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EVEN HAMMING DISTANCE LABELING OF SNAKE GRAPHS

E.ESAKKIAMMAL *
Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai. India.
* Corresponding Author Email: esakkiammal2682@gmail.com

K.THIRUSANGU
Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai. India.
Email: kthirusangu@gmail.com

S.SEETHALAKSHMI
Department of Mathematics, R.V. Govt. Arts College, Chengalpattu, Chennai, India.
Email: seetha0687@gmail.com

Abstract

A function $f: V \to N \cup \{0\}$ is said to be even hamming distance labeling if there exist an induced function $f^* : E \to \{2,4,6, \dots n\}$ such that for every $uv \in E, f^*(uv) = hd([f(u)]_2, [f(v)]_2)$ satisfies the following conditions: (i) For every vertex $v \epsilon V$, the set of all edges incident with v receive distinct even numbers as labels. (ii) For every edge $e = uv$, the adjacent vertices u and v receive distinct labels. The even hamming distance number of a graph G is defined as the least positive integer n such that $2^n - 1 \geq k$, where $k = \max\{f(v)/v \in V\}$ and is denoted by $\eta''_{hd}(G)$. In this paper we obtain the even hamming distance number of Triangular Snake graph, Alternate Triangular Snake, Quadrilateral Snake and Alternate Quadrilateral Snake.

Keywords: Even Hamming Distance Labeling, Even Hamming Distance Number, Triangular Snake Graph, Alternate Triangular Snake, Quadrilateral Snake and Alternate Quadrilateral Snake.

1. INTRODUCTION

A vertex labeling of a graph G is an assignment f of labels to the vertices of G that induces a label for every edge uv depending on the vertex labels $f(u)$ and $f(v)[1]$. we introduced the concept of hamming distance labeling, odd hamming distance labeling and even hamming distance labeling. It has been proved that some Path related graphs admit hamming distance [2] and odd hamming distance labeling [3] and some cycle related graphs admit even hamming distance labeling [4]. In this paper, we show that some Snake related graphs admit even hamming distance labeling and their even hamming distance number were obtained. Here the Triangular Snake T_m is obtained from the Path P_m by replacing each edge of a Path by a Triangle C_3[6]. An Alternate Triangular Snake A(T_m) is obtained from a Path P_m in which every alternate edge is replaced by $C_{3.}$ A Quadrilateral Snake Q_m is obtained from a Path P_m by replacing each edge of P_m by a Cycle C_4 and Alternate Quadrilateral Snake A(Q_m) is obtained from a Path P_m, in which each alternate edge of P_m is replaced by a Cycle [5].

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2. EVEN HAMMING DISTANCE LABELLING OF SOME SNAKE GRAPHS

Theorem 2.1. The Triangular Snake graph $T_{m,}$ m ≥ 2, is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(T_m) = 8$.

Proof: Let us consider the Triangular Snake graph T_m with vertex set $V = \{v_i /0 \leq i \leq 2m\}$ and edge set $E = \{\{v_i v_{i+1} /0 \leq i \leq 2m-1\} \cup \{v_0 v_2\} \cup \{v_{2i} v_{2i+2} /1 \leq i \leq m-1\}\}$. Define a function $f: V \to N \cup \{0\}$ to label the vertices of T_m in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for vertex labeling are explained in the following algorithm.

Procedure: Vertex labeling of Triangular Snake graph T_m.

Input: Triangular Snake graph.

$V \leftarrow \{v_i /0 \leq i \leq 2m\}$

$v_0 \leftarrow 1; v_1 \leftarrow 62;$

for $i = 1\ to\ m$ do

$$v_{2i} \leftarrow \begin{cases} 13 & if\ i \equiv 1(mod\ 4) \\ 242 & if\ i \equiv 2(mod\ 4) \\ 2 & if\ i \equiv 3(mod\ 4) \\ 253 & if\ i \equiv 0(mod\ 4) \end{cases}$$

end for

for $i = 1\ to\ m-1$ do

$$v_{2i+1} \leftarrow \begin{cases} 50 & if\ i \equiv 1(mod\ 4) \\ 1 & if\ i \equiv 2,0(mod\ 4) \\ 61 & if\ i \equiv 3(mod\ 4) \end{cases}$$

end for

end procedure

Output: The labeled vertices of T_m graph.

