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OPTIMIZATION OF MULTI-OBJECTIVE STOCHASTIC SOLID TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS THROUGH WEIBULL DISTRIBUTION

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Abstract

This research focuses on the Solid Transportation Problem characterized by multiple objectives and stochastic parameters with mixed constraints. MSSTPMC (Multi-objective Stochastic Solid Transportation Problem with Mixed Constraints) is a complex logistic optimization challenge arising in various real-world scenarios. This research aims to optimize the time and cost of transportation under probabilistic mixed constraints follows Weibull distribution. MSSTPMC is formulated as a chance constraint programming problem and includes probabilistic constraints to ensure the fulfilment of the Supply, Demand, and Conveyance capacity with specified probabilities. To optimize Multi-objective Optimization Problems, use the Fuzzy Programming Technique and the Global Criteria Method. The effectiveness of the suggested models and techniques for the MSSTPMC under uncertainty is shown by computational results.

Keywords: Multi-objective Stochastic Solid Transportation Problem with mixed constraint (MSSTPMC), Weibull Distribution (WD), Fuzzy Programming Technique, Global Criteria Method, Chance constraint Programming.

1. INTRODUCTION

For transporting goods from various sources to various destinations, the transportation problem plays an essential role. The decision makers may not be aware of the uncertain parameters of transportation problems. The multi-objective stochastic solid transportation problem with mixed constraints (MSSTPMC) is a major problem in current logistics and supply chain management, in which the optimization of multiple conflicting objectives, like transportation cost and transportation time, is further exacerbated by the uncertainty that characterizes real-world systems. In contrast to the previous transportation models, the MSSTPMC includes stochastic parameters of supply, demand, and transport capacity, adding a dimension of complexity that reflects real-world situations across manufacturing to disaster relief supply chains. The transportation problem was first introduced in 1947 [1]. A solution procedure for solving the solid transportation problem is earlier near 1962 [2]. Only when every scalar-valued cost component problem satisfies its prerequisites is the optimal control problem with vector-valued cost also satisfied [3]. The outlines of eight

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distributions of continuous random variables, presenting their admissible regions, probability density functions, cumulative probability functions, parameter estimation, and varied shapes with examples given of their uses [4]. An evolutionary algorithm was suggested that effectively solves the fuzzy solid transportation problem by addressing fuzziness in constraints and identifying a good fuzzy result [5]. The modified Weibull model is expressed as a mixture of uniform and WD [6]. Convert a multi-objective stochastic transportation problem with log-normal random variables into a crisp problem by applying chance-constrained programming and fuzzy programming techniques [7]. This study provides a new method for optimization of MSSTPMC through the Weibull Distribution (WD) to represent probabilistic mixed constraints, which offers a flexible and realistic approach to capture the randomness of logistical parameters. Through the use of chance constraint programming model as a formulation of the problem, this research ensures that the supply, demand, and conveyance capacity needs are met with certain probabilities, thus increasing the solution reliability. Fuzzy programming and the global criteria method are used for providing a balanced approach to optimize multiple objectives, such as cost and time of the problem, and managing the trade-offs between goals. This paper aims to make its contribution to the area by providing an in-depth methodology that not only tackles the stochastic and multi-objective dimension of solid transportation but also illustrates the practical usability of the proposed models through computational outcomes, and proposes a way for more robust and efficient transportation systems under uncertainty.

2. LITERATURE REVIEW

This section includes the literature review of the transportation problem, mixed constraints transportation problem and solid transportation problem. Through stochastic programming and binary variables, the multi-choice stochastic transportation problem was converted into a crisp problem, also illustrated by a numerical example [8]. Obtained an optimal solution of the multi-objective capacitated transportation problem with mixed constraints by using fuzzy programming with linear, exponential, and hyperbolic membership functions [9]. Introduced methods to minimize total fuzzy cost in fully fuzzy fixed charge multi-item STP by handling both balanced and unbalanced cases with fuzzy parameters and decision variables [10]. Developed a STP with mixed constraints in crips, fuzzy and intuitionistic fuzzy programming by using alpha cut sets for defuzzification and also applied genetic algorithm to optimizes the transportation units [11]. Developed a multi-objective capacitated transportation problem with mixed objectives by tackling exact and uncertain inputs, and also demonstrated by a case study [12]. Presented a fuzzy approach with a new exponential membership function to find efficient and compromise results by using LINGO software and illustrated with a numerical example [13]. Introduced a solution procedure by using chance constraint programming and extended fuzzy programming approach to address the MOSTP with gamma distributed random parameters, and also illustrated the model with a numerical example [14]. Formulated a capacitated stochastic transportation problem as multiple objectives optimization model with gamma distribution on supply and demand parameters by using chance constraint

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programming and converted type-2 fuzzy parameters into deterministic form [15]. Proposed a fuzzy methodology by using a hyperbolic membership function to address conflicting objectives in MOSTP with mixed constraints, and also illustrated by a numerical example to demonstrate the function's effectiveness in representing objective satisfaction [16]. Formulated a MOSSTP with Weibull distribution uncertainties in supply, demand, and conveyance capacity through chance constraint programming and applied global criteria and fuzzy programming techniques to obtain an optimal solution of the problem [17]. Based on a hybrid approach of presented a genetic algorithm is presented, using an exponential membership function to optimize the multi-objective stochastic transportation problem by converting stochastic parameters into deterministic values and demonstrating efficacy through a numerical example [18]. Developed three models for the stochastic fuzzy transportation problem by addressing mixed constraints with WD randomness and fuzzy objective function, converting them to deterministic forms by using alpha cut and WD and also did sensitivity analysis for exploring parameter impacts [19]. Explored practical applications of stability theory [20].

