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FUZZIFICATION OF CRISP GRAPHS AND ITS IMPLICATIONS

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Abstract

Traditional fuzzy graph models assign membership values to vertices and edges based on a specific uncertain situation. However, this work investigates a novel approach: representing the situation *as* a graph and deriving fuzziness from the graph's inherent structure. We introduce "ratio labeling" (RL), a new labelling procedure where vertex and edge membership grades are determined by graph parameters. These labels, derived directly from the graph's structure, characterize the graph itself and serve as the basis for examining the admissibility of fuzziness within the graph. This approach allows the study of fuzziness arising from the properties of the graph representing a situation. This paper explores this new idea and examines certain graphs for the admissibility of fuzziness. This topic study the methodologies, properties, and applications of ratio labelling in fuzzy graph identification, focusing on its theoretical foundations and practical implications in solving real-world problems. Furthermore, the proposed ideas are illustrated with several numerical instances. To emphasize the theoretical concept, an application that ensures an effective communication between groups of people in a social media under RL is discussed.

Keywords: Fuzzy Graph, Ratio Labelling, Complete Graph, Complete Bipartite Graph, Cycle, Path.

AMC Subject Classification: 05C72

1. INTRODUCTION

Graph theory plays a fundamental role in modeling relationships and interactions in various real-world problems. Among the numerous extensions of classical graph theory, **fuzzy graph theory** provides a powerful framework for dealing with uncertainties, imprecision, and vagueness inherent in many systems. Fuzzy relations were introduced by Zadeh in 1965. A fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. Kaufmann initially introduced the concept of a fuzzy graph in 1973, building upon Zadeh’s work on fuzzy relations. However, it was Rosenfeld who significantly advanced the theory of fuzzy graphs in 1975 by exploring fuzzy relations defined on fuzzy sets. Rosenfeld provided a more formal and rigorous definition of fuzzy graphs, building upon Kaufmann’s initial work. He introduced the concepts like fuzzy paths, cycles, and connectedness, laying the foundation for further research. In subsequent decades investigations on various types of fuzzy graphs were done, including intuitionistic fuzzy graphs, bipolar fuzzy graphs [4, 12]. Labelling of fuzzy graph was introduced by A. Nagoor Gani and D. Rajalakshmi [9]. The concept of domination in fuzzy graphs was investigated by Somasundaram [10]. The concept of complement of fuzzy graph was investigated by Sunitha and Vijayakumar [11]. The work by Mathew Varkey T K and Sreena T D on evidence labelling of fuzzy graph examines the fuzziness on graphs with a particular labelling [8].

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Applications of the fuzzy graph structures in decision making process, regarding detection of marine crimes and road crimes are presented by Ali N. A. Koam et al. [1]. Asima Razzaque et al. explained the idea of t-intuitionistic fuzzy graphs to analyse complex relationships with multiple factors [7]. Anushree Bhattacharya et al. discussed a fuzzy graph theory approach to a case study problem [3]. Connectivity status of vertices in an intuitionistic fuzzy graph and its application to merging of banks was discussed by Jayanta Bera et al. [6].

These advancements in fuzzy graph theory have sparked significant interest in exploring fuzzy graphs. Labelling the vertices and edges of a graph to examine its "fuzziness" can be approached in a few different ways. In graph theory, this might refer to various forms of uncertainty or imprecision in the relationships between vertices and edges. Blue et al. categorized fuzzy graphs into different types based on various criteria as follows [5].

- "Type I: Crisp vertex set and fuzzy edge set.
- Type II: Crisp vertices and edges with fuzzy connectivity.
- Type III: Fuzzy vertex set and crisp edges.
- Type IV: Crisp graph with fuzzy weights, representing a graph where the vertices and edges have uncertain weights, but the connections are well-defined.
- Type V: Fuzzy set of crisp graphs, involving the fuzzy composition of crisp graphs."

Type IV graphs are particularly useful when the relationships between elements in a graph are clear, but the attributes of these relationships are uncertain. To analyze the "fuzziness" in such graphs, we propose the concept of Ratio Labelling (RL). This technique uses established graph parameters to assign membership values to both the vertices and edges, ranging from 0 to 1. By doing so, RL effectively represents the structural properties of the graph while quantifying the level of fuzziness. This approach is especially beneficial in scenarios where the network's structure is well-understood, but the characteristics of the connections are imprecise or ambiguous. Analyzing fuzziness in a crisp graph through ratio labelling—where vertices are assigned values using σ and edges using μ—reveals that the ability to incorporate fuzziness varies based on the structure of the graph. We examined some of interconnection networks for admissibility of fuzziness using RL [2]. Since ratio labelling promotes strong connectivity among vertices, a ratio-labelled fuzzy graph can represent an efficient communication network or strong interpersonal bonds in a social network.

The novelties and effectiveness in ratio labelling are listed as follows.

1. RL involves assigning labels that represent a ratio or relative value, between two parameters. These ratios can help capture the relative strength, importance, or influence of a node or edge in a graph. For example, an edge between two nodes could be labelled with a ratio that compares the influence or closeness of the two nodes relative to others in the network.

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2. RL can dynamically adjust based on the relative importance of a node or edge compared to others in the graph, allowing for more context-sensitive labelling.
3. By using ratios, one can capture subtle differences in interactions or properties
4. Ratio labelling relies heavily on accurate, high-quality data to compute meaningful ratios. In cases where data is sparse, incomplete, or noisy, the ratios may not be reliable.

The focus of this paper is on introducing ratio labelling and assessing its impact on various traditional graphs including cycles, path, complete graphs, complete bipartite graphs to determine their suitability for accommodating fuzziness and to discuss their properties (see section 3). Moreover, we wish to discuss an application that represents the relationship bonding between the group of people in social media using RL (see section 4).