The induced function $f^* : E \to \{2,4,6, \dots n\}$ for the given function f is defined by $f^*(uv) = hd([f(u)]_2, [f(v)]_2)$, where $uv \in E$. Now the induced edge labels are as follows:

$f^*(v_0v_2) = hd([f(v_0)]_2, [f(v_2)]_2) = 2; f^*(v_0v_1) = hd([f(v_0)]_2, [f(v_1)]_2) = 6.$

$f^*(v_1v_2) = hd([f(v_1)]_2, [f(v_2)]_2) = 4.$

For $2 \leq i \leq 2m-1$

Case (i): If $i \equiv 0(mod2); f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = 6.$

Case (ii): If $i \equiv 1(mod2); f^*(v_iv_{i+1}) = hd([f(v_i)]_2, [f(v_{i+1})]_2) = 2.$

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For $1 \leq i \leq m-1$

Case (i): If$i \equiv 1(mod2)$;$f^*(v_{2i}v_{2(i)+2})$ = $hd([f(v_i)]_2, [f(v_{2(i)+2})]_2)$ =8.

Case (ii): If$i \equiv 0(mod2)$;$f^*(v_{2i}v_{2(i)+2})$ = $hd([f(v_i)]_2, [f(v_{2(i)+2})]_2)$ =4.

From all the above cases, all the adjacent edges receive distinct even numbers as labels. Hence the Triangular Snake graph T_m, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(T_m) = 8, m \geq 2$.

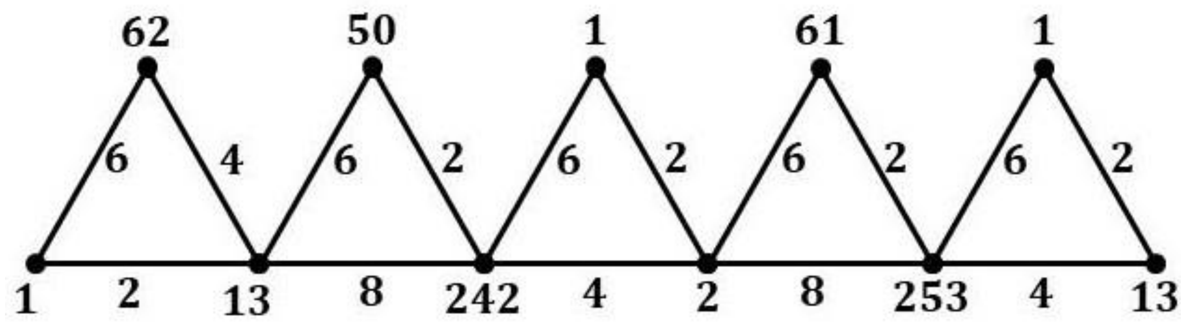

Figure 1: Even Hamming Distance Labeled T_5 graph

Theorem 2.2. The Alternate Triangular Snake graph$A(T_m)$, m ≥ 2, is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(A(T_m)) = 8,$ where m is a positive odd integer and the triangle starts from the first vertex.

Proof: Let us consider the Alternate Triangular Snake graph $A(T_m)$ with vertex set $V = \left\{\{u_i /0 \leq i \leq m\} \cup \left\{v_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor\right\}\right\}$ and edge set $E = \left\{\{u_i u_{i+1} /0 \leq i \leq m-1\} \cup \left\{u_{2i}v_{\frac{2i}{2}}/0 \leq i \leq \lfloor\frac{m}{2}\rfloor\right\} \cup \left\{v_i u_{2i+1} /0 \leq i \leq \lfloor\frac{m}{2}\rfloor\right\}\right\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $A(T_m)$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for vertex labeling are explained in the following algorithm.

