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FREQUENCY ANALYSIS OF FUNCTIONALLY GRADED BI-LAYERED CYLINDRICAL SHELLS WITH RING SUPPORT BY GALERKIN TECHNIQUE

Afsheen Ayub[1] and Muhammad Nawaz Naeem[2]

[1]La Sapienza University of Rome, Italy.
[2]Government College University, Faisalabad, Pakistan.

Abstract
Buckling of vibrating cylindrical shells is an important aspect in aerospace and defense engineering. The proposed study is conducted to craving a method which is easier but still authentic for predicting the natural frequencies (Hz) of bi-layered cylindrical shells with ring support. The ring support is placed arbitrarily along the axis of the shell. It is assumed that the layers of the shell have a uniform thickness. Both layers are contrived independently by functionally graded technique having the constituents, stainless steel, and nickel. The material properties of the com- ponents of functionally graded layers are supervised by volume fraction power-law distribution and assumed to vary continuously and smoothly throughout the thickness of the layers. By interchanging the position of FGM constituents four kinds of cylindrical shells are formulated and its influence on frequency characteristics are analyzed. The expression for strain and curvature–displacement relationships are obtained by utilizing Love's first approximation of linear thin shell theory. Simply supported end conditions are imposed on edges. For numerical approximations, the Galerkin approach is employed to formulate the frequency equation in the form of the eigenvalue problem. The variation in frequency for various shell parameters as; length, height, radius, the width of layers material constituents and the position of the ring supports position are discussed. Effectiveness, validity, and accuracy of the present methodology has proven by comparing the evaluated numerical results with the results available in the open literature.

Keywords: Cylindrical shells, functionally graded materials [FGM], Bi layered, Ring supported, Volume fraction power law, Galerkin approach.

1 Introduction

Cylindrical shells have their special historical importance for being a significant part of structural engineering. These shells give a robust resistance and support to heavy loads, so they are enormously popular than the other types of shells. During the designing of a functional structural shell, the main objective is to make it as thin, light, and low cost as possible with the required properties like a lightweight load barrier or

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heat resistor, etc. The detailed scientific studies of different mechanical and structural aspects of a shell such as geometrical parameters, material properties, vibration characteristics, and mathematical modeling are done by researchers numerically and experimentally for a successful practical application. The 1st remarkable shell theory was presented by Love [6], by customizing few physical terms in Love's thin shell theory many other shells theories were built in. The performance of a shell depends on its frequency which noticeably influenced by the fabricating material and thickness of the shell or its layers. Simply supported edge conditions are employed in this purposed study by considering, Arnold and Warburton [1], and Bing et al. [4] precise discussion on boundary conditions of thin layered cylindrical shells for a better functional structured shell. In the open literature, a wide range of work is available on shells structured by isotropic or orthotropic materials as compared to the shells composed by functionally graded material. The predictable behavior of these pure materials (isotropic and orthotropic) creates constraints for advanced applications and sometimes becomes a cause of scientific problems, failures, or breakouts. An FG material is a mixture of two or more materials, having different chemical or physical properties, the resulting FG material has the best of both materials. They are used for the sake of high thermal gradient, heat resistance and embellish strength in structuring or designing space crafts, defense instruments, industrial and mechanical components to get better performance for prescribed specific purposes. Mahmood et al. [9] said like alloy FGMs have different properties as compared to their parental materials. These mate- rials have the characteristics of gradual variation in composition and material properties, as they are structure over volume as shown in the figure (1).

The material characteristics of FG materials are functions of position and temperature. The material property **P** for functionally graded material is ex- pressed by Yamanouchi [15] as:

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T^1 + P_2T^2 + P_3T^3)$$

Where $P_{-1}, P_0, P_1, P_2 and P_3$ are the coefficients of temperature T(K). T(K) is exclusive expression for material's fabricators in terms of Kelvin. Loy et al.

[7] examined the vibration aspects of a functionally graded circular cylindrical shell for the first time. Sometimes excessive vibration of functional cylindrical shells in aeronautic or defense engineering becomes a cause of failure. To control the vibration, avoid the unwanted composition of materials, stability and overall improve the performance ring support imply to support a shell. Loy et al. [8] also analyzed the role of ring support on an isotropic shell's frequency. Later, Najafizadeh, et al. [10], Arshad et al. [2], [3], Rahimi [11], [5], [12], [14], [13], and [16] studied further aspects of such shells. In the

