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REVIEW ON ALMOST IDEMPOTENT AND MULTIPLICATIVE SUB IDEMPOTENT IN TERNARY SEMIRING

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Abstract

To study Almost Idempotent and Multiplicative Sub Idempotent in Ternary Semiring. We extend the concept of almost idempotent in semiring to Ternary semiring. We discussed, when the additive idempotent and multiplicative idempotent are coinside. In this research work a number equivalent conditions were investigated. We also find some of their interesting results and properties.

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Keywords: Idempotent, Almost Idempotent (AI), Multiplicative Sub Idempotent (MSI), Regular, Ternary Semiring (T.S.R).

1. INTRODUCTION

Initially, let's explore Sen and Bhuniya concept of almost idempotent semiring [10]. The almost idempotent semirings may be considered as a generalization of the idempotent semiring. For more clarity about almost idempotent semiring one who refere[9]. Lehmer[5] introduced the ternary algebraic system, and Lister[6] studied the ternary semiring, a generalization of ternary ring. Dutta and Kar was studied the concept of regular ternary semiring [3].

Furthermore, many researchers investicated the concept in [11, 12]. Vasanthi and Sulochana looked into the fundamental structure of almost idempotent semirings [13, 14]. Some properties of idempotent semirings are discussed in [2, 8]. The area is shaped by the theory of idempotent analysis [4]. Idempotent ternary semirings play a key role in ternary algebraic structure.

In this paper we are interested in almost idempotent ternary semiring in particular, we characterize some special equivalent class of almost idempotent ternary semiring. We discuss the various structure of almost idempotent Ternary semiring in this research.

The main objective of this article almost idempotent element in T and their properties using additive and multiplicative cancellation law. We looked into different ways to characterize almost idempotent ternary semirings.

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2. PRELIMINARIES

Definition 2.1. When a non-empty set T is a semigroup that commutes and addition that satisfies the after requirements, it can be considered a T.S.R. This is achieved by combining a binary operation called addition with a Ternary multiplication symbolized by juxtaposition

(i) $mno \in T$,

(ii) $[mno]pq = m[nop]q = mn[opq]$,

(iii) $[m + n]op = mop + nop$,

(iv) $m[n + o]p = mnp + mop$,

(v) $mn[o + p] = mno + mnp$ $for\ all\ m,n,o,p,q \in T$

Example 2.1. Let the set of all negative integers be Z^-. subsequently combining Ternary multiplication and Binary addition [ ] defined by

$[mno] = mno$ For all $m,n,o \in Z^-, Z^-$ forms a T.S.R.

Definition 2.2. If $m + m^3 = m^3$, then an element 'm' of a T.S.R, T is considered AI. A T.S.R is AI if all of the elements in T are AI.

Example: Let $T = 0, a, 1$ be set. Define Binary operation '+' and '.' as follows

+	0	A	1
0	0	A	1
a	a	A	a
1	1	A	0

·	0	a	1
0	0	0	0
a	0	a	a
1	0	a	1

Definition 2.3. For any m and n in T, if $m + n = m(m + n = n)$, subsequently a Ternary Semigroup $(T, +)$ becomes a left (right) singular.

Definition 2.4. For any element 'm' in a Ternary semiring T, $m + m = m(m^3 = m)$indicates that it is additively idempotent (multiplicatively idempotent)

Definition 2.5. If $m + m^3 = m$, then a member 'm' of a T.S.R in T is regarded to be a MSI.

Definition 2.6. If $mn^2 = m$ $(n^2 m = m, nmn = m)$ for all $m, n \in T$, then a Ternary semiring $(T, .)$ is considered left (right, lateral) singular. If a ternary semiring $(T, .)$ is left, right, and lateral singular, it is referred to as singular.

Definition 2.7. If $m + m^3 = m$ and $m = m + m$ for all $m \in T$, then $(T, +, .)$ viterbi Ternary Semiring, according to the statement.

Definition 2.8. For any $m, n \in T$, a T.S.R $(T, +, .)$ is deemed Ternary Sub idempotent if $m + m^3 + mnm = m$.

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Definition 2.9. For a T.S.R T, an element 'm' is defined as (additive regular) if there exist $n \in T \ni m = mnmnm$ ($m = m + n + m$, for some n is additive 1-inverse of m)

3. AI AND MSI IN TERNARY SEMIRING

Lemma 3.1: If $m \in T$ almost Idempotent then 'm' is additive Idempotent

Proof : Since m is AI $m + m^3 = m$. Using the same condition and additive cancellation law we have $m + m = m$. Therefore 'm' is additive idempotent.