Procedure: Vertex labeling of Alternate Triangular Snake graph$A(T_m)$.

Input: Alternate Triangular Snake graph.

$V \leftarrow \left\{\{u_i /0 \leq i \leq m\} \cup \left\{v_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor\right\}\right\}$

$u_0 \leftarrow 1; v_0 \leftarrow 62;$

for $i = 1\ to\ m$ do

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$$u_i \leftarrow \begin{cases} 13 & if\ i \equiv 1(mod4) \\ 242 & if\ i \equiv 2(mod4) \\ 2 & if\ i \equiv 3(mod4) \\ 253 & if\ i \equiv 0(mod4) \end{cases}$$

end for

for $i = 1\ to\ \lfloor\frac{m}{2}\rfloor$ do

$v_i \leftarrow 1$

end for

end procedure

Output: The labeled vertices of $A(T_m)$ graph.

Now the induced edge labels are as follows:

$f^*(u_0u_1) = hd([f(u_0)]_2, [f(u_1)]_2) = 2$; $f^*(v_0u_1) = hd([f(v_0)]_2, [f(u_1)]_2) = 4$.

For i = 0 to $\lfloor\frac{m}{2}\rfloor$; $f^*(u_{2i}v_{\frac{2i}{2}}) = hd([f(u_{2i})]_2, \left[f\left(v_{\frac{2i}{2}}\right)\right]_2) = 6$.

For i = 1 to $\lfloor\frac{m}{2}\rfloor$ do; $f^*(v_iu_{2i+1}) = hd([f(v_i)]_2, [f(u_{2i+1})]_2) = 2$.

For i = 1 to m-1

Case (i): If $i \equiv 1(mod2)$; $f^*(u_iu_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = 8$.

Case (ii): If $i \equiv 0(mod2)$; $f^*(u_iu_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = 4$.

From all the above cases, all the adjacent edges receive distinct even numbers as labels. Hence the Alternate Triangular Snake graph $A(T_m)$, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(A(T_m)) = 8, m \geq 2$.

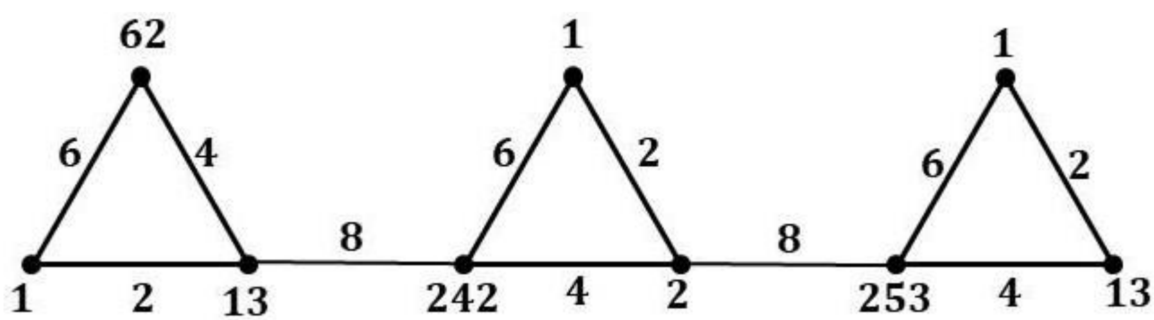

Figure 2: Even Hamming Distance Labeled $A(T_5)$ graph

Theorem 2.3. The Alternate Triangular Snake graph $A(T_m)$, $m \geq 2$, is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(A(T_m)) = 8$, where m is a positive even integer and the Triangle starts from the first vertex.