2. BASIC CONCEPTS

A fuzzy graph G: (σ, μ) is a pair of functions σ: V→[0,1] and μ:V×V→[0,1], where for all x, y ∈V,

$$\mu(x,y) \leq \sigma(x) \wedge \sigma(y) .$$

where ∧ stands for minimum. Also

$$\sigma^* = supp(\sigma) = \{u \epsilon S: \sigma(u) > 0\}. \mu^* = supp(\mu) = \{(u,v) \epsilon S \times S: \mu(u,v) > 0).$$

In G: (σ, μ), the order of G is

$$p = \sum_{x \in S} \sigma(x) .$$

If $\mu(x,y) > 0$ then x and y are called neighbours, x and y are said to lie on the same edge e. The neighbourhood of a vertex $v \in S$ is a set of all vertices which are neighbours of v denoted by $N(v)$ Let G: (σ, μ) be a fuzzy graph. The degree of a vertex v of a fuzzy graph G is defined as $\mathrm{deg_G(v)} = \sum_{\mathrm{u \neq v}} \mathrm{\mu(u,v)}$. In a fuzzy graph G the minimum degree $\delta(\mathrm{G})$, and maximum degree $\Delta(\mathrm{G})$, are defined as follows.

$\delta(\mathrm{G}) = \min\{\mathrm{deg_G(u)}: \text{for all } \mathrm{u} \in \mathrm{V}\}$ and $\Delta(\mathrm{G}) = \max\{\mathrm{deg_G(u)}: \text{for all } \mathrm{u} \in \mathrm{V}\}$. The order of a fuzzy graph $\mathrm{G}(\sigma, \mu)$ is defined to be $\mathrm{O(G)} = \sum_{\mathrm{u \in V}} \sigma(\mathrm{u})$. The size of a fuzzy graph $\mathrm{G}(\sigma, \mu)$ is defined to be $\mathrm{S(G)} = \sum_{\mathrm{(u,v) \in E}} \mu(\mathrm{u,v})$. A fuzzy graph G is said to be regular if for a positive real number k, $\mathrm{deg_G(v)} = \mathrm{k}$, for all $\mathrm{u} \in \mathrm{V}$. In this case, G is called k-regular fuzzy graph.

In a fuzzy graph G (σ, μ), a path is a sequence of distinct vertices v_0, v_1…, vn such that $\mu(v_{i-1}, v_i) > 0, 1 \leq i \leq n$. Here, 'n' is called the length of the path. The consecutive pairs (v_{i-1}, v_i) are called arcs of the path. The strength of the path between two vertices v_1 and v_2 is defined as $\wedge_{i=1}^{n} \mu(v_{i-1}, v_i)$. If u and v are connected using paths of length 'k' then $\mu^k(u,v)$ is defined as

$$\mu^k(u,v) = \sup\{ \mu(u,v_1) \wedge \mu(v_1,v_2) \wedge \ldots \wedge \mu(v_{k-1},v): u, v_1, \ldots, v_{k-1}, v \, \epsilon S\}$$

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If u, v ∈ S the strength of connectedness between u and v is,

$\sup\{\mu^k(u,v): k = 1,2,3, \ldots.\}$, and it is denoted as $CONN_{G-(u,v)}(u,v)$ or $\mu^{\infty}(u,v)$. A fuzzy graph G is connected if $\mu^{\infty}(u,v) > 0$,for all u, v in σ^*.

An arc (u, v) of a fuzzy graph G (σ, μ) is said to be a strong arc if $\mu(u,v) > 0$ and $\mu(u,v) \geq \mu^{\infty}(u,v)$. A path P ($v_0$, v_1,…,v_n) from v_0 to v_n is called a strong path if (v_i,v_{i+1}) is strong for all $1 \leq i \leq n-1$. The edge (u, v) in G (σ, μ) is said to be

(i) α – strong if $\mu(u,v) > CONN_{G-(u,v)}(u,v)$

(ii) β – strong if $\mu(u,v) = CONN_{G-(u,v)}(u,v)$

(iii) δ – arc if $\mu(u,v) < CONN_{G-(u,v)}(u,v)$

A path in a fuzzy graph G (σ, μ) is called an α – strong path if all its edges are α – strong and is called a β – strong path if all its edges are β – strong.

A vertex x, is said to be an isolated vertex if $\mu(u,v) = 0$ for all $u \neq v$.

The fuzzy distance between two vertices u and v is defined as

$$d_f(u,v) = \wedge \sum \{\wedge(\sigma(u), \sigma(v)) \times \mu(u,v)\}.$$

3. MAIN RESULTS

The section discusses the method of labelling the vertices and edges of a given graph using RL (Ratio Labelling). The impact of RL in admitting fuzziness of the given graph is examined. The graphs that are fuzzy under RL can handle fuzzy or uncertain information. Initiated by this the graphs such as paths P_n , cycles C_n, complete graphs K_n and complete bipartite graph $K_{n,m}$ are examined. The complete bipartite graph $K_{n,m}$is fuzzy when $m = n$; however, when $m \neq n$, the graph is fuzzy under some restrictions on degree of the adjacent vertices. The properties such as diameter, eccentricity, strength of the edge of these graphs are discussed. Also, the edges of ratio labelled fuzzy graphs are classified as $\alpha-$ strong, $\beta-$ strong which helps to identify the structure of ratio labelled fuzzy graphs.

