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REACHABILITY DEGREE SUM ENERGY OF GRAPH

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Abstract

Let G be a simple connected graph with p vertices and q edges. In this paper, we introduce the reachability degree sum energy of a graph and obtained reachability degree sum energy of some graphs. Also, we establish the upper and lower bound for this new energy.

Keywords: Energy, Reachability Matrix, Degree Sum Matrix, Reachability Degree Sum Matrix, Reachability Degree Sum Eigen Values.

1. INTRODUCTION

The energy of a simple graph was introduced by Ivan Gutman in 1978[8, 9]. The energy of a graph$G$, denoted by$E(G)$, is defined to be the sum of the absolute value of the eigenvalues of its adjacency matrix (i.e)$E(G) = \sum_{i=1}^{p} |\lambda_i|$. There are many energies based on Distance matrix [3, 7, 11], Laplacian matrix [2], Harary matrix [5] etc.

Let G be a simple connected graph with p vertices and q edges. The reachability matrix $\mathbb{R}(G) = (r_{ij})$ is a square matrix of order p with $r_{ij} = 1$ if t_j is reachable from t_i and 0 otherwise [1]. The Characteristic polynomial of a $\mathbb{R}(G)$ is $\phi(G,\beta) = \det(\mathbb{R}(G) - \beta I)\,, where\ I\ is\ the\ idendity\ matrix$. the roots of the equation $\phi(G,\beta) = 0$ is called the Eigen values of the reachability matrix. Since $\mathbb{R}(G)$ is real and symmetric, its eigenvalues are real numbers and denoted by $\beta_1, \beta_2, \dots, \beta_p$, we label them in non- increasing order$\beta_1 \geq \beta_2 \geq \beta_3 \geq \cdots \geq \beta_p$. The collection of $\mathbb{R}$- eigenvalues is called spectrum of a graph G[4, 10].

Ramane H S et.al introduced the degree sum matrix in 2013 [6, 12, 13]. Let G be a simple connected graph with p vertices and q edges. The degree sum matrix of a graph G is denoted as $DS\,(G)$ is defined as $DSM(G) = [d_{ij}]$ where $d_{ij} = d_i + d_j$ when $i \neq j$ and 0 otherwise.The Characteristic polynomial of $DSM(G)$ is $\phi(G,\alpha) = \det(\mathrm{DSM}(G) - \alpha I)\,, where\ I\ is\ the\ idendity\ matrix$. The roots of the equation $\phi(G,\alpha) = 0$ is called the eigen values of the degree sum matrix. Since $\mathrm{DSM}(G)$ is real and symmetric, its eigenvalues are real numbers and denoted by $\alpha_1, \alpha_2, \dots, \alpha_p$, we label them in non-increasing order$\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \cdots \geq \alpha_p$. Motivated by above studies, In this paper, we will introduce a new energy and index of the graph based on reachability degree sum matrix and also obtain lower and upper bounds for this new energy and new index of the graph G.

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2. REACHABILITY DEGREE SUM MATRIX OF GRAPH

In this section, we introduce a new matrix called reachability degree sum matrix and also, we obtain lemma necessary to find its bounds.

Definition 2.1:

Let G be a simple connected graph with p vertices and q edges. Reachability degree sum matrix, denoted by$\mathbb{R}_{DS}M(G)$, defined by

$$\mathbb{R}_{DS}M(G) = (\gamma_{ij}) = \begin{cases} r_{ij} + d_i + d_j, & t_j \text{ is reachable from } t_i \\ 0 \quad , & \text{otherwise} \end{cases}$$

Where$d_i = degree\ of\ the\ vertex\ t_i$. The Characteristic polynomial of a $\mathbb{R}_{DS}M(G)$ is $\phi(G,\delta) = \det(\mathbb{R}_{DS}M(G) - \delta I)\,, where\ I\ is\ the\ idendity\ matrix$. the roots of the equation $\phi(G,\delta) = 0$ is called the Eigen values of the reachability degree sum matrix. Since $\mathbb{R}_{DS}M(G)$ is real and symmetric, its eigenvalues are real numbers and denoted by $\delta_1, \delta_2, \dots, \delta_p$, we label them in non- increasing order$\delta_1 \geq \delta_2 \geq \delta_3 \geq \cdots \geq \delta_p$. The collection of $\mathbb{R}_{DS}$- eigenvalues is called spectrum of a graph G. The sum of the absolute values of the eigenvalues of $\mathbb{R}_{DS}M(G)$ is known as reachability degree sum energy of a graph G, denoted by $\mathbb{R}_{DS}E(G)$, is defined by $\mathbb{R}_{DS}E(G) = \sum_{i=1}^{p} |\delta_i|$.