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Proof: Let us consider the Alternate Triangular Snake graph $A(T_m)$ with vertex set $V = \left\{\{u_i / 0 \leq i \leq m\} \cup \left\{v_i / 0 \leq i \leq \frac{m}{2} - 1\right\}\right\}$ and edge set $E = \left\{\{u_i u_{i+1} / 0 \leq i \leq m-1\} \cup \left\{u_{2i} v_{\frac{2i}{2}} / 0 \leq i \leq \frac{m}{2} - 1\right\} \cup \left\{v_i u_{2i+1} / 0 \leq i \leq \frac{m}{2} - 1\right\}\right\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $A(T_m)$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling the vertices are explained in the following algorithm.

Procedure: Vertex labeling of Alternate Triangular Snake graph $A(T_m)$.

Input: Alternate Triangular Snake graph.

$$V \leftarrow \left\{\{u_i / 0 \leq i \leq m\} \cup \left\{v_i / 0 \leq i \leq \frac{m}{2} - 1\right\}\right\}$$

$u_0 \leftarrow 1; v_0 \leftarrow 62;$

for $i = 1\ to\ m$ do

$$u_i \leftarrow \begin{cases} 13 & if\ i \equiv 1(mod4) \\ 242 & if\ i \equiv 2(mod4) \\ 2 & if\ i \equiv 3(mod4) \\ 253 & if\ i \equiv 0(mod4) \end{cases}$$

end for

for $i = 1\ to\ \frac{m}{2} - 1$ do

$v_i \leftarrow 1$

end for

end procedure

output: The labeled vertices of $A(T_m)$ graph.

Now the induced edge labels are as follows:

$f^*(u_0 u_1) = hd([f(u_0)]_2, [f(u_1)]_2)$ = 2;$f^*(v_0 u_1) = hd([f(v_0)]_2, [f(u_1)]_2)$ = 4.

For i = 1 to $\frac{m}{2} - 1$;$f^*(v_i u_{2i+1}) = hd([f(v_i)]_2, [f(u_{2i+1})]_2)$ =2.

For i = 0 to $\frac{m}{2} - 1$; $f^*(u_{2i} v_{\frac{2i}{2}}) = hd([f(u_{2i})]_2, \left[f\left(v_{\frac{2i}{2}}\right)\right]_2)$ =6.

For i = 1 to m-1

Case (i): If $i \equiv 1(mod2)$;$f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ = 8.

Case (ii): If $i \equiv 0(mod2)$;$f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ = 4.

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From all the above cases, all the adjacent edges receive distinct even numbers as labels. Hence the Alternate Triangular Snake graph $A(T_m)$, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(A(T_m)) = 8, m \geq 2$.

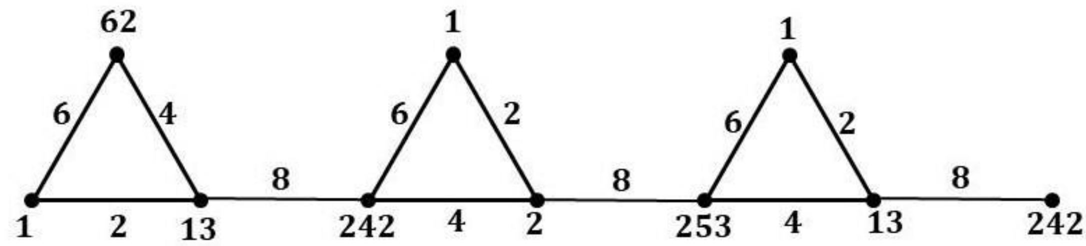

Figure 3: Even Hamming Distance Labeled $A(T_6)$ graph

Theorem 2.4. The Alternate Triangular Snake graph$A(T_m)$, m $\geq$ 2, is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(A(T_m)) = 8$,where m is a positive integer and the triangle starts from the second vertex.

Proof: Let us consider the Alternate Triangular Snake graph $A(T_m)$ with vertex set $V = \left\{\{u_i /0 \leq i \leq m\} \cup \left\{v_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\}\right\}$ and edgeset E= $\left\{\{u_i u_{i+1} /0 \leq i \leq m - 1\} \cup \left\{u_{2i+1}v_{\lfloor\frac{2(i)+1}{2}\rfloor}/0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\} \cup \left\{v_i u_{2i+2} /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\}\right\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $A(T_m)$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for vertex labeling are explained in the following algorithm.