3.1 Definition

Let $G = (V, E)$ be a simple connected graph. The functions, $\sigma: V \rightarrow [0,1]$, and $\mu: E \rightarrow [0, 1]$ that labels the vertices and edges of G, are defined as

$$\sigma(v) = \frac{|N(v)|}{|E|} \quad (1)$$

$$\mu(u,\mathrm{v}) = \frac{\max_{(u,v)\in E}[\sigma(u),\sigma(v)]}{\sum_{v\in V}\sigma(v)} \quad (2)$$

and is called ratio labelling of G . The graph G that is a fuzzy graph due to ratio labelling is called as ratio labelled fuzzy graph (RLFG)

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Example.1

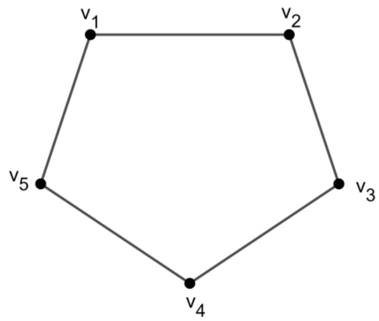

Fig 1

Consider the cycle C_5 with vertex set $V = \{ v_1, v_2, v_3, v_4, v_5\}$.

The vertices are labelled using σ as, $\sigma(v_1) = \frac{|N(v_1)|}{|E|} = \frac{2}{5}$.

Similarly, $\sigma(v_2) = \sigma(v_3) = \sigma(v_4) = \sigma(v_5) = \frac{2}{5}$. The edges are labelled using μ as

$$\mu(v_1, v_2) = \frac{\max[\sigma(v_1),\sigma(v_2)]}{\sum_{v\in V}\sigma(v)} = \frac{\max\left\{\frac{2}{5},\frac{2}{5}\right\}}{5\times\frac{2}{5}} = \frac{1}{5} \ .$$

Similarly, $\mu(v_2, v_3) = \mu(v_3, v_4) = \mu(v_4, v_5) = \mu(v_5, v_1) = \frac{1}{5}$.

Here, $\mu\left(v_i, v_j\right) < \sigma(v_i) \wedge \sigma(v_j)$ for all $1 \leq i, j \leq 5$ and $i \neq j$.

Hence C_5 is a fuzzy graph under RL.

Example.2

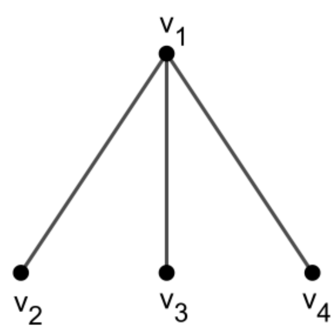

Fig 2

Consider the graph $G = (V, E)$ with vertex set $V = \{ v_1, v_2, v_3, v_4\}$. The vertices of G are labelled using RL as,

$$\sigma(v_1) = \frac{|N(v_1)|}{|E|} = \frac{3}{3} = 1, \sigma(v_2) = \sigma(v_3) = \sigma(v_4) = \frac{1}{3} \text{ and}$$

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The edges are labelled using RL as,

$$\mu(v_1, v_2) = \frac{\max[\sigma(v_1), \sigma(v_2)]}{\sum_{v \in V} \sigma(v)} = \frac{\max\left\{1, \frac{1}{3}\right\}}{3 \times \frac{1}{3} + 1} = \frac{1}{2}, \mu(v_1, v_3) = \mu(v_1, v_4) = \frac{1}{2}.$$

$$\text{Here, } \mu(v_1, v_2) = \frac{1}{2} > \frac{1}{3} = \sigma(v_1) \wedge \sigma(v_2) .$$

Hence G is not a fuzzy under RL.

Remark.

$K_{1,n}$ is not a fuzzy under RL for $n \geq 3$.

3.2 Theorem

For a cycle $C_n(V, E)$, with n vertices and n edges, the vertices and edges are labelled as $\frac{2}{n}, \frac{1}{n}$ respectively by RL.

Proof: In C_n , $|V| = n$, $|E| = n$, and $|N(v)| = 2$, for all $v \in V$. Now, by RL,

$$\sigma(v) = \frac{|N(v)|}{|E|} = \frac{2}{n} \text{ , for all } v \in V$$

$$\sum_{v \in V} \sigma(v) = \sum_{v \in V} \frac{2}{n} = n \times \frac{2}{n} = 2 \text{ and}$$

$$\mu(u, v) = \frac{\max[\sigma(u), \sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{n} \text{ , for all } (u, v) \in E .$$

Hence follows.

3.3 Theorem

For all $n \geq 3$, the cycle $C_n(V, E)$ is a fuzzy graph under ratio labelling.

Proof: By theorem 3.2,

$$\sigma(v) = \frac{2}{n} \text{ , for all } v \in V \text{ , and } \quad \mu(u, v) = \frac{1}{n} \text{ , for all } (u, v) \in E .$$

$$\text{Hence, } \mu(u, v) = \frac{1}{n} < \frac{2}{n} = \sigma(u) \wedge \sigma(v), for\ all\ (u, v) \in E .$$

Hence, C_n is a RLFG.

3.4 Theorem

Let G be a RLFG $C_n(V, E)$. In G the following results holds,

(i) the degree of every vertex is $\frac{2}{n}$

(ii) G is regular

(iii) the size of the graph is 1

Proof: **In C_n, $|V| = n$ and $|E| = n$.**

In G, by theorem 3.2, $\sigma(v) = \frac{2}{n}$, for all $v \in V$, and $\mu(u, v) = \frac{1}{n}$, for all $(u, v) \in E$.

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(i) Now, $deg_G(u) = \sum \mu(u,v)\,, u \neq v$. Since degree of every vertex of C_n is 2, degree of u in G is $\deg_G(u) = 2 \times \frac{1}{n} = \frac{2}{n}$, for all u.

(ii) follows, from (i)

(ii)$S(G) = \sum \mu(u,v)$, for all $(u,v) \in E$.

Since every edge is of equal weight, $S(G) = |E| \times \mu(u,v)$

$$S(G) = n \times \frac{1}{n} = 1$$

3.5 Theorem

Let G be a RLFG $C_n(V,E)$.