Investigated a multi-choice fractional stochastic multi-objective transportation problem with uncertain multi-choice fractional objectives coefficients and normally distributed constraint parameters, using Newton's divided difference interpolation, stochastic programming for crisp transformation, linearization, and fuzzy goal programming [21]. Introduced a linear programming model to optimize warehouse management by analysing [22]. Employed linear programming and dynamic techniques to optimize warehouse resource management [23]. Introduced a novel integrated model for a multi-objective bi-item capacitated transportation problem with fermatean fuzzy multi-choice stochastic mixed constraints, transforming them into deterministic constraints using the alpha-cut technique and improved chance constrained programming method with Newton interpolation and employs the improved global criteria weighted sum method to optimize profits by balancing multiple objectives [24]. Formulated a stochastic solid transportation problem with uncertain supply, demand and conveyance capacity following WD, minimizing transportation cost by chance-constrained programming and alpha cut representation, and presenting four models validated by a numerical example and sensitivity analysis [25].

Applied a multi-dimensional solid transportation problem to optimize a soft drink company's truck distribution in Egypt and minimize fuel costs by considering vehicle capacities, sources and destinations, also achieving a 22% improvement and saving approximately 5 million pounds annually [26]. Proposed a novel solution combining stochastic programming and Pareto distribution to reformulate transportation problems into a tractable mixed-integer optimization model and demonstrate by numerical experiments [27]. Formulated a multi-objective multi-item solid transportation problem with fuzzy parameters like transportation price and transportation time, by using chance constraint modelling to determine the crisp values, and applied techniques to tackle multi-objective programming [28]. Compares recent optimization models for warehouse space allocation and proposed a simple and efficient linear programming model to optimize the objective [29]. Proposed the fuzzy stochastic transportation problem with Lomax

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distribution to optimize hospital length of stay by modelling discharge time uncertainty with the Lomax distribution and reducing bed allocation [30]. Proposed a solution methodology to optimize stochastic solid transportation problem with multiple objectives and multiple items by using gamma distribution [31]. Table 1 shows a comparison of the solution approaches in some current transportation problems in a stochastic environment and the approach that utilized to optimize the MSSTPMC in this article. While all of these prior works placed their reference point in the research society, some challenges exist in those solution methods. The methodology in this paper suggests a straightforward calculation-based transformation of probabilistic constraints into its equivalent deterministic framework and applies approaches to optimize multiple objectives.

Literature review of previous papers is given in mentioned table as follows. Notations used in this paper are presented in Section 3. Formulation of the model for MSSTPMC through chance constraint programming is explained in Section 4. Proposed solution procedure to optimize MSSTPMC models is given in Section 5. Numerical illustrations and comparison of solutions computed by multi-objective optimization techniques are presented in Section 6. Conclusion of the study is given in Section 7. Sections 8 presented the limitation and future scope of the study.

Table 1: Comparison between methodology & methods of various papers

References	Problem's type	Stochastic Parameters	Distribution	Methodology
[7]	Multi-objective stochastic transportation problem	Supply and demand	Log-normal random variables	Chance-constrained programming and fuzzy programming techniques
[9]	Multi-objective capacitated transportation problem with mixed constraints	Supply and demand	Hyperbolic, linear, and exponential programming approach	Fuzzy multi-objective programming
[11]	Solid transportation problem with mixed constraints	Supply, demand and conveyance capacity in fuzzy environment	Interval approximation method,	Meta-heuristic Genetic algorithm
[32]	Multi-choice and stochastic transportation problem	Demand	Galton distribution	Chance constraint programming Newton's divided interpolation
[33]	Multi-objective Linear Fractional Stochastic Transportation Problems	Supply and demand	Normal distribution	Stochastic simulation based Genetic Algorithm
[17]	Multi-objective stochastic solid transportation problem	Supply, demand, and conveyance capacity	Weibull Distribution	Fuzzy goal programming and global criteria method

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[34]	Multi-objective solid transportation problem under stochastic environment	Supply, demand and conveyance capacity	Gamma Distribution	Fuzzy Programming Approach
[24]	Multi-objective Bi-item capacitated transportation problem with fermatean fuzzy multi-choice stochastic mixed constraints	Supply Demand	Normal Distribution	Improved global weighted sum method
[25]	Stochastic solid fuzzy transportation problem with mixed constraints	Supply, demand and conveyance capacity	Weibull distribution	Alpha cut and chance constraint programming
This Research paper	Multi-objective Stochastic Solid Transportation Problem with Mixed Constraints	Supply, demand and conveyance capacity	Weibull distribution	Fuzzy programming and global criteria method

3. NOTATIONS

MSSTPMC involves supply, demand, cost, and conveyance capacity. In this problem, the sources and destinations are treated as mixed constraints. To formulate the mathematical models for MSSTPMC, the following symbols listed below are defined:

i = number of sources for supply in the STP (i=1, 2, …, m)

j = number of possible destinations for demand in the STP (j=1, 2, …, n)

q= number of transportation conveyances (q=1, 2, 3, …, k)