Procedure: Vertex labeling of Alternate Triangular Snake graph $A(T_m)$.

Input: Alternate Triangular Snake graph.

$V \leftarrow \left\{\{u_i /0 \leq i \leq m\} \cup \left\{v_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\}\right\}$

$u_0 \leftarrow 1;$

for $i = 1 \; to \; m$ do

$$u_i \leftarrow \begin{cases} 13 & if \; i \equiv 1(mod4) \\ 242 & if \; i \equiv 2(mod4) \\ 2 & if \; i \equiv 3(mod4) \\ 253 & if \; i \equiv 0(mod4) \end{cases}$$

end for

for i = 0 to $\lfloor\frac{m}{2}\rfloor - 1$

$$v_i \leftarrow \begin{cases} 50 & if \; i \equiv 0(mod2) \\ 61 & if \; i \equiv 1(mod2) \end{cases}$$

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end for

end procedure

Output: The labeled vertices of $A(T_m)$ graph.

Now the induced edge labels are as follows:

$f^*(u_0u_1) = hd([f(u_0)]_2, [f(u_1)]_2)$ = 2.

For i = 0 to $\lfloor\frac{m}{2}\rfloor - 1; f^*(u_{2(i)+1}v_{\lfloor\frac{2(i)+1}{2}\rfloor}) = hd(\left[f\left(u_{2(i)+1}\right)\right]_2, \left[f\left(v_{\lfloor\frac{2(i)+1}{2}\rfloor}\right)\right]_2)$ = 6.

$f^*(v_i\, u_{2i+2}) = hd([f(v_i)]_2, [f(u_{2i+2})]_2)$ = 2.

For i = 1 to m-1

Case (i): If $i \equiv 1(mod2); f^*(u_iu_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ = 8.

Case (ii): If $i \equiv 0(mod2); f^*(u_iu_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ = 4.

From all the above cases, all the adjacent edges receive distinct even numbers as labels. Hence the Alternate Triangular Snake graph $A(T_m)$, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(A(T_m)) = 8, m \geq 2$.

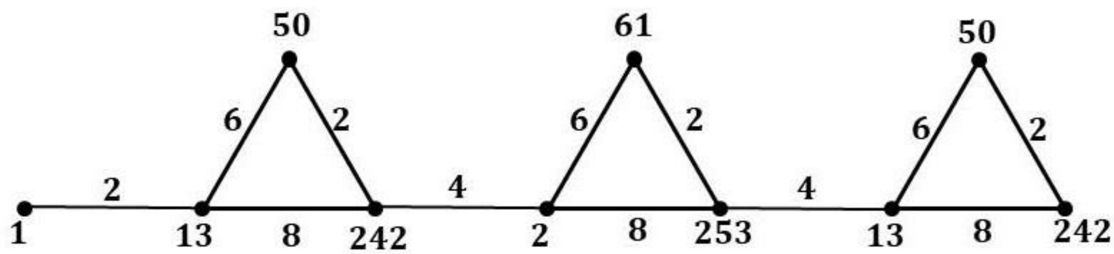

Figure 4: Even Hamming Distance Labeled $A(T_6)$ graph

Theorem 2.5. The Quadrilateral Snake graph $Q_m, m \geq 2$ is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(Q_m) = 8$.

Proof: Let us consider the Quadrilateral Snake graph Q_m with vertex set

$V = \left\{\{u_i \,/\, 0 \leq i \leq m\} \cup \{v_{i,}w_i \,/ 0 \leq i \leq m-1\}\right\}$ and edge set

$E = \left\{\{u_i\, u_{i+1} \,/ 0 \leq i \leq m-1\} \cup \{u_iv_i \,/ 0 \leq i \leq m-1\} \cup \{u_i\, w_{i-1} \,/ 1 \leq i \leq m\} \cup \{v_i\, w_i \,/\right.$ $\left. 0 \leq i \leq m-1\}\right\}$. Define a function $f: V \to N \cup \{0\}$ to label the vertices of Q_m in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling the vertices are explained in the following algorithm.