Lemma 2.1:

Let G be a connected graph of order p and let $\delta_1, \delta_2, \dots, \delta_p$ be eigenvalues of $\mathbb{R}_{DS}M(G)$. Then

$$\sum_{i=1}^{p} \delta_i = 0 \quad \& \quad \sum_{i=1}^{p} {\delta_i}^2 = 2 \sum_{1 \leq i < j \leq p} (r_{ij} + d_i + d_j)^2$$

Proof:

We know that $\sum_{i=1}^{p} \delta_i$ is equal to the trace of a matrix and also the reachability degree sum matrix of a graph is defined as

$\mathbb{R}_{DS}(G) = (\gamma_{ij})$ where $\gamma_{ij} = r_{ij} + d_i + d_j$ when $i \neq j$ and 0 otherwise.

Now, $\sum_{i=1}^{p} \delta_i = trace\,(\mathbb{R}_{DS}(G)) = \sum_{i=j=1}^{p} \gamma_{ij} = 0$.

Moreover, for $i = 1,2,\dots,p$, the $(i,i)th$ entry of $(\mathbb{R}_{DS}(G))^2$ is equal to $\sum_{j=1}^{p} (\gamma_{ij})(\gamma_{ji}) = \sum_{i=1}^{p} (\gamma_{ij})^2$

$$\sum_{i=1}^{p} {\gamma_i}^2 = trace\ (\mathbb{R}_{DS}(G))^2 = \sum_{i=1}^{p} \sum_{j=1}^{p} (\gamma_{ij})^2 = 2 \sum_{1 \leq i < j \leq p} (r_{ij} + d_i + d_j)^2.$$

Hence the result.

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3. BOUNDS FOR REACHABILITY DEGREE SUM ENERGY OF GRAPH

In this section, we obtain bounds for reachability degree sum energy of a graph$G$.

Theorem 3.1:

If G be a (p,q) connected graph, then

$$\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}\le \mathbb{R}_{DS}E(G)\le\sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}.$$

Proof:

By Cauchy-Schwartz inequality,

$$\left(\sum_{i=1}^{p}a_ib_i\right)^2\le\left(\sum_{i=1}^{p}{a_i}^2\right)\left(\sum_{i=1}^{p}{b_i}^2\right)$$

Consider, $a_i=1$ and $b_i=|\delta_i|$, then

$$\left(\sum_{i=1}^{p}|\delta_i|\right)^2\le\ p\left(\sum_{i=1}^{p}{\delta_i}^2\right)$$

$$\mathbb{R}_{DS}E(G)^2\le 2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2$$

$$\mathbb{R}_{DS}E(G)\le\sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}.$$

Which gives the required upper bound for $\mathbb{R}_{DS}E(G)$.

We can easily obtain the inequality,

$$\left(\mathbb{R}_{DS}E(G)\right)^2=\left(\sum_{i=1}^{p}|\delta_i|\right)^2\ge\sum_{i=1}^{p}|\delta_i|^2=2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2$$

$$\mathbb{R}_{DS}E(G)\ge\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}.$$

Which gives the required lower bound for $\mathbb{R}_{DS}E(G)$.

Hence the result.

Theorem 3.2:

Let G be a (p,q) connected graph and let Δ be the absolute value of the determinant of the reachability degree sum matrix $\mathbb{R}_{DS}M(G)$ of a graph then

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$$\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2+p(p-1)\Delta^{\frac{2}{p}}}\le \mathbb{R}_{DS}E(G)\le \sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}.$$

Proof:

By theorem 3.1, we have upper bound for $\mathbb{R}_{DS}E(G)$.