$z^h(x)$ = hth objective function (h=1, 2, 3, …, H)

c_{ijq}^h = unit transportation cost in the hth objective function

x_{ijq} = amount of product shipment from source i to destination j through different transportation modes

a_i = quantity of products supplied, available at the source i

b_j = quantity of products demand at destination j

u_q= the maximum quantity of products that can be shipped through qth mode of transportation

P_{a_i}= probabilities for supply a_i

P_{b_j}= probabilities for demand b_j

P_{u_q}= probabilities for conveyance capacity u_q

μ_{a_i}= shape parameter for supply a_i following WD

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μ_{b_j}= shape parameter for demand b_j following WD

μ_{u_q}= shape parameter for conveyance capacity u_q following WD

β_{a_i}= scale parameter for supply a_i following WD

β_{b_j}= scale parameter for supply b_j following WD

β_{u_q}= scale parameter for conveyance capacity u_q following WD

δ_{a_i}= location parameter for supply a_i following WD

δ_{b_j}= location parameter for demand b_j following WD

δ_{u_q}= location parameter for conveyance capacity u_q following WD

4. FORMULATION OF MODELS FOR MSSTPMC

The Weibull distribution describes long-standing real-life phenomena. Waloddi Weibull, Swedish physicist, examines how statistics can be applied to various problems. Both simple and complex distribution examples are provided. Since then, it has been widely applied in many fields.

The three-parameter WD is an extension of the standard Weibull distribution since it incorporates location parameter (δ) in addition to shape (η) and scale (β) parameters. this extension provides more flexibility toward modelling different kinds of data distributions, particularly in reliability analysis and risk assessment related to MSSTPMC.

Three-parameter WD is followed by a random variable r. The Probability Density Function (PDF) of WD is given by:

$$f(r) = \frac{\mu}{\beta}\left(\frac{r-\delta}{\beta}\right)^{\mu-1} e^{-\left(\frac{r-\delta}{\beta}\right)^{\mu}}, \quad (1)$$

& Cumulative distribution function of WD function is

$$F(r) = 1 - e^{-\left(\frac{r-\delta}{\beta}\right)^{\mu}}, \quad (2)$$

Where$f(r) \geq 0, r \geq 0$ or δ, $\mu > 0, \beta > 0, -\infty < \delta < \infty$. Here μ, β & δ are shape, scale, and location parameters respectively. The WD incorporates the failure rate function, that describes the frequency of failures in engineered components.

This distribution is better in reflecting the randomness of the problem in transportation times or cots, especially when there is a non-zero minimum threshold (represented by δ). By incorporating these parameters into a stochastic modelling framework, decision-makers perform better optimizations of transportation strategies and policies under varying levels of uncertainty and risk.

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4.1. Chance Constraint Programming Model For MSSTPMC.

In the process of converting the mixed constraints of the suggested stochastic programming model to their deterministic equivalents, a closed-form expression for the quantiles of a probability distribution function must be obtained. The availability of a closed-form expression for a quantile from the WD forms another basis for using that distribution. The chance constrained programming model for MSSTPMC is described as bellow:

(P) Minimize $z^h = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k} c_{ijq}^h x_{ijq}, \qquad h = 1, 2, \dots, H,$ (3)

Subject to constraints,

$$P(\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \geq a_i) \geq p_{a_i}, \qquad i \in s_1, \tag{4}$$

$$P(\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = a_i) \geq p_{a_i}, \qquad i \in s_2, \tag{5}$$

$$P(\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \leq a_i) \geq p_{a_i}, \qquad i \in s_3, \tag{6}$$

$$P(\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \geq b_j) \geq p_{b_j}, \qquad j \in t_1, \tag{7}$$

$$P(\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = b_j) \geq p_{b_j}, \qquad j \in t_2, \tag{8}$$

$$P(\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \leq b_j) \geq p_{b_j}, \qquad j \in t_3, \tag{9}$$

$$P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \geq u_q) \geq p_{u_q}, \qquad q \in v_1, \tag{10}$$

$$P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = u_q) \geq p_{u_q}, \qquad q \in v_2, \tag{11}$$

$$P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \leq u_q) \geq p_{u_q}, \qquad q \in v_3, \tag{12}$$

$x_{ijq} \geq 0,$ for every $i = 1\ to\ m,\ j = 1\ to\ n$ and $q = 1\ to\ k$ (13)

Where p_{a_i}, p_{b_j} and p_{u_q} are given probabilities. Random variables such as supply a_i, demand b_j and conveyance capacity u_q parameters follow WD. Parameters of WD were assumed for a_i which have three parameters as shape (μ_{a_i}), scale (β_{a_i}) & location (δ_{a_i}) parameters. Similarly, the parameters were defined for the WD of b_j & u_q. The probabilistic constraints for the supply a_i form source i are represented by constraints (4) to (6). These are defined, with a specified probability P_{a_i}, the total quantities of goods shipped from supply source i that must be allocated as either exactly, at least, or at most a_i units. Partitions of $i(i = 1,2,3 \dots, m)$ are s_1, s_2 and s_3. Similarly, the probabilistic

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constraints for the demand b_j at destination j are represented by constraints (7) to (9). These are defined, with a specified probability P_{b_j}, the total quantities of goods needed at the destination j that must be received as either exactly, at least, or at most b_j units. Partitions of $j(j = 1,2,3 \ldots, n)$ are t_1, t_2 and t_3. Constraints (10) to (12) relate to the desired conveying capacity u_q for q transportation modes. These are defined, with a specified probability P_{u_q}, the total quantities of goods needed to be transported from origin to destination through different modes of transportation which either be exactly, at least, or at most u_q units. Partitions of $q(q = 1,2,3 \ldots, k)$ are v_1, v_2 and v_3. For the transportation of goods, the transportation cost is denoted as c_{ijq}.