(i) Every edge is a strong edge

(ii) Every edge is β- strong

(iii) Any path is a strong path

Proof: **By theorem 3.2, $\boldsymbol{\sigma(v) = \frac{2}{n}}$, for all $\boldsymbol{v \in V}$, and $\boldsymbol{\mu(u,v) = \frac{1}{n}}$, for all $\boldsymbol{(u,v) \in E}$ in G.**

(i) In G, every edge is of same weight, $\mu(u,v) = CONN_{G-(u,v)}(u,v)$. Hence every edge is a strong edge.

(ii) By (i) it follows that every edge is β- strong

(iii) follows from(i)

Hence follows.

3.6 Theorem

The diameter of a ratio labelled Cycle $C_n(V,E)$ is $\left\lfloor \frac{n}{2} \right\rfloor \times \frac{2}{n^2}$.

Proof: For the cycle C_n, by theorem 3.2,

$$\sigma(v) = \frac{2}{n} \text{ for all } v \text{ and } \mu(u,v) = \frac{1}{n} \text{ for all } (u,v) \in E.$$

Distance between any two vertices in a cycle C_n varies from i= 1 to $\left\lfloor \frac{n}{2} \right\rfloor$.

Fuzzy distance between two vertices,

$$d_f(u,v) = \wedge \textstyle\sum_1^i \{\wedge(\sigma(u),\sigma(v)) \times \mu(u,v)\} \text{ when u and v}$$

are at a distance i in C_n.

$$= \wedge \sum_1^i \left[\frac{2}{n} \times \frac{1}{n}\right] for\ all\ u,v$$

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$$= \begin{cases} \frac{2}{n^2} \ for\ i = 1 \\ 2 \times \frac{2}{n^2} \ for\ i = 2 \\ \cdots \\ \left\lfloor \frac{n}{2} \right\rfloor \times \frac{2}{n^2} \quad for\ i = \left\lfloor \frac{n}{2} \right\rfloor \end{cases}$$

The eccentricity of ratio labelled C_n,

$$e_f(v) = \vee\{d_f(u, v)\}, for\ all\ u \in V$$

$$= \left\lfloor \frac{n}{2} \right\rfloor \times \frac{2}{n^2}$$

$$diam_f(C_n) = \max\{e_f(v)\}, for\ all\ v$$

$$= \left\lfloor \frac{n}{2} \right\rfloor \times \frac{2}{n^2}$$

This completes the proof.

3.7 Theorem

For a path $P_n(V, E)$ with n vertices, n-1 edges, the vertices and edges are labelled as $\sigma(v) = \begin{cases} \frac{1}{n-1} \quad for\ pendant\ vertices \\ \frac{2}{n-1}\ for\ the\ internal\ vertices \end{cases}$ and $\mu(u, v) = \frac{1}{n-1}$ for all (u, v) in P_n by RL.

Proof: In P_n , $|V| = n$,$|E| = n - 1$,

and $|N(v)| = \begin{cases} 1, for\ pendant\ vertices \\ 2, for\ internal\ vertices \end{cases}$.

Now,

$$\sigma(v) = \frac{|N(v)|}{|E|} = \begin{cases} \frac{1}{n-1}, for\ pendant\ vertices \\ \frac{2}{n-1}, for\ internal\ vertices \end{cases}.$$

As there are two pendant vertices in a path P_n ,

$$\sum_{v \in V} \sigma(v) = 2 \times \frac{1}{n-1} + (n-2) \times \frac{2}{n-1} = 2 .$$

$$\mu(u, v) = \frac{\max[\sigma(u), \sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{n-1}, \text{ for all } (u, v) \in E .$$

Hence follows.

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3.8 Theorem

For all $n \geq 2$, the path $P_n(V, E)$ is a fuzzy graph under RL.

Proof: By theorem 3.7, $\sigma(v) = \frac{|N(v)|}{|E|} = \begin{cases} \frac{1}{n-1}, for\ pendant\ vertices \\ \frac{2}{n-1}, for\ internal\ vertices \end{cases}$ and

$$\mu(u, v) = \frac{1}{n-1}, \text{ for all } (u, v) \in E \ .$$

Case (i)

When (u, v) is an edge incident with pendent vertex?

$$\mu(u, v) = \frac{1}{n-1} = \sigma(u) \wedge \sigma(v)$$

Case (ii)

When (u, v) is an edge incident with non-pendent vertices,

$$\mu(u, v) = \frac{1}{n-1} < \frac{2}{n-1} = \sigma(u) \wedge \sigma(v) \ .$$

From Case(i) and (ii),

$$\mu(u, v) \leq \sigma(u) \wedge \sigma(v), \qquad for\ all\ (u, v) \in E$$

Hence follows.

3.9 Theorem

Let G be a RLFG $P_n.(V, E)$ Then

(i) minimum degree of G= $\delta(G) = \frac{1}{n-1}$

(ii) maximum degree of G =$\triangle(G) = \frac{2}{n-1}$

(iii) the size of the graph is 1

Proof: **In G, by theorem 3.7,** $\boldsymbol{\sigma(v)} = \begin{cases} \boldsymbol{\frac{1}{n-1}, for\ pendant\ vertices} \\ \boldsymbol{\frac{2}{n-1}, for\ internal\ vertices} \end{cases}$ **and**

$$\mu(u, v) = \frac{1}{n-1}, \text{ for all } (u, v) \in E \ .$$

(i) Now, $deg_G(u) = \sum \mu(u, v), u \neq v$.

$$deg_G(u) = \begin{cases} \frac{1}{n-1}, for\ pendant\ vertices \\ \frac{2}{n-1}, for\ internal\ vertices \end{cases}$$

minimum degree of G = $\delta(G) = \frac{1}{n-1}$

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(ii) maximum degree of G $= \triangle(G) = \frac{2}{n-1}$

(iii) $S(G) = \sum \mu(u,v)$, for all $(u,v) \in E$. Since every edge is of equal weight, $S(G) = |E| \times \mu(u,v)$

$$S(G) = (n-1) \times \frac{1}{n-1} = 1$$

Hence follows.