Procedure: Vertex labeling of Quadrilateral Snake graph Q_m.

Input: Quadrilateral Snake graph.

$V \leftarrow \left\{\{u_i \,/ 0 \leq i \leq m\} \cup \{v_{i,}w_i \,/ 0 \leq i \leq m-1\}\right\}$

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$u_0 \leftarrow 0; v_0 \leftarrow 15;\ w_0 \leftarrow 48;$

for $i = 1\ to\ m$ do

$$u_i \leftarrow \begin{cases} 3 & if\ i \equiv 1(mod\ 4) \\ 60 & if\ i \equiv 2(mod\ 4) \\ 12 & if\ i \equiv 3(mod\ 4) \\ 51 & if\ i \equiv 0(mod\ 4) \end{cases}$$

end for

for $i = 1\ to\ m - 1$ do

$$v_i \leftarrow \begin{cases} 252 & if\ i \equiv 1(mod\ 4) \\ 0 & if\ i \equiv 0,2(mod\ 4) \\ 243 & if\ i \equiv 3(mod\ 4) \end{cases} ;\ w_i \leftarrow \begin{cases} 195 & if\ i \equiv 1(mod\ 4) \\ 63 & if\ i \equiv 0,2(mod\ 4) \\ 204 & if\ i \equiv 3(mod\ 4) \end{cases}$$

end for

end procedure

Output: The labeled vertices of Q_m graph.

Now the induced edge labels are as follows:

For $0 \le i \le m - 1, f^*(v_i w_i) = hd([f(v_i)]_2, [f(w_i)]_2) = 6$.

Case (i): If $i \equiv 0(mod2)$

$f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = 2$;$f^*(u_i v_i) = hd([f(u_i)]_2, [f(v_i)]_2) = 4$

Case (ii): If $i \equiv 1(mod2)$

$f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2) = 6$;$f^*(u_i v_i) = hd([f(u_i)]_2, [f(v_i)]_2) = 8$

For $1 \le i \le m$

Case (i): If $i \equiv 1(mod2)$;$f^*(u_i w_{i-1}) = hd([f(u_i)]_2, [f(w_{i-1})]_2) = 4$

Case (ii): If $i \equiv 0(mod2)$;$f^*(u_i w_{i-1}) = hd([f(u_i)]_2, [f(w_{i-1})]_2) = 8$

From all the above cases, all the adjacent edges receive distinct even numbers as labels. Hence the Quadrilateral Snake graph Q_m, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(Q_m) = 8, m \ge 2$.

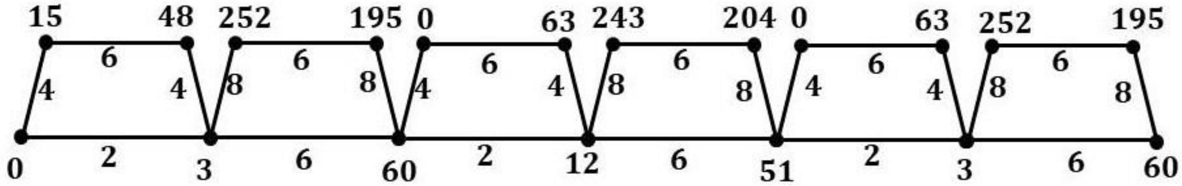

Figure 5: Even Hamming Distance Labeled $A(T_6)$ graph

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Theorem 2.6. The Alternate Quadrilateral Snake graph $A(Q_m), m \geq 2,$ is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(A(Q_m)) = 6,$ where Q_m starts from the first vertex.