Remark 3.1. Converse need not to be true in Lemma (3.1), which is illustrated in the following example:

Example 3.1. $T = \{i, j, k,\}$ with following expansions which is Ternary Semiring Lemma (3. 1)

+	i	j	k
i	i	j	i
j	j	j	k
k	i	k	k

·	i	j	k
i	j	i	k
j	j	i	j
k	i	j	k

'i' is additive idempotent but not almost Idempotent in T.

Lemma 3.2. If $m \in T$ Multiplicatice Sub Idempotent then 'm' is additive Idempotent.

Proof: Since $m \in T$ is Multiplicative Sub Idempotent

$$m + m^3 = m$$

Using the same conditions and additive cancellation law in both sides we have $m + m = m$

∴ 'm' is additive Idempotent.

Lemma 3.3. $m \in T$ is multiplicative Sub Idempotent, then these are equal

(i) 'm' is additive Idempotent

(ii) 'm' is multiplicative Idempotent

Proof: (i) ⇒ (ii) since m is additive Idempotent $m + m = m$,

$$m + m + m^3 = m + m$$

(ii)⇒ (i) Since m is multiplicative Idempotent $m^3 = m$

$$m^3 = m + m^3$$
$$m + m = m$$

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Theorem 3.1 For $m \in T$ is almost idempotent if and only if 'm' is multiplicative sub idempotent

Proof : Since $m + m^3 = m^3$

By Lemma 3.1, we have $m + m(m+m)m = m^3$

$$m + m^3 + m^3 = m^3$$

$$m + m^3 = m^3$$

$\therefore$ m is multiplicative Sub Idempotent $\forall\, m \in T$

Conversely, By Lemma 4.3, we obtain $m^3 + m^3 = m^3$

Therefore 'm' is almost Idempotent for all $m \in T$

Theorem 3.2 If $m \in T$ is almost idempotent element in T.S.R, then these are equal

(i) 'm' is additive idempotent

(ii) 'm' is multiplicative idempotent

Proof: Let $m \in T$ is an almost idempotent $m + m^3 = m^3$
(i)⇒(ii) since 'm' is additive idempotent

We have $m + m^3 = m^3$

$$m + m + m^3 = m(m+m)m$$

Using additve cancellation and additive idempotent we have $m = m^3$

Therefore 'm' is multiplicative Idempotent.

From (ii) ⇒(i) Let 'm' is multiplicative idempotent $m = m^3$

$$m + m = m$$

$\therefore$ 'm' is additive Idempotent.

We give the converse part of theorem (4.9) in [13] p.299 as following:

Theorem 3.3. Let $(T, +, .)$ be almost idemotent T.S.R then these are equivalent

(i) T is Viterbi Ternary Semiring

(ii) $(T, .)$ is Idempotent

Proof: (i) ⇒(ii) From theorem (4.9) in [13] page.299

(ii)⇒(i)

Straight forward theorem (3.2)

Theorem 3.4. Let $(T, +, .)$ be multiplicative sub Idempotent T.S.R then these are equivalent

(i) 'T' is Viterbi Ternary Semiring

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(ii) 'T' is multiplicative Idempotent

Proof: From Theorem (3.1) which is obvious.

Proposition 3.1. Let $m \in T$ is an AI and Additive left Singular then 'm' is additive regular.

Proof: Since $m + n = m$ and AI we have $m + n + m + m^3 = m^3$

Therefore 'm' is additive regular.

Proposition 3.2. Let $m \in T$ is an almost idempotent and is additive regular then 'm' is additive left Singular

Proof: Since Definition of AI and additive regular we have,

$$m + n + m + m^3 = m^3$$

$$m + n + m^3 = m + m^3$$

$$m + n = m$$

Therefore 'm' is additive left singular.

Corollary 3.1. Suppose $m \in T$ almost Idempotent and additive regular in Ternary Semiring then an element 'm' is additive idempotent.

Proof: Since 'm' is A.I and $m + n + m = m,$

$$m + n + m + m^3 = m^3$$

$$m + n + m + m^3 + m^3 = m^3$$

$$m + m + m^3 = m + m^3$$

∴ 'm' is additive idempotent.

Corollary 3.2. Consider T almost idempotent T.S.R. If $(T, +)$ is left singular them T is additive Idempotent.

Proof: Let 'm' is almost idempotent and additive left singular in T we obtain 'm' is additive idempotent therefore $(T, +)$ is additive idempotent.

