying a good from several sources or origins to various destinations.

The normal simplex methods are not appropriate for resolving issues with transportation due to its unique structure. It takes a unique approach to solve these problems. Transportation problem (TP) is a term that refers to an issue that arises while supplying goods from distinct sources to different destinations.

It was first introduced by Hitchcock in 1941 then later by Koopmans 1947, [6]]. There is a single objective related to these transportation problems. But, under actual conditions, every organisation aims to achieve a number of goals while preparing for the transportation of products. In order to decide how to achieve various goals simultaneously, Lee et al. in 1973 applied the goal programming approach, [9]].

Multi-objective linear programming can be resolved by Zeleny in 1974 created a non-dominated fundamental feasible result, [20]]. An alternate method to find for every non dominated solution for multi-objective problems (MOTP) was described by Diaz in 1978, which is based on how satisfied you are with how closely any compromise solution comes to the ideal one, [2]].

A method to determine all non-dominated solutions for MOTP was created by Diaz in 1979, [3]]. Isermann in 1979 created various methods to obtain a collection of effective solutions, [7]]. Gupta et al. in 1983 use the multi-criteria simplex method to solve the MOTP, [4]]. Two interactive techniques for solving MOTP were presented by Ringuest et al. in 1987, [18]].

Different methods to solve MOTP were developed by Kasana et al. in 2000 and Bai et al. in 2011, [8]], [1]]. Pandian et al. in 2011 and Qudoods et al. in 2013b entrenched dripping method for solving transportation problem which is bi-objective and solved bi-objective transportation problem (TP) using lexicographic programming and the MMK technique, [15]], [17]]. For a multi-objective chance constraint capacitated TP, Gupta et al. in 2013 identified a compromise solution, [5]].

A method for determining the solution of MOTP using interval parameters was devised by Yu et al. in 2014, [19]19]]. Multi-choice MOTP and MOTP with interval aim were solved by Maity et al. (2014, 2016a) using the utility function method, [10]], [12]]. Maity et al. in (2015, 2016b) created methods for solving MOTP with non-linear costs, multi-choice demands, and price reliability in unpredictable surroundings, [11]11]], [13]].

Nomani et al. in 2017 created an examined technique established on goal programming to acquire a compromise Multi-objective transportation problem result, in which a new model is provided to achieve a varied result in accordance with the preference of DM (decision maker), [14]]. Suggested preservation technology in two-warehouse inventory model [21]].

Suggested the real-life applications of stability theory [22]]. In Supply chain management involve all the parties directly or indirectly. In Supply Chain Management procedure involving various parties like –Manufacturer, Supplier, retailer. These parties providing the products to customer [23]], [24]].

Proposed the formulation of solution procedure for stochastic solid transportation problem with mixed constraint under stochastic environment [25]]. Proposed a methodology to tackle solid transportation problem involving multiple objectives and multiple products under fuzzy environment [27]]. Suggested an approach to help numerous enterprises and Transportation problem [26]].

Developed an efficient technique for arranging various commodities in a warehouse [28]]. Offered valuable insight for business to optimize their supply chain processes and improve warehouse layout [30]]. Suggested a method to solve stochastic solid transportation problem with multi-objective multi-item by using gamma distribution [29]].

Suggested the General Characteristic Equation for Eigen Values of graph [31]]. Proposed an approach for the inventory model for quadratic demand [32]]

2. PRELIMINARIES

This section contains some general definitions.

- a) A solution which is not dominated to the MOTP (multi-objective transportation problem) given by a feasible solution $\bar{x} = \{\bar{x}_{ij}\} \in X$ iff, there is no other feasible vector $x = \{x_{ij}\} \in X$ such that

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij}^h x_{ij} \leq \sum_{i=1}^m \sum_{j=1}^n c_{ij}^h \bar{x}_{ij} \text{ for all } h$$

$$\text{and } \sum_{i=1}^m \sum_{j=1}^n c_{ij}^h x_{ij} \neq \sum_{i=1}^m \sum_{j=1}^n c_{ij}^h \bar{x}_{ij} \text{ for } h$$

\bar{x} is said to be systematic if hold this connection. It gives an inferior or dominated solution if \bar{x} is not structured (Ringuest & Rinks,1987), [18]].