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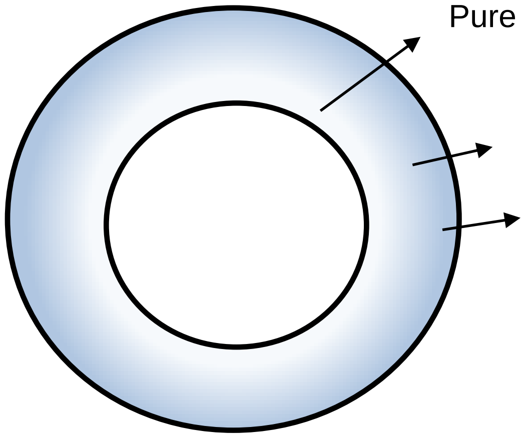

Figure 1: Functionally graded material

current proposed study, the thickness of the shell comprised of two layers, both are assumed functionally graded, comprised of materials; stainless and nickel. Stainless steel is a model material due to attractive qualities of corrosion resistance, low maintenance, staining, and familiar luster. Layers are assumed thin, uniform, linearly elastic and independent. It is being assumed that the layers of shell precisely bounded in the transverse direction at the interface of between two layers with no slip. Thus, the deformation at layer's interface in the shell remains continuous. Four sorts of cylindrical shells are created by exchanging the constituents of FGMs. The alteration of FGM constituents and its effects on natural frequency has been analyzed and discussed. The volume fraction law is utilized for the distribution and control of the material properties of FGM ingredients. Limited work on cylindrical shell having ring support has been observed in the literature review. While ring support has a significant impact on the performance of a functional structural component. To analyze the frequency and avoid excessive vibration, the position of the ring support is taken arbitrarily and its influence on the vibration characteristics is examined and discussed. The first approximation of linear thin shell theory of Love is imposed to get expressions for curvature and strain displacement relationship. By modifying material parameters of the shell as Poisson ratio, mass density, Young's modulus and geometrical parameters the problem is framed into a system of differential equations. Several Finite element methods are used to get approximate solutions to mathematical problems that framed for physical realities such as Differential Quadrature Method (DQM), Rayleigh-Ritz method, Rung-Kutta method, Wave Propagation approach etc. In the current study, the Galerkin technique is adopted for numerical results evaluation. The adopted technique for the present evaluation is the Galerkin approach.

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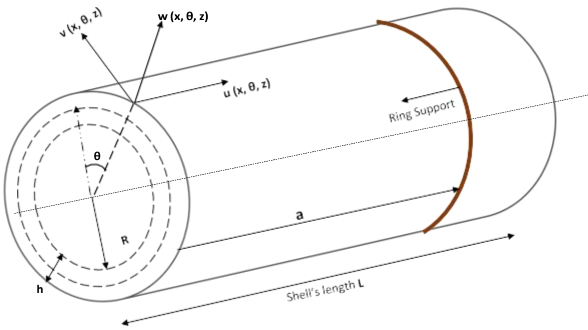

Figure 2: Bi-layered cylindrical shell's configuration

Usually, this technique is employed to determine the coefficients of the power series solution of ordi- nary or partial differential equations (ODE / PDE). In the current study, its adoption converts the system of the equation of motion into partial differential equations (PDEs). The frequency equations are then framed into an eigenvalue problem. By using MATLAB coding numeral values for the frequency parameter

$$\Omega = \omega \mathbb{R} \sqrt{(1-\upsilon^2)\frac{\rho}{\mathrm{E}}}$$

are obtained and then, compared with the results available in the reviewed literature. Correlations of estimated results with those accessible in literature are made to check the legitimacy and precision of the present methodology.

1.1 Theoretical formulation

Assume that the geometrical aspects of presented cylindrical shells are length L, mean radius R and thickness h. (u, v, w) be the deformations defined at the middle surface concerning the (x, θ, z) coordinate system as elaborate in the figure(2), where u(x, θ, t) is the axial deformation function, v(x, θ, t) is the function of circumferential deformation and w(x, θ, t) is transverse deformation function. Material parameters of the shell under discussion are defined by employing the volume fraction power Law as:

$$E\ (\text{Young's modulus}) = E(z) = (E_1 - E_2)\left(\frac{2z-h}{2h}\right)^N + E_2$$

$$\nu\ (\text{Poisson ratio}) = (\nu_1 - \nu_2)\left(\frac{2z-h}{2h}\right)^N + \nu_2$$

$$\rho\ (\text{Mass density}) = (\rho_1 - \rho_2)\left(\frac{2z-h}{2h}\right)^N + \rho_2$$

ere the subscripts 1 and 2 are respectively material parameters of the composer materials (say M_1 and M_2). The fundamental connection among the stress and

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strain expressions are portrayed by the summed-up Hook's law:

$\{\sigma\} = [Q]\,\{e\}$ (1)