Remark 3.2. In similar manner we can prove for additive right singular also.

Theorem 3.5. Suppose that $m \in T$ is additive and multiplicative Idempotent in Ternary Semiring iff 'm' is almost Idempotent.

Proof: Since 'm' is additive idempotent $m + m = m$ and multiplicative idempotent $m^3 + m$ we have 'm' is almost idempotent.

Conversely,

Let 'm' is AI we have 'm' is additive idempotent

To prove 'm' is multiplicative idempotent

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Since 'm' is almost idempotent and also using additive idempotent we obtain 'm' is multiplicative idempotent.

Theorem 3.6. Let $m \in T$ is almost Idempotent and multiplivative Idempotent in Ternary Semiring $\forall\ m \in T$ and $n \geq 1$ then the followring holds:

(i) $m + m^{2n+1} = m$

(i(a)) $m^{2n+1} + m = m$

(ii) $m^{2n+1} = m$

Proof: (i) since 'm' is AI and Idempotent we have $m + m^3.m^2 = m$

$m + m^5 = m$ --------------------------------(3.1)

In similar manner $m + m^7 = m$, in general we have $m + m^{2n+1} = m$, where $n \in N$

(i(a)) Which is obvious from (i)

ii) Form Eqn (3.1) we have $m + m^5 = m^3$

$\Rightarrow m + m^5 = m + m + m^3$

$\Rightarrow m^5 = m^3 = m$

$\Rightarrow m^5 = m$

$\Rightarrow m^7 = m^9 = m^{11} \ldots m^{2n+1} = m$

Hence $m^{2n+1} = m,$ where $n \in N$

Theorem 3.7. If T is almost Idempotent T.S.R, then these are equal

(i) 'T' is additive Left Singular

(ii) 'T' is additive Right Singular

Proof: (i) ⇒ (ii)

Let $m \in T$ is left singular, Now, $m + n = m$ for some $n \in T$

By the condition T is left singular and almost idempotent we have

$\Rightarrow m + n + m + m^3 = m + m + m^3$ we obtain

Therefore T is additive Right singular

(ii)⇒(i) T is additive Right singular and almost idempotent we have

$\Rightarrow n + m + n + m^3 = n + m + m^3$ we obtain 'T' is additive left singular.

Theorem 3.9. Let T is almost Idempotent Ternary Semiring, then these are equal

(i) T is additive Left Singular

(ii) $(T, +, .)$ is Ternary Sub Idempotent

Proof : (i)) ⇒(ii)

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Since definition (2.2) and (2,3) we have $m + \; m^3 \; = \; m^3 + mnm$

$$m^3 \;\; = \;\; m^3 + mnm \text{ --------------------------------(3.2)}$$

$$m + \; m^3 + mnm + mnm = \;\; m^3 + mnm \; [from \; 3.2]$$

$$m + m^3 + m(n + m)m = m^3$$

$$m + m^3 + mnm = m$$

$\therefore T$ is Ternary sub Idempotent.

(ii))⇒(i)

Let 'm' is Ternary sub idempotent we have

$$m^3 + mnm = m \text{ --------------------------------(3.3)}$$

$$m + m^3 + mnm = m^3 + mnm \;\; [from \; Equation \; 3.3]$$

$$m \;\; + m + m \;\; = \;\; m + m^3 + mnm$$

$$m + m \;\; = \;\; m^3 + mnm$$

$$m^3 + mnm + \;\; m = \;\; m^3 + mnm \;\; [from \; Equation \; 3.3]$$

$$mnm + m = mnm$$

$$mnm + m^3 + mnm = mnm \; [from \; Equation \; 3\,.3]$$

$$m \;\; (n + \;\; m + n) \;\; m = mnm$$

$$n + m \;\; + n = \;\; n \text{ --------------------------------(3.4)}$$

$$n + m + m^3 + mnm + n = n$$

$$n + m \;\; + m + n = n \; [from \; Equation \; 3.3]$$

$$n \;\; + m + \;\; m + n \;\; = \;\; n + \;\; m \;\; + n \; [from \; Equation \; 3.4]$$

$$m + \;\; m \;\; + n = \;\; m + n$$

$$m + m + n \;\; = \;\; m + n + m + n$$

$$m + m \;\; = \;\; m + \;\; n + \;\; m$$

$$m \;\; = \;\; m + \;\; n$$

Therefore $(T, \;\; +)$ is left singular.

Theorem 3.10. Let T is multiplicative sub idempotent T.S.R, then these are equivalent

(i) T is additive Left Singular

(ii) $(T, +, \; .)$ is Ternary Sub Idempotent

Proof : From Theorem (3.1) which is obvious.