The objective is to determine the quantity x_{ijq} transported from i (source) to j (destination), while minimizing the overall cost of transportation to meet the mixed constraints of supply, demand, and conveyance capacity. Only one of the random variables $a_i, b_j, or\ u_q$ is treated as uncertain in the first three scenarios. All three random variables are considered uncertain in the fourth scenario.

4.2. Case I- Only Supply a_i (I=1 To M) Is Uncertain

For probabilistic constraint $P(\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \geq a_i) \geq p_{a_i}$, $i \in s_1$, obtained deterministic supply constraint $\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \geq \delta_{a_i} + \beta_{a_i}\{-\ln(1 - P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, i \in s_1$ by using WD with three parameters μ_{a_i}, β_{a_i} & δ_{a_i} in [25].

For probabilistic equality constraint $P(\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = a_i) \geq p_{a_i}$, $i \in s_2$ obtained a deterministic supply constraint $\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = \delta_{a_i} + \beta_{a_i}\{-\ln(1 - P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, i \in s_2$ by using WD with three parameters μ_{a_i}, β_{a_i} & δ_{a_i} in [25].

For probabilistic constraint $P(\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \leq a_i) \geq p_{a_i}$, $i \in s_3$ obtained a deterministic supply constraint $\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \leq \delta_{a_i} + \beta_{a_i}\{-\ln(P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, i \in s_3$ by using WD with three parameters μ_{a_i}, β_{a_i} & δ_{a_i} in [25].

So, for deterministic multi-objective solid transportation problem with mixed constraints can be converted into the stochastic problem by assuming supply constraint is uncertain.

MSSTPMC -A:

Minimize $z^h = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k} c_{ijq}^h x_{ijq}, \qquad h = 1,2,\ldots,H,$ (14)

Subject to constraints,

$$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \geq \delta_{a_i} + \beta_{a_i}\{-\ln(1 - P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, \qquad i \in s_1, \tag{15}$$

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$$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = \delta_{a_i} + \beta_{a_i}\{-\ln(1-P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, \quad i \in s_2, \tag{16}$$

$$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \le \delta_{a_i} + \beta_{a_i}\{-\ln(P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, \quad i \in s_3, \tag{17}$$

$$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \ge b_j, \quad j \in t_1, \tag{18}$$

$$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = b_j, \quad j \in t_2, \tag{19}$$

$$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \le b_j, \quad j \in t_3, \tag{20}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \ge u_q, \quad q \in v_1, \tag{21}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = u_q, \quad q \in v_2, \tag{22}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \le u_q, \quad q \in v_3, \tag{23}$$

$$x_{ijq} \ge 0, \text{ for every } i, j \text{ and } q, \tag{24}$$

4.3. Case II-Only Demand b_j (J=1 Ton) Is Uncertain

For probabilistic constraint $P(\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \ge b_j) \ge p_{b_j}$, $j \in t_1$ obtained deterministic constraint $\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \ge \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(1-P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, j \in t_1$ by using WD with three parameters μ_{b_j}, β_{b_j} & δ_{b_j} in [25].

For probabilistic equality constraint $P(\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = b_j) \ge p_{b_j}$, $j \in t_2$ obtained deterministic constraint $\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, j \in t_2$ by using WD with three parameters μ_{b_j}, β_{b_j} & δ_{b_j} in [25].

For probabilistic constraint $P(\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \le b_j) \ge p_{b_j}, j \in t_3$ obtained a deterministic constraint $\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \le \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, j \in t_3$ by using WD with three parameters μ_{b_j}, β_{b_j} & δ_{b_j} in [25].

Hence, the multi-objective stochastic solid transportation problem with mixed constraints can be formulated by assuming the demand constraint is uncertain.

MSSTPMC -B:

$$\text{Minimize } z^h = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k} c_{ijq}^{h} x_{ijq}, \quad h = 1,2,\dots,H, \tag{25}$$

Subject to constraints,

$$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \ge a_i, \quad i \in s_1, \tag{26}$$

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$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = a_i, \qquad i \in s_2,$ (27)

$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \leq a_i, \qquad i \in s_3,$ (28)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \geq \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(1 - P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, \quad j \in t_1$ (29)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, \qquad j \in t_2$ (30)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \leq \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, \qquad j \in t_3$ (31)

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \geq u_q, \qquad q \in v_1,$ (32)

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = u_q, \qquad q \in v_2,$ (33)

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \leq u_q, \qquad q \in v_3,$ (34)

$x_{ijq} \geq 0,$ for every i, j *and* $q,$ (35)

4.4. Case III-Only Conveyance Capacity u_q (Q=1 To K) *Is* Uncertain

For probabilistic constraint $P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \geq u_q) \geq p_{u_q},\ q \in v_1$ obtained deterministic constraint $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \geq \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(1 - P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, q \in v_1$ by using WD with three parameters μ_{u_q}, β_{u_q} & δ_{u_q} in [25].