Where, $\{\sigma\} = (\sigma_x, \sigma_\theta, \sigma_{x\theta})^t$ and $\{e\} = (e_x, e_\theta, e_{x\theta})^t$, are the component form along the vectors x, θ and the plan- xθ. And Q, the reduced stiffness matrix is as follow

$$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

here Q_{ij} (i, j = 1, 2 and 6) are assumed as interpreted by Rahimi [11], [5] and [3]. Moreover,

$$e_x = e_1 + z\,K_1, \qquad e_\theta = e_2 + zK_2, \qquad e_{x\theta} = e_{12} + 2z\,K_{12}$$

are the strain and curvature displacements, details can be seen in [13] and Love's approximation of linear thin shell theory [6]. The system of equation of motion are also determined by Love's approximation to uniform thin cylindrical shells, which are:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} = \rho_t \frac{\partial^2 u}{\partial t^2} \qquad (2)$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} = \rho_t \frac{\partial^2 v}{\partial t^2} \qquad (3)$$

$$\frac{\partial M_x}{\partial x} + \frac{2}{R}\frac{\partial^2 M_\theta}{\partial \theta^2} - \frac{N_\theta}{R} = \rho_t \frac{\partial^2 w}{\partial t^2} \qquad (4)$$

Here

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$$\{N_x, N_\theta, N_{x\theta}\} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta})\ dz \tag{5}$$

$$\{M_x, M_\theta, M_{x\theta}\} = \int_{-h/2}^{h/2} z\,(\sigma_x, \sigma_\theta, \sigma_{x\theta})\ dz \tag{6}$$

$$\rho_t = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\ dz \tag{7}$$

are internal combined resultant forces, moment resultants and the mass density per unit length. To frame the general problem, let us make successive substitutions of (5)-(7) in (2)-(4), which on simplification gives,

$$\begin{pmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{pmatrix} e_1 \\ e_2 \\ \tau \\ \kappa_1 \\ \kappa_2 \\ \gamma \end{pmatrix}$$

Where A_{ij}, B_{ij} and D_{ij} (i, j = 1, 2, 6) represent the extensional, coupling and bending stiffness respectively and are as defined by S. H. Arshad et al. in [**3**].

1.2 Problem formulation for bi-layered

Since in the present scheme, cylindrical shells are assumed to comprise of two layers of uniform thickness and are fabricated by FG-materials. These layers are superbly fortified along the transverse vector at their interface with no-slip and their deformation is consistent over the layers' interface. So the extensional, coupling and bending stiffness moduli for (i,j=1, 2, 6) are described as:

$$A_{ij} = A_{ij}^{inn} + A_{ij}^{out}, \qquad B_{ij} = B_{ij}^{inn} + B_{ij}^{out}, \qquad D_{ij} = D_{ij}^{inn} + D_{ij}^{out} \tag{10}$$

where the superscripts [inn] and [out] stands for the stiffness moduli respectively of inner and outer layers. The entries of stiffness matrices are evaluated as:

$$\left(A_{ij}^{inn}, B_{ij}^{inn}, D_{ij}^{inn}\right) = \int_{-\frac{h}{2}}^{0} Q_{ij}^{inn}(1, z, z^2)\ dz \tag{11}$$

$$\left(A_{ij}^{out}, B_{ij}^{out}, D_{ij}^{out}\right) = \int_{0}^{h/2} Q_{ij}^{out}(1, z, z^2)\ dz \tag{12}$$

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By taking,

$$Q_{11}^{inn/out} = \frac{E(z)^{inn/out}}{1-\nu(z)^2} = Q_{22}^{inn/out}, \quad (13)$$

$$Q_{12}^{inn/out} = \frac{\nu(z)E(z)^{inn/out}}{1-\nu(z)^2}, \quad (14)$$

$$Q_{66}^{inn/out} = \frac{E(z)^{inn/out}}{2(1+\nu(z))}. \quad (15)$$

Different configurations of shells are formulated by altering the constituents of functionally graded material; stainless steel (SS) and nickel(N) as enlisted in table 1. Simply supported (S-S) boundary conditions, $v = w = N_x = M_x = 0$ and the

Types	Type I	Type II	Type III	Type IV
Inner Layer	SS-N	SS-N	N-SS	N-SS
Outer Layer	N-SS	SS-N	N-SS	SS-N

Table 1: Types of shells by altering constituents

following displacement fields are used for further evaluation of the problem.