- b) Each objective would concurrently reach its minimum as a outcome of the multi-objective transportation problem's ideal solution

$$Z_h^* = \min Z_h = \min \sum_{i=1}^m \sum_{j=1}^n c_{ij}^h x_{ij}$$

As a result, the vector $Z^* = (Z_1^*, Z_2^*, \dots, Z_l^*)$ is an ideal solution. The MOTP (multi-objective transportation problem) has an ideal solution only for feasible extreme point x^* such that $Z^* = (Z_1^*, Z_2^*, \dots, Z_l^*) = Z_l(x^*)$. For each of the sub problems this would mean

$$\min Z_h = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^h x_{ij} \quad h= 1,2,\dots, k$$

Subject to

$$AX \leq B \quad X \geq 0 \quad i=1,2,\dots,k$$

Every Z_k optimises by at least one similar extreme point, hence a compromise solution must be obtained (Ringuest & Rinks,1987), [18]].

- c) A solution $\bar{x} = \{\bar{x}_{ij}\} \in X$ is an optimal compromise solution of MOTP which has always used by the decision maker, has taken into consideration all criteria used in the MOTP.

3. MODEL REPRESENTATION

Every organiser in real-world circumstances usually aims to accomplish several objectives at once while arranging for the delivery of products.

Thus, MOTP was created by researchers to achieve a number of objectives. Similar to a traditional TP(transportation problem), in MOTP (multi-objective transportation problem), amount of the product(x_{ij}) has to be shifted from origin i ($i = 1, 2, \dots, m$) to target/destinations j ($j = 1, 2, \dots, n$) along the price c_{ij} and their capacities are p_1, p_2, \dots, p_m and q_1, q_2, \dots, q_n respectively, where c_{ij} may be total shipment time, transportation cost/least transportation risk and so on.

The price of transportation, cost of damage, price of security/total shipping time and so on is to be correlated with the h objectives Z_1, Z_2, \dots, Z_h .

The MOTP (multi-objective transportation problem) (η) has the following mathematical model.

$$\min Z_h = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^h x_{ij}, \quad h=1, 2, \dots, k$$

Subject to constraints

$$\sum_{i=1}^m x_{ij} = p_i, p_i \geq 0, \quad (i = 1, 2, \dots, m)$$

$$\sum_{j=1}^n x_{ij} = q_j, q_j \geq 0, \quad (j = 1, 2, \dots, n)$$

Where vectors $Z_h = \{Z_1, Z_2, \dots, Z_k\}$ of k objective functions, the superscript on c_{ij}^k and subscript on Z_h are used to determine how many number of objective functions are there ($k=1, 2, \dots, K$), while keeping its generality, it will be supposed throughout this entire research paper that $p_i > 0 \quad \forall i, q_j > 0 \quad \forall j, c_{ij}^h \geq 0 \quad \forall (i, j)$ and $\sum_{i=1}^m p_i = \sum_{j=1}^n q_j$.

Due to its unique design of the MOT (multi-objective transportation) model, table 1 represents the problem (η).

Consider the problem (η) is always balanced, that is $\sum_{i=1}^m p_i = \sum_{j=1}^n q_j$. We can easily make it balanced if it is not balanced.

Table1: Representation of the problem

Destination→ Origin↓	D ₁	D ₂	D _n	Supply(p _i)
O ₁	c_{11}^1 c_{11}^2 . . . c_{11}^h	c_{12}^1 c_{12}^2 . . . c_{12}^h	\dots \dots . . . \dots	c_{1n}^1 c_{1n}^2 . . . c_{1n}^h	p ₁
O ₂	c_{21}^1 c_{21}^2 . . . c_{21}^h	c_{22}^1 c_{22}^2 . . . c_{22}^h	\dots \dots . . . \dots	c_{2n}^1 c_{2n}^2 . . . c_{2n}^h	p ₂
.	.	.	\dots	.	.
.	.	.	\dots	.	.
.	.	.	\dots	.	.
O _m	c_{m1}^1 c_{m1}^2 . . . c_{m1}^h	c_{m2}^1 c_{m2}^2 . . . c_{m2}^h	\dots \dots . . . \dots	c_{mn}^1 c_{mn}^2 . . . c_{mn}^h	p _m
Demand(q _j)	q ₁	q ₂	...	q _n	