Now, we show that the lower bound for $\mathbb{R}_{DS}E(G)$ then this will finish the proof.

By definition of reachability degree sum energy,

$$\left(\mathbb{R}_{DS}E(G)\right)^2=\left(\sum_{i=1}^{p}|\delta_i|\right)^2=\sum_{i=1}^{p}|\delta_i|^2+2\sum_{1\le i<j\le p}|\delta_i|\left|\delta_j\right|$$

$$=2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2+\sum_{i\ne j}|\delta_i|\left|\delta_j\right|$$

From Arithmetic – Geometric Mean Inequality, we have,

$$\frac{1}{p(p-1)}\sum_{i\ne j}|\delta_i|\left|\delta_j\right|\ge\left(\prod_{i\ne j}|\delta_i|\left|\delta_j\right|\right)^{\frac{1}{p(p-1)}}$$

$$=\left(\prod_{i=1}^{p}|\delta_i|^{2(p-1)}\right)^{\frac{1}{p(p-1)}}=\Delta^{\frac{2}{p}}$$

$$\sum_{i\ne j}|\delta_i|\left|\delta_j\right|\ge\Delta^{\frac{2}{p}}$$

Which gives

$$\left(\mathbb{R}_{DS}E(G)\right)^2\ge 2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2+p(p-1)\Delta^{\frac{2}{p}}$$

$$\mathbb{R}_{DS}E(G)\ge\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2+p(p-1)\Delta^{\frac{2}{p}}}\ .$$

Hence the result.

4. REACHABILITY DEGREE SUM ENERGY OF SOME GRAPHS

In this section, we obtain reachability degree sum energy for some graphs.

Theorem 4.1:

Reachability degree sum energy of path graph P_p is $\mathbb{R}_{DS}E\left(P_p\right)=2(5p-12)\ \forall\ p\ge 4$.

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Proof:

Consider a path graph P_p with p vertices. The reachability degree sum matrix $\mathbb{R}_{DS}M(P_p)$ is

$$\mathbb{R}_{DS}M(P_p) = \begin{pmatrix} 0 & 4 & 4 & \cdots & 4 & 3 \\ 4 & 0 & 5 & \cdots & 5 & 4 \\ 4 & 5 & 0 & \cdots & 5 & 4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 4 & 5 & 5 & \cdots & 0 & 4 \\ 3 & 4 & 4 & \cdots & 4 & 0 \end{pmatrix}$$

Let us find the spectrum of $\mathbb{R}_{DS}M(P_p)$ using the relation,

$$\phi(P_p, \delta) = \det(\mathbb{R}_{DS}M(P_p) - \delta I)\,, where\ I\ is\ the\ idendity\ matrix.$$

$$\phi(P_p, \beta) = \begin{vmatrix} -\delta & 4 & 4 & \cdots & 4 & 3 \\ 4 & -\delta & 5 & \cdots & 5 & 4 \\ 4 & 5 & -\delta & \cdots & 5 & 4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 4 & 5 & 5 & \cdots & -\delta & 4 \\ 3 & 4 & 4 & \cdots & 4 & -\delta \end{vmatrix} = 0$$

Hence, the spectrum of $\mathbb{R}_{DS}M(P_p)$ is

$$\begin{pmatrix} -3 & -5 & \frac{5p-12\pm\sqrt{25p^2-52p+68}}{2} \\ 1 & p-3 & 1 \end{pmatrix}.$$

The reachability energy $\mathbb{R}_{DS}E(P_p)$ can be determined as follows: $\mathbb{R}_{DS}E(P_p) = \sum_{i=1}^{p}|\delta_i|$

$= (|-3| \times 1) + (|-5| \times (p-3)) + \left(\left|\frac{5p-12+\sqrt{25p^2-52p+68}}{2}\right| \times 1\right) + \left(\left|\frac{5p-12-\sqrt{25p^2-52p+68}}{2}\right| \times 1\right)$

$= 3 + 5p - 15 + 5p - 12 = 10p - 24 = 2(5p - 12).$

$$\mathbb{R}_{DS}E(P_p) = 2(5p - 12).$$

This completes the proof.