$u(x, \theta, t) = A_m U(x) \cos n\theta \cos \omega t,$ (16)

$v(x, \theta, t) = B_m V(x) \sin n\theta \cos \omega t,$ (17)

$w(x, \theta, t) = C_m (x-a) W(x) \cos n\theta \cos \omega t,$ (18)

where A_m, B_m, C_m are the constants, denoting the amplitudes of vibration in x, θ and z directions, whereas a is the arbitrary ring support's position, m is the axial wave number, n shows the circumferential wave numbers and ω (rad s^{-1}) is the natural angular frequency for the bi-layered cylindrical shell. The functions U(x), V(x) and W(x) are chosen as:

$$U(x) = \cos\frac{m\pi x}{L}, \qquad V(x) = W(x) = \sin\frac{m\pi x}{L} \quad (19)$$

By considering, (16), evaluate the expressions (13) and (10), (11), (12), step by step. Then by substituting the obtained values in (9), we get a system of ODE's (ordinary differential equations). By doing some mathematical computation and simplifications, we retrieved the following system in three unknowns.

$$L_{11}A_m + L_{12}B_m + L_{13}C_m = -\omega^2 \rho t U(x) A_m, \quad (20)$$
$$L_{21}A_m + L_{22}B_m + L_{23}C_m = -\omega^2 \rho t V(x) B_m, \quad (21)$$
$$L_{31}A_m + L_{32}B_m + L_{33}C_m = -\omega^2 \rho t (x-a) W(x) C_m, \quad (22)$$

where the expressions for L_{ij} (i, j=1,2,3) are coefficients of A_m, B_m and C_m in terms of $U(x)$, $V(x)$ and $W(x)$ and their ordinary derivatives. To determine the natural frequencies, the Galerkin method is employed. For which, a new system of differential equations is retrieved by multiplying the system of Differential Equations (20), (21) and (22) by U(x), V(x) and (x-a) W(x) respectively. Simplify the definite integral from 0 to L with respect to x, the differential equations turned up to a simultaneous system of homogeneous equations on considered Fourier coefficients. Thus, we obtained the following generalized eigenvalue problem,

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \end{bmatrix} = -\omega^2 \rho_t \begin{bmatrix} I_2 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_{12} \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \end{bmatrix}$$

Where the expressions for c_{ij} (i, j = 1, 2, 3) and I_k (k = 1, 2, 3...14) are precisely described in appendix section. The numerical results of the frequency parameter are obtained by MATLAB coding and verified by comparing with results accessible in literature.

3 Results and Discussion

The current audit gives readers an analytical view of almost all aspects of a shell that has impact on frequency. In this section we discussed the convergence of the Galerkin approach, the impact of ring support and its position on the frequencies of cylindrical shells. Moreover, the influence of variation in physical parameters; L, R, h, the circumferential wave number m, and the power-law exponents "p, q" on vibration characteristics of the shells are analyzed. By employing all the said assumptions, edge conditions, and Galerkin approach, numerical results are evaluated and discussed to check the efficiency, accuracy, and validity of the proposed technique. Material properties are briefly enlisted in the table 2 for the volume fraction power law of FGM distribution.

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Coefficients	Stainless Steel			Nickel		
	$E(Nm^{-2})$	v	$\rho(Kgm^{-3})$	$E(Nm^{-2})$	v	$\rho(Kgm^{-3})$
P_0	$201.04x10^9$	0.3262	8166	$223.95x10^9$	0.31	8900
$P{-1}P_1$	0	0	0	0	0	0
$P{-2}$	$3.079x10^{-4}$	$-2.002x10{-4}$	0	$-2.794x10{-4}$	0	0
P_2	$-6.534x10^{-7}$	$3.797x10^{-7}$	0	$-3.998x10{-9}$ 0	0	0
	0	0	0	$2.05098x10^{11}$	0	0
P_3	$2.07788x10^{11}$	0.31776	8166		0.31	8900

Table 2: Mechanical properties of FG constituents

3.1 Convergence of Glarkin approach

The precision and convergence of the applied strategy with Warburton (1953), Loy and Lam (1997) and G.H. Rahimi (2011) are shown in table 3.

m	n=2			
	Warburton	Loy and Lam	Rahimi	Present
1	2046.8	2050.7	2043.8	2046.401
2	5637.6	5643.3	5635.4	5637.189
3	8935.3	8941.3	8932.5	8933.449
4	11405	11416.9	11407.5	11407.794
5	13245	13262	13253.2	13253.019
	n=3			
1	2199.3	2195.1	2,195.10	2199.049
2	4041.9	4035.5	4,035.50	4041.29
3	6620	6614.6	6,614.60	6619.232
4	9124	9121	9,121.00	9124.109
5	11357	11359	11,359.00	11360.774

Table 3: Convergence of Galerkin approach

The shell configurations are considered as; $L = 8in$, $R = 2in$, $h = 0.1in$, $E = 30 \text{ X } 10^6 lbfin^{-2}$, $v = 0.3$, $\rho = 7.35 \text{ X}10^{-4} lbfs^2 in^{-4}$. It can easily observe that the obtained results are accurate and valid. Well-agreement is seen with the compared ones. So, the proposed approach is a convergent approach see figure 3.