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Proposition 3.3. Let T be a almost Idempotent and $(T, +)$ is left singular then $(T, .)$ is Idempotent

Proof. (i) since $m + m^3 = m^3$ and $m + n = m$

By Corollary 3.2, we have

$$m + m(m + m)m = m^3$$

$$m^3 + m^3 = m + m^3 \text{ --------------------------------(3.5)}$$

$$m^3 + m^3 = m + m^3$$

$\therefore$ $(T, .)$ is Idempotent.

Remark 3.3. The only if condition need not be true for proposition (3.3) which is illustrated in the following Example (3.2)

Example 3.2. Let $T = \{o, p, q\}$

+	o	p	q
o	o	p	q
p	o	p	q
q	o	p	q

$\cdot$	o	p	q
o	p	p	q
p	p	p	p
q	o	p	q

(i) $o + q = q \neq o$

(ii) $q + o = o \neq q$, but o, q are idempotents.

Theorem 3.11. For $m \in T$ is almost Idempotent then these are similar

(i) 'm' is additive idempotent

(ii) $m^3 + m = m$ for all $m \in T$

Proof : (i) $\Rightarrow$ (ii)

$m + m^3 = m^3$ which implies $m + m(m + m)m = m^3$

Hence $m^3 + m = m^3$

(ii))$\Rightarrow$(i) Take $m^3 + m = m^3$

Hence $m + m = m \quad \forall m \in T$

Theorem 3.12. For $m \in T$ is multiplicative sub idempotent then these are equal

(i) 'm' is additive idempotent

(ii) $m^3 + m = m \ \forall \ m \in T$

Proof: From Theorem (3.1) which is obvious.

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Proposition 3.4. Let $m \in T$ is almost idempotent and $(T, +)$ left singular then $m^3 + m = m$.

Proof: Since m is almost Idempotent and additive left singular we have,

$$m + m \ \ (m + n) \ \ m = \ \ m^3$$

$$m^3 + mnm = \ \ m^3 \text{ --------------------------------(3.6)}$$

$$m^3 \ + \ m(n \ + \ m)m \ = \ m^3$$

$$m^3 + mnm + m^3 \ = \ m^3 \text{ --------------------------------(3.7)}$$

Equation (3.7), we have $m^3 + m^3 = m^3$

Hence, $m^3 + m = m$ for all $m \in T$

Remark 3.4. Suppose $m \in T$, If 'm' satisfying $m^3 \ + m = m$ then

$m + m = m \ \ \forall \ m \in T$.

Remark 3.5. The converse of Proposition (3.4) need not be which can be obvious from Example (3.2)

Theorem 3.13. Let T is almost Idempotent T.S.R and $(T, +)$ is left singular then the following holds

(i) $m^{4n+1} = m, \forall m \in T \ and \ n \geq 1$.

(ii) $m + m^{4n+1} = m, \forall \ m \in T and \ n \geq 1$.

Proof: (i) By Proposition (3.4) we $have \ m + m^3 \ = \ m^3 + \ m^4$

$$m \ = m^5 \text{ --------------------------------(3.8)}$$

$m \ .m^4 = \ m$

$m^5.m^4 = \ m$

$$m^9 = \ m \text{ --------------------------------(3.9)}$$

Generalizing Equation (3.8), Equation (3.9), we obtain $m^{4n+1} = m$.

Hence $m^{4n+1}m, \forall \ \ m \in T, n \geq 1$

(ii) Since 'm' is A1

$$m + (m^3 + m)m^2 = m^3 \ [proposition \ 3.4]$$

$$m + m^5 = m \text{ --------------------------------(3.10)}$$

$$m + (m + m^5)m^4 = m$$

$$m + m^5 + m^9 = m \text{ [from Equation (3.9)]}$$

$$m + m^9 = m \text{ --------------------------------(3.11)}$$

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Generalizing Equation (3.10) and (3.11) we obtain

$$m + m^{4n+1} = m$$

Hence, $m + m^{4n+1} = m, \forall\, m \in T\ and\ n \geq 1$

Theorem 3.14. Let T be an almost Idempotent T.S.R and additive right singular then multiplicative singular

Proof: (i) since 'm' is AI and 'n' is additive right singular

$m + mnm + m^3 = m^3$

$mnm + m + m^3 = m^3$

$mnm + m = m$

$mnm + m = n + m$

$mnm = n$

Therefore $(T, .)$ is lateral singular.