For probabilistic equality constraint $P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = u_q) \geq p_{u_q},\ q \in v_2$ obtained deterministic constraint $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, q \in v_2$ by using WD with three parameters μ_{u_q}, β_{u_q} & δ_{u_q} in [25].

For probabilistic constraint $P(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \leq u_q) \geq p_{u_q}, q \in v_3$ obtained deterministic constraint $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \leq \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, q \in v_3$ by using WD with three parameters μ_{u_q}, β_{u_q} & δ_{u_q} in [25].

Hence, a deterministic multi-objective stochastic solid transportation problem with mixed constraints is given by assuming the demand constraint is uncertain.

MSSTPMC -C:

Minimize $z^h = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k} c_{ijq}^{h} x_{ijq}, \qquad h = 1,2,\dots,H,$ (36)

Subject to constraints,

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$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \geq a_i, \qquad i \in s_1,$ (37)

$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = a_i, \qquad i \in s_2,$ (38)

$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \leq a_i, \qquad i \in s_3,$ (39)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \geq b_j, \qquad j \in t_1,$ (40)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = b_j, \qquad j \in t_2,$ (41)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \leq b_j, \qquad j \in t_3,$ (42)

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \geq \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(1 - P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, \qquad q \in v_1$ (43)

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, \qquad q \in v_2$ (44)

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \leq \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, \qquad q \in v_3$ (45)

$x_{ijq} \geq 0$, for every i, j *and* q, (46)

4.5. Case IV- Supply a_i, Demand b_j and Conveyance Capacity u_q are Uncertain

In this case, we have assumed that all three independent random variables supply a_i, demand b_j and conveyance capacity u_q following the WD. The deterministic mathematical model for MSSTPMC is obtained by combining the above three cases and using the quantile of the WD.

MSSTPMC -D:

Minimize $z^h = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k} c_{ijq}^{h} x_{ijq}, \qquad h = 1,2,\dots,H,$ (47)

Subject to constraints,

$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \geq \delta_{a_i} + \beta_{a_i}\{-\ln(1 - P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, \qquad i \in s_1,$ (48)

$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} = \delta_{a_i} + \beta_{a_i}\{-\ln(1 - P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, \qquad i \in s_2,$ (49)

$\sum_{j=1}^{n}\sum_{q=1}^{k} x_{ijq} \leq \delta_{a_i} + \beta_{a_i}\{-\ln(P_{a_i})\}^{\frac{1}{\mu_{a_i}}}, \qquad i \in s_3,$ (50)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \geq \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(1 - P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, \qquad j \in t_1$ (51)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} = \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, \qquad j \in t_2$ (52)

$\sum_{i=1}^{m}\sum_{q=1}^{k} x_{ijq} \leq \delta_{b_j} + \beta_{b_j}\left\{-\ln\left(P_{b_j}\right)\right\}^{\frac{1}{\mu_{b_j}}}, \qquad j \in t_3$ (53)

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$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \geq \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(1 - P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, \quad q \in v_1 \tag{54}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} = \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, \quad q \in v_2 \tag{55}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijq} \leq \delta_{u_q} + \beta_{u_q}\left\{-\ln\left(P_{u_q}\right)\right\}^{\frac{1}{\mu_{u_q}}}, \quad q \in v_3 \tag{56}$$

$$x_{ijq} \geq 0, \text{for every } i, j \text{ and } q, \tag{57}$$

5. SOLUTION PROCEDURE FOR THE MSSTPMC: A MULTI-OBJECTIVE OPTIMIZATION PERSPECTIVE

This section includes the solution procedure for optimizing MSSTPMC, which follows WD.

Step 1: Problem (P) converted to 4 models based on its constraint.

Step 2: Formulate the model MSSTPMC-A (14) to (24), where the a_i supply constraint is an uncertain, demand constraint b_j and conveyance capacity u_q remains certain. We use WD to convert an uncertain probabilistic supply into a deterministic (15) to (17).

Step 3: Then, for the formulation of model MSSTPMC-B (25) to (35), where the b_j demand constraint is an uncertain, supply constraint a_i and conveyance capacity u_q remains certain. We use WD to convert an uncertain probabilistic demand into a deterministic (29) to (31).

Step 4: To form the model MSSTPMC-C (36) to (46), where the u_q conveyance capacity is an uncertain, supply constraint a_i and demand b_j remains certain. We use WD to convert an uncertain probabilistic conveyance capacity into a deterministic constraint (43) to (45).

Step 5: For the deterministic model MSSTPMC-D (47) to (57), where the b_j demand constraint, supply constraint a_i and conveyance capacity u_q taken as uncertain. We use WD to convert all uncertain probabilistic constraints into deterministic constraints (48) to (56).

Step 6: Various methods have existed to obtain the compromise solutions for such types of models. This study implements two approaches, the fuzzy programming technique and the global criteria method, to generate solutions for all these models of the multi-objective stochastic solid transportation problem with mixed constraints.