3.10 Theorem

The diameter of a ratio labelled path graph $P_n(V,E)$ is $(n-2) \times \frac{2}{(n-1)^2}$.

Proof: For the path graph P_n, under RL,

$$\sigma(v) = \frac{|N(v)|}{|E|} = \begin{cases} \frac{1}{n-1}, for\ pendant\ vertices \\ \frac{2}{n-1}, for\ internal\ vertices \end{cases}$$

$$\mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{n-1}, \text{ for all } (u,v) \in E.$$

Fuzzy distance between two vertices,

$$d_f(u,v) = \wedge \sum \{ \bigwedge (\sigma(u), \sigma(v)) \times \mu(u,v) \}$$

In a path P_n, $\max\{d_f(u,v)\}$ is attained between the pendant vertices.

$$\mathrm{Max}\{d_f(u,v)\} = \frac{1}{(n-1)^2} + \frac{2}{(n-1)^2} + \frac{2}{(n-1)^2} + \cdots + \frac{2}{(n-1)^2} + \frac{1}{(n-1)^2}, (n-1\ terms)$$

$$= \frac{2}{(n-1)^2} + (n-3)\frac{2}{(n-1)^2}$$

$$= (n-2)\frac{2}{(n-1)^2}$$

The eccentricity of ratio labelled path P_n,

$$e_f(v) = \vee\{d_f(u,v)\}, for\ all\ u \in V$$

$$= \frac{2(n-2)}{(n-1)^2}, for\ all\ v$$

$$diam_f(P_n) = \max\{e_f(v)\}, for\ all\ v$$

$$= \frac{2(n-2)}{(n-1)^2}$$

This completes the proof.

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3.11 Theorem

In a complete graph $K_n(V,E)$, for every $v \in V$, $\sigma(v) = \frac{2}{n}$ and for all $(\mathrm{u},\mathrm{v}) \in K_n$, $\mu(u,v) = \frac{1}{n}$ by RL.

Proof: In K_n , $|V| = n$,$|E| = \frac{n(n-1)}{2}$, and $|N(v)| = n-1$, for all $v \in V$.

$$\text{Hence,}\sigma(v) = \frac{|N(v)|}{|E|} = \frac{n-1}{\frac{n(n-1)}{2}} = \frac{2}{n} \text{ , for all n.}$$

$$\sum_{v \in V} \sigma(v) = n \times \frac{2}{n} = 2.$$

$$\boldsymbol{\mu(u,v)} = \frac{\mathbf{max}[\boldsymbol{\sigma(u),\sigma(v)}]}{\sum_{\boldsymbol{v} \in \boldsymbol{V}} \boldsymbol{\sigma(v)}} = \frac{\mathbf{1}}{\boldsymbol{n}} \text{, for all, } \boldsymbol{(u,v) \in E},$$

Hence follows.

3.12 Theorem

Every complete graph $K_n(V,E)$ is a fuzzy graph under RL.

Proof: By theorem 3.11,

$\sigma(v) = \frac{2}{n}$, for all n and $\mu(u,v) = \frac{1}{n}$, for all, $(u,v) \in E$

$$\mu(u,v) = \frac{1}{n} < \frac{2}{n} = \sigma(u) \wedge \sigma(v), for\ all\ (u,v) \in E..$$

Hence follows.

3.13 Theorem

Let G be a RLFG $K_n(V,E)$. In G the following results holds,

(i) the degree of every vertex is $\frac{n-1}{n}$

(ii) G is regular

(iii) the size of the graph is $\frac{n-1}{2}$

Proof: **In G, by theorem 3.11,** $\boldsymbol{\sigma(v)} = \frac{\mathbf{2}}{\boldsymbol{n}}$ **, for all** $\boldsymbol{v \in V}$ **, and**

$\mu(u,v) = \frac{1}{n}$, for all $(u,v) \in E$.

(i) Now, $deg_G(u) = \sum \mu(u,v), u \neq v$. Since degree of every vertex of K_n is $n-1$, $deg_G(u) = (n-1) \times \frac{1}{n} = \frac{n-1}{n}$, for all $u \in G$.

(ii) (ii) follows, from (i)

(iii) $S(G) = \sum \mu(u,v)$, for all $(u,v) \in E$

Since every edge is of equal weight, $S(G) = |E| \times \mu(u,v)$

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$$S(G) = \frac{n(n-1)}{2} \times \frac{1}{n} = \frac{n-1}{2}$$

Hence follows.

3.14 Theorem

Let G be a RLFG $K_n(V, E)$. In G the following results holds.

(i) Every edge is a strong edge

(ii) Every edge is β- strong

(iii) Every path is a strong path

Proof: In G, by theorem 3.11, $\boldsymbol{\sigma(v) = \frac{2}{n}}$ **, and for all** $\boldsymbol{v \in V}$**,** $\boldsymbol{\mu(u,v) = \frac{1}{n}}$ **, for all** $\boldsymbol{(u,v) \in E}$**.**

(i) In G, every edge is of same weight, $\mu(u,v) = CONN_{G-(u,v)}(u,v)$. Hence every edge is a strong edge.

(ii) By (i) it follows that every edge is β- strong

(iii) From (i), (iii) follows

This completes the proof.

3.15 Theorem

The diameter of a ratio labelled complete graph $K_n(V, E)$ is $\frac{2}{n^2}$.