4. ALGORITHM FOR THE SUGGESTED APPROACH

In order to solve MOP (multi-objective problems), finding an effective solution that approaches the ideal solution is necessary. We have here suggested a simple method for finding a uniquely effective solution, which results in a compromise solution. To continue with the suggested method, the steps listed below must be performed:

Step I:

Create Table 1 by converting the provided TP into a problem-solving format (η).

Step II:

Determine

Row max(maximum) cost (μ) as $\mu_i^h = \max(c_{ij}^h)$, for fixed i, $1 \leq j \leq n$ and $1 \leq h \leq k$,

Column max(maximum) cost (τ) as $\tau_j^h = \max(c_{ij}^h)$, for fixed j, $1 \leq i \leq m$ and $1 \leq h \leq k$,

here $\mu = \{\mu_1^1, \dots, \mu_1^h; \dots; \mu_m^1, \dots, \mu_m^h\}$ and $\tau = \{\tau_1^1, \dots, \tau_1^k; \dots; \tau_n^1, \dots, \tau_n^k\}$. Table 2 represents these sets μ and τ in multi-objective transportation.

Table 2: Represents the sets of multi-objective transportation

Destination → Origin ↓	D ₁	D ₂	D _n	Supply(p _i)	μ
O ₁	c_{11}^1	c_{12}^1	...	c_{1n}^1	p ₁	μ_1^1
	c_{11}^2	c_{12}^2	...	c_{1n}^2		μ_1^2
	·	·	·	·		·
	·	·	·	·		·
	c_{11}^k	c_{12}^k	...	c_{1n}^k		μ_1^k
O ₂	c_{21}^1	c_{22}^1	...	c_{2n}^1	p ₂	μ_2^1
	c_{21}^2	c_{22}^2	...	c_{2n}^2		μ_2^2
	·	·	·	·		·
	·	·	·	·		·
	c_{21}^k	c_{22}^k	...	c_{2n}^k		μ_2^k
·	·	·	...	·	·	·
·	·	·	...	·	·	·
·	·	·	...	·	·	·
O _m	c_{m1}^1	c_{m2}^1	...	c_{mn}^1	p _m	μ_m^1
	c_{m1}^2	c_{m2}^2	...	c_{mn}^2		μ_m^2
	·	·	·	·		·
	·	·	·	·		·
	c_{m1}^k	c_{m2}^k	...	c_{mn}^k		μ_m^k
Demand(q _j)	q ₁	q ₂	...	q _n		
τ	τ_1^1	τ_2^1	...	τ_n^1		
	τ_1^2	τ_2^2	...	τ_n^2		
	·	·	·	·		
	·	·	·	·		
	τ_1^k	τ_2^k	...	τ_n^k		

Step III:

Select

$$Q = \max_{1 \leq i \leq m, 1 \leq j \leq n} (\mu_i^h, \tau_j^h), \forall h.$$

Step IV:

In the table there are many cells select the one C (cell) which has Q as its objective value, if more than one such cells is there than select that one cell which has maximum cost of objective value other than Q.

Step V:

Select that cell which has minimum $\left(\sum_{i=1}^m c_{ij}^h, \text{ for stable } j \right)$ in the corresponding row/column of the cell selected in step IV.

Step VI:

Do the max(maximum) allocation you can, to that cell you choose in Step V and when supplies/demands are satisfied then eliminate that row/column.

Step VII:

For the remaining origins and destinations, repeat Steps 3 to 6 until all supply or demand conditions are not satisfied. We now analyse the suggested strategy using this following example:

5. EXAMPLE

Step I:

Consider the MOTP (multi-objective transportation problem) like table 1 in a tabular format.