Proof: Let us consider the Alternate Quadrilateral Snake graph $A(Q_m)$ with vertex set $V = \left\{\{u_i\,/0 \leq i \leq\ m\} \cup \left\{v_i\,/0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\} \cup \left\{w_i\,/0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\}\right\}$ and edge set $E = \left\{\{u_i\, u_{i+1}\,/0 \leq i \leq\ m-1\} \cup \left\{u_{2i}\, v_{\frac{2(i)}{2}}/0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\} \cup \left\{v_i\, w_i\,/0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\} \cup \left\{w_i\, u_{2i+1}\,/\ 0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\}\right\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $A(Q_m)$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling the vertices are explained in the following algorithm

Procedure: Vertex labeling of Alternate Quadrilateral Snake graph $A(Q_m)$.

Input: Alternate Quadrilateral Snake graph.

$V \leftarrow \left\{\{u_i\,/0 \leq i \leq\ m\} \cup \left\{v_i\,/0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\} \cup \left\{w_i\,/0 \leq i \leq\ \lfloor\frac{m-1}{2}\rfloor\right\}\right\}$

$u_0\ \leftarrow 0; v_0\ \leftarrow 15; w_0\ \leftarrow 48;$

for $i = 1\ to\ m$ do

$$u_i\ \leftarrow \begin{cases} 3 & if\ i \equiv 1(mod\ 4) \\ 60 & if\ i\ \equiv 2(mod\ 4) \\ 12 & if\ i \equiv 3(mod\ 4) \\ 51 & if\ i\ \equiv 0(mod\ 4) \end{cases}$$

end for

for $i = 1\ to\ \left\lfloor\frac{m-1}{2}\right\rfloor$do

$v_i\ \leftarrow 0;\ w_i\ \leftarrow 63;$

end for

end procedure

output: The labeled vertices of $A(Q_m)$ graph.

Now the induced edge labels are as follows:

For $i = 0\ to\ m-1$

Case (i): If $i \equiv 0(mod2)$;$f^*(u_iu_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ =2 .

Case (ii): If $i\ \equiv 1(mod2)$;$f^*(u_iu_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ =6.

For $i = 0\ to\ \lfloor\frac{m-1}{2}\rfloor$;$f^*(u_{2i}\, v_{\frac{2(i)}{2}}) = hd([f(u_{2i})]_2, \left[f\left(v_{\frac{2(i)}{2}}\right)\right]_2)$ =4

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$f^*(v_iw_i) = hd([f(v_i)]_2, [f(w_i)]_2)$ =6;$f^*(w_iu_{2i+1}) = hd([f(w_i)]_2, [f(u_{2i+1})]_2)$ =4

From all the above cases, all the adjacent edges receive distinct labels. Hence the alternate quadrilateral snake graph $A(Q_m)$, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(A(Q_m)) = 6, m \geq 2$.

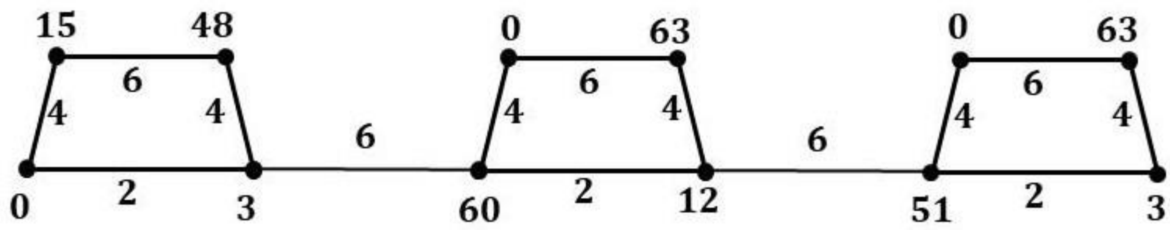

Figure 6: Even Hamming Distance Labeled $A(T_5)$ graph

Theorem 2.7. The Alternate Quadrilateral Snake graph $A(Q_m), m \geq 2$ is an even hamming distance labeled graph and the even hamming distance number is $\eta''_{hd}(A(Q_m)) = 6$, where Q_m starts from the second vertex.