Proof: For the complete graph K_n, under ratio labelling,

$$\sigma(v) = \frac{|N(v)|}{|E|} = \frac{2}{n} \text{ for all } v \in V.$$

$$\mu(u,v) = \frac{\max[\sigma(u), \sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{n} \text{, for all } (u,v) \in E,$$

Fuzzy distance between two vertices,

$$d_f(u,v) = \wedge \sum \{ \bigwedge (\sigma(u),\ \sigma(v)) \times \mu(u,v) \}$$

$$= \wedge \left[\frac{2}{n} \times \frac{1}{n} \right] for\ all\ u, v$$

$$= \frac{2}{n^2}\ for\ all\ u, v.$$

The eccentricity of ratio labelled K_n,

$$e_f(v) = \vee \{ d_f(u,v) \}, for\ all\ u \in V$$

$$= \quad \frac{2}{n^2}, for\ all\ v \in V$$

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$$diam_f(K_n) = \max\{e_f(v)\}, for\ all\ v$$
$$= \frac{2}{n^2}$$

Hence follows.

3.16 Theorem

In a complete bipartite graph $K_{n,n}(V,E)$, for every $v \in V$, $\sigma(v) = \frac{1}{n}$ and for all

$(u,v) \in \mathrm{E}$, $\mu(u,v) = \frac{1}{2n}$ by RL.

Proof: In $K_{n,n}$, $|V| = 2n$, where $|V_1| = |V_2| = n$, $|E| = n \times n$, $|N(v)| = n$, for all $v \in V$. Now,

$$\sigma(v) = \frac{|N(v)|}{|E|} = \frac{1}{n} \text{ , for all n.}$$

$$\sum_{v \in V} \sigma(v) = 2n \times \frac{1}{n} = 2 \text{ .}$$

$$\mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{2n} \text{ ,for all } (u,v) \in E.$$

Hence follows.

3.17 Theorem

Every complete bipartite graph $K_{n,n}(V,E)$ is a fuzzy graph under RL.

Proof: By theorem 3.16,

$$\sigma(v) = \frac{1}{n}, \text{ for all } v \in V \text{ and } \mu(u,v) = \frac{1}{2n} \text{ ,for all } (u,v) \in E.$$

$$\text{Hence, } \mu(u,v) = \frac{1}{2n} < \frac{1}{n} = \sigma(u) \wedge \sigma(v), for\ all\ (u,v) \in E \text{ .}$$

Hence follows.

3.18 Theorem

Let G be a RLFG $K_{n,n}(V,E)$. In G the following results holds,

(i) the degree of every vertex is $\frac{1}{2}$

(ii) G is regular

(iii) the size of the graph is $\frac{n}{2}$

Proof: **In G, by theorem 3.16,** $\boldsymbol{\sigma(v) = \frac{1}{n}}$ **, for all** $\boldsymbol{v \in V}$ **, and**

$$\mu(u,v) = \frac{1}{2n} \text{ , for all } (u,v) \in E.$$

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(i) Now, $deg_G(u) = \sum \mu(u,v), u \neq v$. Since degree of every vertex of $K_{n,n}$ is n, $deg_G(u) = n \times \frac{1}{2n} = \frac{1}{2}$, for all $u \in G$.

(ii) follows, from (i)

(iii) $S(G) = \sum \mu(u,v)$, for all $(u,v) \in E$

Since every edge is of equal weight, $S(G) = |E| \times \mu(u,v)$

$$S(G) = n^2 \times \frac{1}{2n} = \frac{n}{2}$$

Hence follows.

3.19 Theorem

Let G be a RLFG $K_{n,n}(V,E)$. In G the following results hold.

(i) Every edge is a strong edge

(ii) Every edge is β- strong

(iii) Every path is a strong path

Proof: In G, by theorem 3.16, $\sigma(v) = \frac{1}{n}$, for all $v \in V$, and $\mu(u,v) = \frac{1}{2n}$, for all $(u,v) \in E$.

(i) In G, every edge is of same weight,

$$\mu(u,v) = CONN_{G-(u,v)}(u,v).$$

Hence every edge is a strong edge.

(ii) By (i) it follows that every edge is β- strong

(iii) From(i) , (iii) follows

Hence follows.

3.20 Theorem

The diameter of a ratio labelled complete bipartite graph $K_{n,n}(V,E)$ is $\frac{1}{n^2}$.

Proof: For the complete bipartite graph $K_{n,n}$, under ratio labelling,

$$\sigma(v) = \frac{|N(v)|}{|E|} = \frac{1}{n} \text{ for all } v.$$

$$\mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{2n} \text{ , for all } (u,v) \in E,$$

Fuzzy distance between two vertices,

$$d_f(u,v) = \wedge \sum \{ \bigwedge (\sigma(u), \sigma(v)) \times \mu(u,v) \}$$

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In a complete bipartite graph $K_{n,n}$, $\max\{d(u,v)\}$ is 2. Hence in G,

$$\max\{d_f(u,v)\} = \frac{1}{n} \times \frac{1}{2n} + \frac{1}{n} \times \frac{1}{2n}$$
$$= \frac{2}{2n^2} = \frac{1}{n^2}$$

The eccentricity of ratio labelled $K_{n,n}$,

$$e_f(v) = \vee\{d_f(u,v)\}, for\ all\ u \in V$$
$$= \frac{1}{n^2}, for\ all\ v$$
$$diam_f(K_{n,n}) = \max\{e_f(v)\}, for\ all\ v$$
$$= \frac{1}{n^2}$$

Hence follows.