In similar manner we can prove (ii) and (iii)

Theorem 3.15. Let T be a multiplicative sub T.S.R and additive right singular then multiplicative singular

Proof. From Theorem (3.1) which is obvious.

We already discussed the equivalent for 'm' is almost idempotent in (3, 9). Here we add one more condition that regular we have the following.

Theorem 3.16. Let $m \in T$ is almost Idempotent and regular, $n \in m\{1\}$ the subsequent are equal

(i) 'm' is additive idempotent

(ii) 'm' is lateral singular

(iii) 'm' is idempotent

Proof: since $m \in T$ is almost idempotent and regular which implies

$m + m^3 = m^3\ and\ mnmnm = m$ for some $n \in m\ \{1\}$

(i))⇒(i)

$$mnmnm + mmm = mmm$$

$$m(nmn + m)m = mmm$$

$$nmn + m = m + m$$

$$nmn = m$$

Therfore 'm' is lateral singular.

(ii))⇒(iii))

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Consider $nmn = m$

$$nmnmnmn = m$$

$$mmm = m$$

Therfore 'm' is idempotent.

(iii))⇒(i)

Since $m^3 = m$ and $m + m^3 = m^3$

$$m + m + m^3 = m + m^3$$

Therfore 'm' is additive idempotent.

Theorem 3.17. Let $m \in T$ Multiplicative sub idempotent and regular, $n \in m$ {1} then these are similar

(i) $m + m = m$

(ii) $nmn = m$

(iii) $m^3 = m$

Proof: From Theorem (3.1) which is obvious.

Theorem 3.18. For $m \in T$ is almost idempotent then these are equal

(i) $n + m = n$

(ii) $mnm = m$

(iii) $m^2 n = m$

(iv) $nm^2 = m \; \forall m, n \in T$

Proof : (i))⇒(ii)

Since 'm' is AI and $n + m = m$ we have $m + mnm + m^3 = m^3$

$$mnm + m^3 = m + m^3$$

$$mnm = m$$

Conversely,

Consider $mnm = m$, since 'm' is AI we obtain$n + m = m$

Similaely we can proved (i))⇒(iii) and (i))⇒((iv).

Theoren 3.19. For $m \in T$ is multiplicative sub idempotent then the followring are equivalent

(i) $n + m = n$

(ii) $mnm = m$

(iii) $m^2 n = m$

(iv) $nm^2 = m \; \forall \; m, n \in T$

Proof. From Theorem (3.1) which is obvious.

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Theorem 3.20. Let T be almost Idempotent T.S.R and additive right singular the subsequent are equal

(i) $mnmnm = m, \forall n \in m\{1\}$

(ii) $mnm = m\ for\ all\ n\ \in m\{1\}$

Proof. Since 'm' is AI and right singular (i))⇒(ii)

Consider $mnmnm = m$

$$m^3nm + mnmnm = m$$

$m^3nm = n$ --------------------------------(3.12)

$$(m + m^3)nm = n$$

$$mnm + m^3nm = n$$

$mnm + n = n\ (from(3.12)$

(ii))⇒(i)

Which is obvious.

Theorem 3.21. Let T be multiplicative sub Idempotent T.S.R and additive right singular then these are equal

(i) $mnmnm = m, n$ is 1 inverse of m

(ii) $mnm = m, n$ is 1 inverse of m

Proof: From Theorem (3.1) which is obvious.

Corollary 3.3. Consider T is almost Idempotent T.S.R and additive Right singular then T is regular.

Remark 3.6. In the above Corollary (3.3) converse need not to be true in the following example (3.2) put $m = \sigma, n = q$, we have $(T, +)$ is not right singular.

Theorem 3.22. If $m \in T\ is$ a multiplicative sub idempotent in a ternary semiring then $m + m^{2n+1} = m$ for all $m \in T$ and $n \geq 1$

Proof: Since m is multiplicative sub idempotent we have

$m\ +\ (m + m^3)\ m^2\ =\ m$

$m + m^5 =\ m$ --------------------------------(3.13)

$m + m^5 + m^7 = m\ +\ m^3$

$m + m^7 =\ m$ --------------------------------(3.14)

In general Equation (3.13), (3.14) we have the following $m + m^{2n+1} = m$ for all $m \in T\ and\ n \geq 1$

4. CONCLUSION

In this research work, Almost idempotent and multiplicative sub idempotent in ternary semirings are defined and their properties have been obtained. The charaterization

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of Idempotent may be extended to Natural Idempotent ternary semirings.

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