Example 4.1:

Bounds of Reachability degree sum energy of path graph is given in Table 4.1.

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Table 4.1. Bounds of Reachability Degree Sum Energy of Path Graph

Vertices p	$\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$	$\mathbb{R}_{DS}E(P_p)$	$\sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$
4	14	16	28
5	18.973	26	42.426
6	23.958	36	58.685
⋮	⋮	⋮	⋮
p	$\sqrt{25p^2-61p+40}$	$2(5p-12)$	$\sqrt{p(25p^2-61p+40)}$

Theorem 4.2:

Reachability degree sum energy of Cycle graph C_p is $\mathbb{R}_{DS}E(C_p) = 10(p-1)\ \forall\, p \ge 3$.

Proof:

Consider a cycle graph C_p with p vertices. The reachability degree sum matrix $\mathbb{R}_{DS}M(C_p)$ is

$$\mathbb{R}_{DS}M(C_p) = \begin{pmatrix} 0 & 5 & 5 & \cdots & 5 & 5 \\ 5 & 0 & 5 & \cdots & 5 & 5 \\ 5 & 5 & 0 & \cdots & 5 & 5 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 5 & 5 & 5 & \cdots & 0 & 5 \\ 5 & 5 & 5 & \cdots & 5 & 0 \end{pmatrix}$$

Let us find the spectrum of $\mathbb{R}_{DS}M(C_p)$ using the relation,

$$\phi(C_p,\delta) = \det(\mathbb{R}_{DS}M(C_p) - \delta I)\,, where\ I\ is\ the\ idendity\ matrix.$$

$$\phi(C_p,\beta) = \begin{vmatrix} -\delta & 5 & 5 & \cdots & 5 & 5 \\ 5 & -\delta & 5 & \cdots & 5 & 5 \\ 5 & 5 & -\delta & \cdots & 5 & 5 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 5 & 5 & 5 & \cdots & -\delta & 5 \\ 5 & 5 & 5 & \cdots & 5 & -\delta \end{vmatrix} = 0$$

Hence, the spectrum of $\mathbb{R}_{DS}M(C_p)$ is

$$\begin{pmatrix} 5(p-1) & -5 \\ 1 & p-1 \end{pmatrix}.$$

The reachability energy $\mathbb{R}_{DS}E(C_p)$ can be determined as follows: $\mathbb{R}_{DS}E(C_p) = \sum_{i=1}^{p}|\delta_i|$

$\mathbb{R}_{DS}E(C_p) = (|5(p-1)| \times 1) + (|-5| \times (p-1)) = 10p - 10 = 10(p-1).$

$\mathbb{R}_{DS}E(C_p) = 10(p-1).$

This completes the proof.

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Example 4.2:

Bounds of Reachability degree sum energy of cycle graph is given in Table 4.2.

Table 4.2. Bounds of Reachability Degree Sum Energy of Cycle Graph

Vertices p	$\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$	$\mathbb{R}_{DS}E\left(C_p\right)$	$\sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$
3	12.247	20	21.213
4	17.320	30	34.641
5	22/360	40	50
⋮	⋮	⋮	⋮
p	$5\sqrt{p(p-1)}$	$10(p-1)$	$5p\sqrt{p-1}$

Theorem 4.3:

Reachability degree sum energy of complete graph K_p is $\mathbb{R}_{DS}E\left(K_p\right) = 2(2p^2 - 3p + 1)$ $\forall\, p \geq 3$.

Proof:

Consider a complete graph K_p with p vertices. The reachability degree sum matrix $\mathbb{R}_{DS}M\left(K_p\right)$ is

$$\mathbb{R}_{DS}M\left(K_p\right) = \begin{pmatrix} 0 & 2p-1 & 2p-1 & \cdots & 2p-1 & 2p-1 \\ 2p-1 & 0 & 2p-1 & \cdots & 2p-1 & 2p-1 \\ 2p-1 & 2p-1 & 0 & \cdots & 2p-1 & 2p-1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2p-1 & 2p-1 & 2p-1 & \cdots & 0 & 2p-1 \\ 2p-1 & 2p-1 & 2p-1 & \cdots & 2p-1 & 0 \end{pmatrix}$$