5.1. Fuzzy Programming Technique

Decision-makers need to optimize conflicting objectives simultaneously in multi-objective programming. We require a compromise or PO solution because of the nature of conflicting objectives, not all objectives have a single optimal solution. Various methods like the weighted sum approach, goal programming, ∈-constraint method, fuzzy programming approach, fuzzy goal programming technique, and global criteria method

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are given in the above literature. Except for the fuzzy programming technique, all of the techniques mentioned in the above require prior knowledge regarding the decision maker's objectives, such as weights and goals to optimize the issue. Hence, we use the fuzzy programming technique for finding a compromise solution. The steps are given below.

Step 1: Obtain an ideal solution of the model by assuming a single objective at once. The Pareto ideal solution point for the deterministic model is formed by a collection of optimal results for all the different objectives.

Step 2: For all the objectives, form the pay-off matrix in the following manner:

$$\text{Pay-off matrix} = \begin{bmatrix} .. & z^1 & z^2 & z^3 & \ldots & z^h \\ x_{ijq}^1 & z^1(x_{ijq}^1) & z^2(x_{ijq}^1) & z^3(x_{ijq}^1) & \ldots & z^h(x_{ijq}^1) \\ x_{ijq}^2 & z^1(x_{ijq}^2) & z^2(x_{ijq}^2) & z^3(x_{ijq}^2) & \ldots & z^h(x_{ijq}^2) \\ x_{ijq}^3 & z^1(x_{ijq}^3) & z^2(x_{ijq}^3) & z^3(x_{ijq}^3) & \ldots & z^h(x_{ijq}^3) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ x_{ijq}^h & z^1(x_{ijq}^h) & z^2(x_{ijq}^h) & z^3(x_{ijq}^h) & \ldots & z^h(x_{ijq}^h) \end{bmatrix},$$

For the hth objective function, x_{ijq}^h; i=1to m; j=1 to n and h=1 to H is the individual optimum solution.

Step 3: Determine lower bound (z_l^h) and upper bound (z_u^h) for each objective function by using the payoff matrix.

Step 4: For each objective, define a function of linear membership as

$$\mu_h(z^h) = \begin{cases} 1, & if\ z^h \le z_l^h, \\ \frac{z_u^h - z^h(x)}{z_u^h - z_l^h}, & if z_l^h \le z^h \le z_u^h, \\ 0, & if\ z^h \ge z_u^h. \end{cases} \quad (58)$$

Step 5: Formulate the equivalent deterministic model as follows:

Max λ (59)

Subject to

$\lambda \le \mu_h(z^h) = \frac{z_u^h - z^h(x)}{z_u^h - z_l^h}$, h=1 to H, (60)

With the problem's initial set of constraints. (61)

5.2. Global Criteria Method

Most of the multi-objective optimization problems are solved by using weighting factors. For determining the weighting elements for each objective, a clear proof of the favouritism shown to a certain target is required. The global criteria method is a multi-objective optimization approach applied when no explicit preference or weighting is placed on objectives, for example, minimizing transportation time and cost in a multi-objective

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transportation problem. In contrast to weighting factor-based methods that necessitate direct prioritization, the global criteria method provides equal priority for all objectives and minimizes a global criterion function, usually in terms of Makowski's Lp-metric (example, p=2 for Euclidean distance). This objective function measures the normalized deviation from ideal results, combining objectives into a single scalar measure. By minimizing the sum of squared relative deviations or distance to individual optimal values, the global criteria method produces compromise results that satisfy all objectives efficiently, making it easy and robust when preferences are ambiguous.

The MSSTPMC model proposed in this study is a multiple-objective optimization model, solved by using a global criteria method; MSSTPMC with WD can be converted into its equivalent crisp form as follows.

Minimize G(x)

S. t. $x \in D,$

$G(x) = Min\{\sum_{h=1}^{H}(\frac{z_h(x)-z_h(x^*)}{z_h(x^*)})^e\}^{\frac{1}{e}}$, h=1, 2..., H, (62)

Subject to,

$z^h(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{k} c_{ijq}^{h} x_{ijq},$ (63)

and constraints (48) to (56),

$x_{ijq} \geq 0,$ for every $i, j \text{ and } q,$ (64)

Where, $z_h(x^*)$ is the q-th objective function's value at its individual optimal x^* and $z_h(x)$ is the h-th objective function. The relative importance of each goal is expressed using the integer exponent $e(1 \leq e \leq \infty)$. When $e = 1$ represents that all deviations have an equal significance. Higher deviations are given more weight when $e = 2$ because the deviations are weighted equally in [3]. For $e > 2$ the greatest deviations are given much more weight.