Table 3: Multi-objective transportation problem

Destination Origin → ↓	D ₁	D ₂	D ₃	Supply (p _i)
O ₁	3 5	4 2	5 2	8
O ₂	3 4	5 4	2 3	5
O ₃	5 3	1 3	2 2	2
Demand (q _j)	7	4	4	

Step II:

Determine row max(maximum) cost μ as

$$\mu_1^1 = \max\{3, 4, 5\} = 5, \mu_1^2 = \max\{5, 2, 2\} = 5,$$

$$\mu_2^1 = \max\{3, 5, 2\} = 5, \mu_2^2 = \max\{4, 4, 3\} = 4,$$

$$\mu_3^1 = \max\{5, 1, 2\} = 5, \mu_3^2 = \max\{3, 3, 2\} = 3.$$

And column max(maximum) cost τ as

$$\tau_1^1 = \max\{3, 3, 5\} = 5, \tau_1^2 = \max\{5, 4, 3\} = 5,$$

$$\tau_2^1 = \max\{4, 5, 1\} = 5, \tau_2^2 = \max\{2, 4, 3\} = 4,$$

$$\tau_3^1 = \max\{5, 2, 2\} = 5, \tau_3^2 = \max\{2, 3, 2\} = 3.$$

Table 4 indicates μ and τ .

Step III:

Determine

$$Q = \max_{1 \leq i \leq m, 1 \leq j \leq n} (\mu_i^h, \tau_j^h), \forall h = \max\{5, 4, 3\} = 5. \text{ Initial process for proposed algorithm shows table 4.}$$

Table 4: Initial process

Destination Origin → ↓	D ₁	D ₂	D ₃	Supply (p _i)	μ
O ₁	3 5	4 2	5 2	8	5 5
O ₂	3 4	5 4	2 3	5	5 4
O ₃	5 3	2 2	2 2	2	5 3
Demand (q _j)	7	4	4		
τ	5 5	5 4	5 3		

Step IV:

One of the cells $c_{11}^h, c_{13}^h, c_{22}^h, c_{31}^h$ had objective values 5, although we have to choose only one cell. Hence, using the suggested method, from chosen cell, for another objective c_{22}^h has maximum cost 4. Now we choose c_{22}^h .

Step V:

Though c_{32}^h has $\min(1+3=4)$ cost, so we make minimum $\{2, 4\}=2$ allocation in the cell c_{32}^h and eliminate the row 3rd in table 4 for which origin O_3 is fulfilled,

Once more, for rows and columns that are left then the value of Q is

$$Q = \max_{1 \leq i \leq m, 1 \leq j \leq n} (\mu_i^h, \tau_j^h), \forall h = \max\{5, 4, 3\} = 5,$$

Then again perform Steps III to V for columns and rows that are not eliminated, then 4 is the second allocation we get is in the cell c_{23}^h and eliminate 3rd column, for which D_3 demand is fulfilled.

Similarly, through repeating the process from step III to V we get allocations for rows and columns that are left, then we get 3rd, 4th and 5th allocations as 2, 1 and 6 in the cells c_{12}^h, c_{21}^h and c_{11}^h respectively, this is shown in table 5. Hence, the effective solution that was found from table 5 is (39, 56).

Table 5: Effective solution for the above algorithm

Destination → Origin ↓	D ₁	D ₂	D ₃	Supply (p _i)	μ
O ₁	3 5 (6)	4 2 (2)	5 2	8	5 5
O ₂	3 4 (1)	5 4	2 3 (4)	5	5 4
O ₃	5 3	1 3 (2)	2 2	2	5 3
Demand (q _j)	7	4	4		
τ	5 5	5 4	5 3		

6. CONCLUSION

From above examples we analysed that our proposed technique gives unique solution. Compromise solution is given by obtained unique solution, which will choose by DM (decision maker). So, there is no requirement to get more than one effective solution. This algorithm can be used easily to solve multi-objective transportation problem (MOTP) on large scale and in less time, better decisions can be made. So, applying it to each decision maker may be more appropriate.

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