Let us find the spectrum of $\mathbb{R}_{DS}M\left(K_p\right)$ using the relation,

$$\phi\left(K_p,\delta\right) = \det(\mathbb{R}_{DS}M\left(K_p\right) - \delta I)\,, where\ I\ is\ the\ idendity\ matrix.$$

$$\phi\left(K_p,\beta\right) = \begin{vmatrix} -\delta & 2p-1 & 2p-1 & \cdots & 2p-1 & 2p-1 \\ 2p-1 & -\delta & 2p-1 & \cdots & 2p-1 & 2p-1 \\ 2p-1 & 2p-1 & -\delta & \cdots & 2p-1 & 2p-1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2p-1 & 2p-1 & 2p-1 & \cdots & -\delta & 2p-1 \\ 2p-1 & 2p-1 & 2p-1 & \cdots & 2p-1 & -\delta \end{vmatrix} = 0$$

Hence, the spectrum of $\mathbb{R}_{DS}M\left(K_p\right)$ is

$$\begin{pmatrix} 2p^2-3p+1 & -(2p-1) \\ 1 & p-1 \end{pmatrix}.$$

The reachability energy $\mathbb{R}_{DS}E\left(K_p\right)$ can be determined as follows:

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$$\mathbb{R}_{DS}E\left(K_p\right) = \sum_{i=1}^{p} |\delta_i| = (|2p^2 - 3p + 1| \times 1) + \left(|-(2p-1)| \times (p-1)\right)$$

$= 2p^2 - 3p + 1 + 2p^2 - p - 2p + 1 = 4p^2 - 6p + 2 = 2(2p^2 - 3p + 1).$

$\mathbb{R}_{DS}E\left(K_p\right) = 2(2p^2 - 3p + 1).$

This completes the proof.

Example 4.3:

Bounds of Reachability degree sum energy of complete graph is given in Table 4.3.

Table 4.3. Bounds of Reachability Degree Sum Energy of Complete Graph

Vertices p	$\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$	$\mathbb{R}_{DS}E\left(K_p\right)$	$\sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$
3	12.247	20	21.213
4	24.248	42	48.490
5	40.249	72	90
⋮	⋮	⋮	⋮
p	$\sqrt{p(4p^3-8p^2+5p-1)}$	$2(2p^2-3p+1)$	$\sqrt{p^2(4p^3-8p^2+5p-1)}$

Theorem 4.4:

Reachability degree sum energy of wheel graph W_p is $\mathbb{R}_{DS}E\left(W_p\right) = 14(p-1)\ \forall\ p \ge 4$.

Proof:

Consider a wheel graph W_p with p vertices. The reachability degree sum matrix $\mathbb{R}_{DS}M\left(W_p\right)$ is

$$\mathbb{R}_{DS}M\left(W_p\right) = \begin{pmatrix} 0 & 7 & 7 & \cdots & 7 & p+4 \\ 7 & 0 & 7 & \cdots & 7 & p+4 \\ 7 & 7 & 0 & \cdots & 7 & p+4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 7 & 7 & 7 & \cdots & 0 & p+4 \\ p+4 & p+4 & p+4 & \cdots & p+4 & 0 \end{pmatrix}$$

Let us find the spectrum of $\mathbb{R}_{DS}M\left(W_p\right)$ using the relation,

$$\phi\left(W_p, \delta\right) = \det(\mathbb{R}_{DS}M\left(W_p\right) - \delta I)\,, where\ I\ is\ the\ idendity\ matrix.$$

$$\phi\left(W_p, \beta\right) = \begin{vmatrix} -\delta & 7 & 7 & \cdots & 7 & p+4 \\ 7 & -\delta & 7 & \cdots & 7 & p+4 \\ 7 & 7 & -\delta & \cdots & 7 & p+4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 7 & 7 & 7 & \cdots & -\delta & p+4 \\ p+4 & p+4 & p+4 & \cdots & p+4 & -\delta \end{vmatrix} = 0$$