6. SOLUTION FOR A REAL-WORLD PROBLEM RELATED TO LOGISTIC

To address the complexities of multiple objective transportation problems, supply and demand uncertainties in the box-making industry, a multi-objective optimization technique was utilized to simultaneously minimize the cost and time of transportation. The study aimed to achieve an optimal balance between reducing transportation cost and time while ensuring trucks were adequately capacitated to accommodate varying shipment quantities to avoid underutilization. Using a model that considers stochastic supply and demand patterns, the technique adopted a global criterion approach and fuzzy programming technique to optimize the cost and time objectives simultaneously. These techniques enhanced operational efficiency and increased flexibility to change market demands, justifying the necessity for balanced multi-objective optimization in sustainable logistics for the box-making industry. A company has two supplier industries and four distribution centres. Industry S_1 has a capacity of making at least $a_1$units and industry S_2 has a capacity for making products at most $a_2$units. Similarly, the distribution centre D_1

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has a demand capacity at most b_1 units, the distribution centre D_2 has a demand capacity exact b_2 units and the distribution centre D_3 has a demand capacity at most b_3 units. The box industry supplier offers appropriate transportation means. The issue accounts for two conveyances U_1 and U_2 with different loading capacities: the capacity of 1st conveyance is at least $u_1$units and the capacity of 2nd conveyance is at most u_2 units. Per-unit transportation cost from each industry to each destination for h-th objective function is represented as c_{ijq}^h . The supply of the box-making industry is uncertain because various elements bring variability and unpredictability to the supply, including the availability of raw material, labour availability, defects in raw material, machinery breakdown, and weather conditions. When the supply is unstable, we can simply explain why the availability of manufacturing is probabilistic. Then the probability of having the necessary number of cartons available for (a_1) supply is denoted as P_{a_1}. Similarly, the probability for (a_2) supply is denoted P_{a_2}.

The demand for the box-making industry is uncertain because of fluctuating consumer behaviour, market disruptions, and delays in deliveries. For demand b_1, the probability of anticipated demand will be met P_{b_1}. Similarly for demand b_2 and b_3, probabilities are P_{b_2} and P_{b_1} respectively. Same as above the conveyance's capacities are uncertain due to port congestion and road closures. The probability that two conveyances have available capacities are P_{u_1} and P_{u_2}. Based on forecasting and observations, the decision maker can select these probabilities. Using the information, the distances between all origins and destinations. The Cost of fuel consumption (c_{ijq}^1) per vehicle is given in Tables 2 and cost of transportation per carton is given in table 3. The required transportation time (c_{ijq}^2) per vehicle is given in Tables 4 and time of transportation per carton is given in table 5.

Table 2: Per vehicle transportation cost (in rupees)

	D_1		D_2		D_3	
	U_1	U_2	U_1	U_2	U_1	U_2
S_1	41.0847	36.7599	49.9257	44.6703	10.9212	9.7716
S_2	45.2451	40.4825	54.6061	48.8582	9.3612	8.3757

Table 3: Per carton transportation cost (in rupees)

	D_1		D_2		D_3	
	U_1	U_2	U_1	U_2	U_1	U_2
S_1	0.6847	0.5487	0.8321	0.6667	0.1820	0.1459
S_2	0.7541	0.6042	0.9101	0.7292	0.1560	0.1250

Table 4: Per vehicle transportation time (in seconds)

	D_1		D_2		D_3	
	U_1	U_2	U_1	U_2	U_1	U_2
S_1	1200	1380	1380	1560	420	600
S_2	1380	1560	1560	1740	360	540

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Table 5: Per carton transportation time (in seconds)

	D_1		D_2		D_3	
	U_1	U_2	U_1	U_2	U_1	U_2
S_1	20	20.597	23	23.2836	7	8.9552
S_2	23	23.2836	26	25.9702	6	8.0597

In this section, the nominal values of certain constants supply for this numerical are given as
$a_1 = 500$, $a_2 = 600$, demand as $b_1 = 450$, $b_2 = 250$, $b_3 = 300$, and capacity of conveyance as $u_1 = 60$, $u_2 = 67$. Also, arbitrary probabilities are provided as $P_{a_1} = 0.90$, $P_{a_2} = 0.85$, $P_{b_1} = 0.30$, $P_{b_2} = 0.50$, $P_{b_3} = 0.40$, $P_{u_1} = 0.19$, and $P_{u_2} = 0.20$. Consider WD parameters have different values as $\mu_{a_i} = \mu_{b_j} = \mu_{u_q} = 2$, $\beta_{a_i} = \beta_{b_j} = \beta_{u_q} = 2$, and $\delta_{a_1} = 499, \delta_{a_2} = 597,\ \delta_{b_1} = 448,\ \delta_{b_2} = 249,\ \delta_{b_3} = 294,\ \delta_{u_1} = 57, \delta_{u_2} = 66.$ Obtain a deterministic form of all the probabilistic constraints by using equations (48) to (56).

Four models are formed, based on their constraints through problem (P) by using step 2. For model A only supply constraints are considered as uncertain, demand and conveyance capacity are considered as certain by using step 2. For model B only demand constraints are considered uncertain, supply and conveyance capacity are considered certain by using step 3. For model C, only conveyance capacity constraints are considered as uncertain, supply and demand are considered as certain by using step 4. For model D, supply, demand and conveyance capacity are considered as uncertain by using step 5. By step 6, all models A, B, C and D are solved by using two techniques fuzzy programming and the global criteria method then obtained optimal solution is obtained by Lingo software.

6.1. Computational Solutions by both Techniques

Two procedures are utilized from Section 5 to solve MSSTPMC. In this section, we obtain the optimal solutions of the problem given in Section 6 by using fuzzy programming and global criteria method. Table 6 represents the optimal solutions of the four models which are obtained after using fuzzy programming technique. Table 7 represents the optimal solutions obtains by using global criteria method of the given problem for 4 stochastic models.

Table 6: Optimal solutions through the fuzzy programming technique.