Proof: Let us consider the Alternate Quadrilateral Snake graph $A(Q_m)$ with vertex set

$V = \left\{\{u_i /0 \leq i \leq m\} \cup \left\{v_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\} \cup \left\{w_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\}\right\}$ and edge set

$E = \left\{\{u_i u_{i+1} /0 \leq i \leq m-1\} \cup \left\{u_{2i+1} v_i/0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\} \cup \left\{v_i w_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\} \cup \left\{w_i u_{2i+2} /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\}\right\}$. Define a function $f: V \rightarrow N \cup \{0\}$ to label the vertices of $A(Q_m)$ in such a way that $f(u) \neq f(v)$ for any two adjacent vertices and the procedure for labeling the vertices are explained in the following algorithm

Procedure: Vertex labeling of Alternate Quadrilateral Snake graph $A(Q_m)$.

Input: Alternate Quadrilateral Snake graph.

$V \leftarrow \left\{\{u_i /0 \leq i \leq m\} \cup \left\{v_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\} \cup \left\{w_i /0 \leq i \leq \lfloor\frac{m}{2}\rfloor - 1\right\}\right\}$

$u_0 \leftarrow 0;$

for $i = 1$ *to* m do

$$u_i \leftarrow \begin{cases} 3 & if\ i \equiv 1(mod\ 4) \\ 60 & if\ i \equiv 2(mod\ 4) \\ 12 & if\ i \equiv 3(mod\ 4) \\ 51 & if\ i \equiv 0(mod\ 4) \end{cases}$$

end for

for $i = 0$ *to* $\lfloor\frac{m}{2}\rfloor - 1$do

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$w_i \leftarrow 0;$

$v_i \leftarrow \begin{cases} 12 & if\ i \equiv 0(mod\ 2) \\ 3 & if\ i \equiv 1(mod\ 2) \end{cases}$

end for

end procedure

output: The labeled vertices of $A(Q_m)$ graph.

Now the induced edge labels are as follows:

For $i = 0\ to\ m - 1$

Case (i): If $i \equiv 0(mod2)$;$f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ =2 .

Case (ii): If $i\ \equiv 1(mod2)$; $f^*(u_i u_{i+1}) = hd([f(u_i)]_2, [f(u_{i+1})]_2)$ =6.

For $i = 0\ to\ \lfloor\frac{m}{2}\rfloor - 1$;$f^*(u_{2i+1}\, v_i) = hd([f(u_{2i+1})]_2, [f(v_i)]_2)$ =4.

$f^*(v_i w_i) = hd([f(u_i)]_2, [f(w_i)]_2)$ =2;$f^*(w_i u_{2i+2}) = hd([f(w_i)]_2, [f(u_{2i+2})]_2)$ =4 .

From all the above cases, all the adjacent edges receive distinct even numbers as labels. Hence the Alternate Quadrilateral Snake graph $A(Q_m)$, admits even hamming distance labeling and the even hamming distance number is $\eta''_{hd}(A(Q_m)) = 6, m \geq 2$.

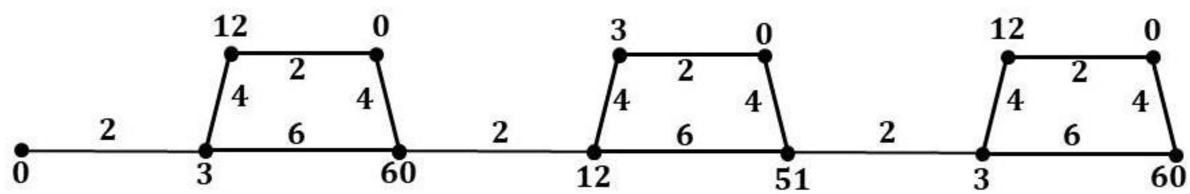

Figure 7: Even Hamming Distance Labeled $A(T_6)$ graph

CONCLUSION

In this paper, we proved the existence of the even hamming distance labeling of some snake related graphs and their even hamming distance number were obtained.

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