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Hence, the spectrum of $\mathbb{R}_{DS}M\left(W_p\right)$ is

$$\begin{pmatrix} -7 & \frac{7(p-1)\pm\sqrt{4p^3+81p^2-34p+48}}{2} \\ p-1 & 1 \end{pmatrix}.$$

The reachability energy $\mathbb{R}_{DS}E\left(W_p\right)$ can be determined as follows:

$$\mathbb{R}_{DS}E\left(W_p\right)=\sum_{i=1}^{p}|\delta_i|$$

$$=\left(|-7|\times(p-1)\right)+\left(\left|\frac{7(p-1)+\sqrt{4p^3+81p^2-34p+48}}{2}\right|\times 1\right)+\left(\left|\frac{7(p-1)-\sqrt{4p^3+81p^2-34p+48}}{2}\right|\times 1\right)$$

$$=7(p-1)+7(p-1)=14(p-1).$$

$$\mathbb{R}_{DS}E\left(C_p\right)=14(p-1).$$

This completes the proof.

Example 4.4:

Bounds of Reachability degree sum energy of wheel graph is given in Table 4.4.

Table 4.4. Bounds of Reachability Degree Sum Energy of Wheel Graph

Vertices p	$\sqrt{2\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$	$\mathbb{R}_{DS}E\left(W_p\right)$	$\sqrt{2p\sum_{1\le i<j\le p}\left(r_{ij}+d_i+d_j\right)^2}$
4	33.166	42	66.332
5	42.308	56	94.604
6	51,672	70	126.570
⋮	⋮	⋮	⋮
p	$\sqrt{p(2p^2+65p-17)}$	$14(p-1)$	$p\sqrt{2p^2+65p-17}$

CONCLUSION

In this paper, we have introduced and obtained the reachability degree sum energy of graph of order$p$. Also, we have found its bounds of this energy of graph of order$p$. Further, we obtained reachability degree sum energy of some graphs such as path graph, cycle graph, complete graph and wheel graph.

References

1) Bondy J A, Murty U S R, Graph theory with applications, The Mac Millan Press LTD, London and Basingstoke,1979.

2) Bo Zhou and Ivan Gutman, On Laplacian energy of graphs, MATCH Comm. Math compute chem. 57(2007) 211-220.

3) Buckley F, Harary F, Distance in Graphs, Addison-Wesley, Redwood, 1990.

4) Cvetkovic D M, Doob M, Sachs H, Spectra of graphs, Academic Press New York 1980.

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5) Gungor A D, Cevik A S, On the Harary energy and Harary Estrada index of a graph, MATCH commun. Math. Comput. Chem, 2010,64,282-296

6) Hande S P, Jog S R, Revankar D S, Bounds for the degree sum eigen value and degree sum energy of a common neighborhood graph, Int. J. Graph theory, vol.1 3(2013), 131-136.

7) Indulal G, Ivan Gutman, Vijaykumar A, on the distance energy of a graph, MATCH Commun. Math. Comput. Chem. 60 (2008) 461-472.

8) Ivan Gutman, The energy of a graph, ber.math. Statist. Sekt. Forschangsz. Graz. 103, 1- 22,1978.

9) Ivan Gutman, The Energy of a Graph: Old and New Results, in: A. Betten, A. Kohnert, R. Laue, A. Wassermann (Eds.), Algebraic Combinatorics and Applications, Springer-Verlag, Berlin, 2001, pp. 196-211.

10) Janezic D, Milicevic A, Nikolic S, Trinajstic, Graph theoretical matrices in chemistry, Univ. Kragujevac, Kragujevac, 2007.

11) Ramane H S, Revankar D S, Gutman I, Rao S B, Acharya B D, Walikar H B, Bounds for the Distance Energy of a Graph, Kragujevac J. Math. 31 (2008) 59-68.

12) Ramane H S, Revankar D S, Patil J B, Bounds for the Degree sum eigenvalues and Degree sum Energy of a Graph, International journal of Pure and applied mathematical sciences, 6(2), 161-167, 2013.

13) Sudhir R Jog, Raju kotambari, Degree sum energy of some graphs, Annals of Pure and Applied Mathematics, Vol. 11, No. 1, 2016, 17-27.