Models	Cost (rupees)	Time (seconds)	Shipment	Flow of units
Model A	255.8424	7811.535	$x_{121} = 216.4999, x_{122} = 33.5001,$ $x_{131} = 252.0349, x_{231} = 47.9651$	550
Model B	255.6126	7796.709	$x_{121} = 217.1652, x_{122} = 33.49988,$ $x_{131} = 249.3349, x_{231} = 46.09455$	546.095
Model C	255.6669	7809.710	$x_{121} = 215.7586, x_{122} = 34.24136,$ $x_{131} = 250, x_{231} = 50$	550
Model D	255.5379	7798.963	$x_{121} = 216.3936, x_{122} = 34.27147,$ $x_{131} = 251.3697, x_{231} = 44.05970$	546.094

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Table 7: Optimal solutions through the global criteria method.

Models	Cost (rupees)	Time (seconds)	Shipment	Flow of units
Model A	250.3359	7820.977	$x_{121} = 183.2024, x_{122} = 66.79759,$ $x_{131} = 252.0349, x_{231} = 47.96515$	550
Model B	250.1061	7806.512	$x_{121} = 183.8677, x_{122} = 66.79742,$ $x_{131} = 249.3349, x_{231} = 46.09455$	546.095
Model C	250.0481	7819.345	$x_{121} = 181.7818, x_{122} = 68.21823,$ $x_{131} = 250, x_{231} = 50$	550
Model D	249.9055	7808.621	$x_{121} = 182.3347, x_{122} = 68.33042,$ $x_{131} = 251.3697, x_{231} = 44.0597$	546.095

Table 8 contrasts the two methods' optimal solutions for MSSTPMC, emphasizing how they impact the proposed models. It indicates that the global criteria method tends to have lower transportation costs than the fuzzy programming techniques. The global criteria method provides quicker transportation time for models B and D than the fuzzy programming technique.

Transportation time through fuzzy programming technique is shorter for the other two models A and C than global criteria method. Interestingly, although the overall shipments to satisfy demand is the same for all models. The optimal values are different based on the solution approach employed.

Table 8: Results comparison

MSSTPMC models	Parameters	Fuzzy programming	Global criteria
Model A	Unit flow (no. of cartons shipped)	550	550
	Cost (rupees)	255.8424	250.3359
	Time (seconds)	7811.535	7820.977
Model B	Unit flow (no. of cartons shipped)	546.095	546.095
	Cost (rupees)	255.6126	250.1061
	Time (seconds)	7796.709	7806.512
Model C	Unit flow (no. of cartons shipped)	550	550
	Cost (rupees)	255.6669	250.0481
	Time (seconds)	7809.710	7819.345
Model D	Unit flow (no. of cartons shipped)	546.094	546.095
	Cost (rupees)	255.5379	249.9055
	Time (seconds)	7798.963	7808.621

7. CONCLUSION

This research has considerably addressed the complexity of the Multi-objective Stochastic Solid Transportation Problem with Mixed Constraints (MSSTPMC) by using the WD to deal with stochastic parameters and developed it as a chance constraint programming problem.

This captures the probabilistic occurrences of supply, demand and capacity of conveyance to fulfil them with desired probabilities.

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The use of the fuzzy programming methodology and global criteria method has been found critical in maximizing the conflicting goals of transportation cost minimization and time savings under mixed constraints.

The computational findings of a numerical example emphasize the feasibility and effectiveness of the proposed models and solution methods, confirming their ability to handle uncertainty in real-world logistics environments.

Remarkably, the comparison of the fuzzy programming and global criteria methods, as shown in the results, makes clear-cut trade-offs, with the global criteria method tending to deliver lower costs and the fuzzy programming technique performing better in minimizing transportation durations for certain models.

These observations reflect the generality of the proposed methods in accepting a variety of logistical needs. This research not only provides a new framework to address MSSTPMC but also makes it easier for further study to generalize such models for other probability distributions or introduce other goals and restrictions, hence broadening their applications to other transportation and logistics problems.

8. LIMITATIONS AND FUTURE SCOPE

The limitations of this research are presented in this section. Only supply, demand, and conveyance capacity are considered as uncertain. But in a real-world scenario, transportation time and cost are also affected because of traffic congestion.

Here, we consider only three parameters to be uncertain, but the delivery of perishable items such as fruits, vegetables, vaccines, and other breakable materials requires various factors influencing total cost and time of transportation, and thus needs to be introduced as parameters with suitable constraints in these models.

Future work for this study, our suggested methodology to solve other complex optimization and logistics problems, such as vehicle routing, warehouse location models, and supply chain system design, where stochastic and mixed constraints are prevalent.

The approach could also be adapted for inventory control systems, particularly those facing uncertain demand, such as economic order quantity models. In addition, it could be useful in data envelopment analysis to evaluate performance in the transportation system when uncertainty exists.

Other probability distributions could be applied, e.g., normal, gamma, and exponential. In addition to the WD to make them more widely applicable to the range of real-world problems. Investigation of more advanced methods of multi-objective optimization, e.g., genetic algorithm or weighted sum method, would enhance solution effectiveness.

Moreover, merging real-time data and dynamic constraints, such as changing demand or external environment, could make the model more robust for realistic logistics use. Extending the framework with sustainability targets, such as reducing carbon emissions or testing it in new industries like drone transportation. Cost and time parameters could also be considered as uncertain.

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