3.21 Theorem

In a complete graph $K_{m,n}(V,E)$, $m \neq n$, for every $v \in V$,

$$\sigma(v) = \begin{cases} \frac{1}{m}, & for\ v \in V_1 \\ \frac{1}{n}, & for v \in V_2 \end{cases} \text{ and } (u,v) \in E \quad \mu(u,v) = \begin{cases} \frac{1}{2m}, & for\ m < n \\ \frac{1}{2n}, & for\ m > n \end{cases} \text{ by RL.}$$

Proof: In $K_{m,n}$, $|V| = m + n$, where $|V_1| = m, |V_2| = n$, $|E| = m \times n$, and

$$|N(v)| = \begin{cases} n & if\ v \in V_1 \\ m & if\ v \in V_2 \end{cases}.$$

Now, $\sigma(v) = \frac{|N(v)|}{|E|} = \begin{cases} \frac{1}{m} & if\ v \in V_1 \\ \frac{1}{n} & if\ v \in V_2 \end{cases}$.

$$\sum_{v \in V} \sigma(v) = m \times \frac{1}{m} + n \times \frac{1}{n} = 2 .$$

Case (i) For $m < n$, $\mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{2m}$, for all $(u,v) \in E$.

Case (ii) For $m > n$, $\mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v \in V} \sigma(v)} = \frac{1}{2n}$, for all $(u,v) \in E$.

Hence, $\mu(u,v) = \begin{cases} \frac{1}{2m} & for\ m < n \\ \frac{1}{2n} & for\ m > n \end{cases}$

Hence follows.

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3.22 Theorem

The necessary and sufficient condition for a complete bipartite graph $K_{m,n}(V,E)$ to be a fuzzy graph for $m \neq n$ under ratio labelling is that

(i) for every $m < n, n \leq 2m$

(ii) for every $m > n, m \leq 2n$

Proof: By theorem 3.21, $\sigma(v) = \begin{cases} \frac{1}{m} \; if \; v \in V_1 \\ \frac{1}{n} \; if \; v \in V_2 \end{cases}$. and $\mu(u,v) = \begin{cases} \frac{1}{2m} \; for \; m < n \\ \frac{1}{2n} \; for \; m > n \end{cases}$ for all $(u,v) \in E$

When $m < n, \mu(u,v) = \frac{1}{2m} < \frac{1}{n} = \sigma(u) \wedge \sigma(v) \; only \; when \; n \leq 2m.$

When $> n, \mu(u,v) = \frac{1}{2n} < \frac{1}{m} = \sigma(u) \wedge \sigma(v) \; only \; when \; m \leq 2n.$

Hence follows.

3.23 Theorem

Let G be a RLFG $K_{m,n}(V,E)$, the following results holds

(i) the degree of every vertex is given by

$$\text{for } m < n, \; deg_G(u) = \begin{cases} \frac{n}{2m}, u \in V_1 \\ \frac{1}{2}, \; u \in V_2 \end{cases}$$

$$\text{for } m > n, \; deg_G(u) = \begin{cases} \frac{1}{2}, u \in V_1 \\ \frac{m}{2n}, \; u \in V_2 \end{cases}$$

(ii) the size of the graph is

$$S(G) = \begin{cases} \frac{n}{2}, m < n \\ \frac{m}{2}, \; m > n \end{cases}$$

Proof: In G, by theorem 3.21, $\sigma(v) = \begin{cases} \frac{1}{m} \; if \; v \in V_1 \\ \frac{1}{n} \; if \; v \in V_2 \end{cases}$.and

$$\mu(u,v) = \begin{cases} \frac{1}{2m} \; for \; m < n \\ \frac{1}{2n} \; for \; m > n \end{cases} \text{ for all } (u,v) \in E$$

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(i) Now, $deg_G(u) = \begin{cases} \sum \mu(u,v), & u \in V_1 \\ \sum \mu(u,v), & u \in V_2 \end{cases}, u \neq v.$

Since degree of every vertex of $K_{m,n}$, is $n\ for\ u \in V_1$ and $m, for\ u \in V_2$

$$\text{In G, for } m < n,\ deg_G(u) = \begin{cases} n \times \frac{1}{2m}, u \in V_1 \\ m \times \frac{1}{2m}, \ u \in V_2 \end{cases},$$

$$= \begin{cases} \frac{n}{2m}, u \in V_1 \\ \frac{1}{2}, \ u \in V_2 \end{cases}$$

$$\text{In G, for } m > n, \text{ in G, } deg_G(u) = \begin{cases} n \times \frac{1}{2n}, u \in V_1 \\ m \times \frac{1}{2n}, \ u \in V_2 \end{cases},$$

$$= \begin{cases} \frac{1}{2}, u \in V_1 \\ \frac{m}{2n}, \ u \in V_2 \end{cases}$$

(ii) $S(G) = \sum \mu(u,v)$, for all $(u,v) \in E$

For $m < n$, Since every edge is of equal weight,

$$S(G) = |E| \times \mu(u,v)$$

$$S(G) = mn \times \frac{1}{2m} = \frac{n}{2}$$

For $m > n$, Since every edge is of equal weight,

$$S(G) = |E| \times \mu(u,v)$$

$$S(G) = mn \times \frac{1}{2n} = \frac{m}{2}$$

Hence follows.

3.24 Theorem

The diameter of a RLFG $K_{m,n}(V,E)$ is $\frac{1}{mn}$.

Proof: **For the complete bipartite graph $K_{m,n}$ under RL, by theorem 3.21,**

$$\sigma(v) = \frac{|N(v)|}{|E|} = \begin{cases} \frac{1}{m} & if\ v \in V_1 \\ \frac{1}{n} & if\ v \in V_2 \end{cases} \text{ for all } v \text{ and } \mu(u,v) = \begin{cases} \frac{1}{2m} & for\ m < n \\ \frac{1}{2n} & for\ m > n \end{cases} \text{ for all } (u,v) \in E$$

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Case(i) For $m < n$, $K_{m,n}$ is a fuzzy graph under RL when $n \leq 2m$

$$\text{For all } (u,v) \in E, \quad \mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v\in V}\sigma(v)} = \frac{1}{2m}.$$

Fuzzy distance between two vertices is given by,

$$d_f(u,v) = \wedge \sum \{\bigwedge (\sigma(u),\ \sigma(v)) \times \mu(u,v)\}$$

In a complete bipartite graph $K_{m,n}$, $\max\{d(u,v)\}$ is 2 for any $u,v \in V$

Hence in RLFG $K_{m,n}$, $\text{Max}\{d_f(u,v)\} = \frac{1}{n} \times \frac{1}{2m} + \frac{1}{n} \times \frac{1}{2m}$

$$= \frac{2}{2mn} = \frac{1}{mn}$$

Case(ii) For $m > n$, $K_{m,n}$ is a fuzzy graph under RL when $m \leq 2n$

$$\mu(u,v) = \frac{\max[\sigma(u),\sigma(v)]}{\sum_{v\in V}\sigma(v)} = \frac{1}{2n}, \text{ for all } (u,v) \in E.$$

Hence for any $u,v \in V$, $\text{Max}\{d_f(u,v)\} = \frac{1}{m} \times \frac{1}{2n} + \frac{1}{m} \times \frac{1}{2n}$

$$= \frac{2}{2mn} = \frac{1}{mn}$$

The eccentricity of ratio labelled $K_{m,n}$

$$e_f(v) = \vee\{d_f(u,v)\}, for\ all\ u \in V$$

$$= \frac{1}{mn}, for\ all\ v \quad \text{from case (i) and (ii)}$$

$$diam_f(K_{m,n}) = \max\{e_f(v)\}, for\ all\ v$$

$$= \frac{1}{mn}$$

This completes the proof.

4. APPLICATION OF RATIO LABELLED FUZZY GRAPHS

The relationship between individuals on social media can be analyzed using ratio labelling. In this context, the individuals in a family or friend group on social media are represented as vertices, and the communication between two individuals is represented by edges. The level of interaction determines closeness of relationship rather than being connected online. Such a friendship analysis was done for a group of friends in social media. Their chats and responses were examined and analyzed using RL. The admissibility of fuzziness is verified using the definition $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$. Graphs that allow fuzziness indicate a strong communication bond between individuals. In this study, a group of friends—Sasi, Mathu, Uma, Sudha, Hema, Joe, Rani, Priya, and Sharmila—

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are considered. The communication between them is analyzed, and a corresponding graph is constructed.

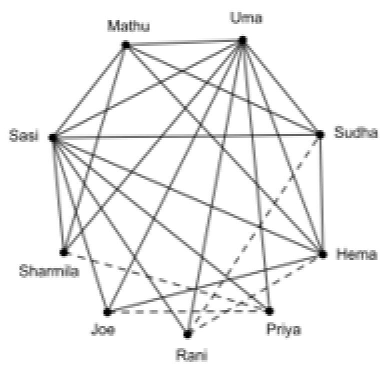

Fig 3

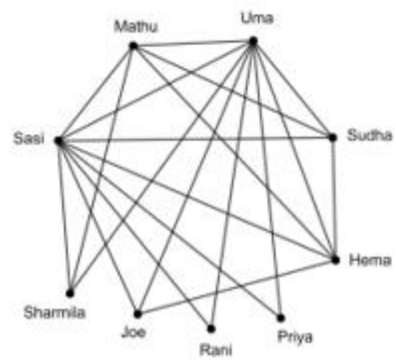

Fig 4

The graph in Fig. 3 is ratio labelled as,

$$\sigma(Sasi) = \frac{8}{20} = \sigma(Uma);\ \sigma(Mathu) = \sigma(Hema) = \frac{5}{20};$$

$$\sigma(Sudha) = \frac{4}{20}; \sigma(Sharmila) = \sigma(Joe) = \frac{3}{20}; \sigma(Priya) = \sigma(Rani) = \frac{2}{20}.$$

For any x, $\neq Sasi, Uma$,

$$\mu(Sasi, x) = \mu(Uma, x) = \frac{8}{40}; \mu(Mathu, x) = \mu(Hema, x) = \frac{5}{40};$$

Now,$\mu(Sasi, Priya) = \frac{8}{40} = \frac{4}{20} > \frac{2}{20} = \sigma(Sasi) \wedge \sigma(Priya)$, which violates the definition of fuzzy graph. This indicates that in a group of nine friends, the individual Priya and some other individuals are not as closely connected with the rest of the group as Sasi and Uma are. This weakens the overall relationship bond between the individuals. Therefore, the graph does not qualify as a fuzzy graph under ratio labelling.

Also, ratio labelling does not expect that every individual communicates with everyone else at all times, which is impractical in reality. It admits fuzziness to some extent, for example, Hema is less communicative when compared to Sasi and Uma, for whom $\mu(Sasi, Hema) = \frac{8}{40} = \frac{4}{20} < \frac{5}{20} = \sigma(Sasi) \wedge \sigma(Priya)$. Thus, fuzziness is neither entirely rejected nor fully accepted by ratio labelling; instead, it is constrained by specific limits based on the degree of the vertices in accommodating fuzziness.

As we are discussing uncertainty, the communication between the people may vary depending on the situation and time. On one such situation, the communication between the friends increases and leads to few more adjacent vertices like (Joe, Priya), (Sudha, Rani), (Sharmila, Priya), (Rani, Hema) thereby the graph becomes fuzzy under RL, see Figure 4.

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5. CONCLUSION

This paper examines the graph for admittance of fuzziness under RL. This paper defines and explores RL and characterized the graph that admits fuzziness. Also, we have examined strength of connectedness of the path of RLFG. It is noted that all regular graphs are RLFG. The effectiveness of the theoretical result has been demonstrated through an application.

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