[1]*The Mechanical properties of functionally graded constituents are evaluated for Tc=300K

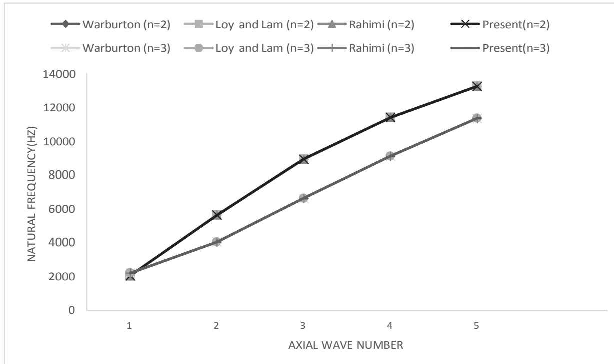

Figure 3: Convergence of Galerkin approach

3.2 Impact of ring support

For the analysis shell's parameters under SS end conditions are considered as L = 20m, h = 0.02m, and R = 1m. The power law exponents p, q =0.5. The axial and circumferential wave number m, n is considered to have value 1. It is observed from evaluated data in table 4 that shell attains its maximum frequency (Hz) when ring support is placed at mid of the shell(longitudinally) and mini- mum values are obtained when the ring support is placed at the ends. It is also observed that frequencies are symmetric to the center. All four types of shell's frequencies follow each other perhaps the shell of type two has comparatively a bit higher frequency than the others. Figure 4 shows that frequencies (Hz) moves upward speedily when the ring supports position vary from x=0 to 0.3 and for x= 0.4, 0.5, 0.6, gradual increase is observed, the frequency curve bend downward with respect to the ring support's movement towards end of the shell. Thus, the behavior of frequencies for a two layered functionally graded ring sup- ported shell follow the same as of Rahimi and Loy's analysis. Figure 5 describes the natural frequency's behavior of FGM bi-layered shells under the conditions: having no ring support, ring support at one end and ring support placed at the middle of the shell. The dominant effect of ring support is observed. The shell with ring support has stable frequency behavior.

3.3 Variation versus shell's configurations

The figure 6 gives a quick view that shell attains higher frequencies (Hz) as number of circumferential waves increased. A jump is seen for n=1 to n=2. It also seen that the curves of all four types of shell behave similarly. Figure 7 depicts that shell's frequency has inverse relation with L, means increase of length produce low frequencies. Also,
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minor or no variation is observed for L 5. Another view of this data demonstrates that for L= 1, 2 frequency curves initially bent down from its maximum frequency, attain minimum value

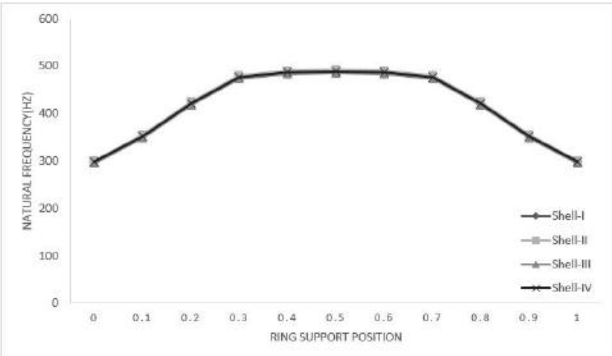

Figure 4: Variation of frequencies versus ring support's position of 4-types of shells

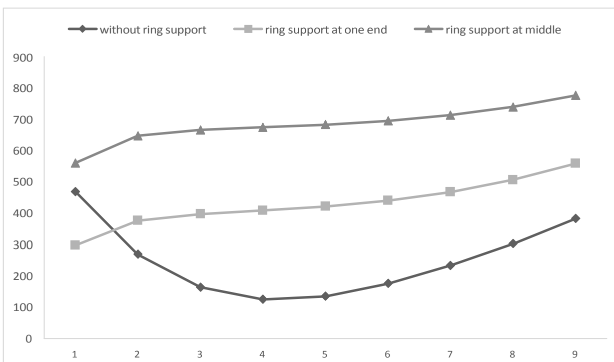

Figure 5: Frequency variation of shell-I in three special cases

x	Shell-I	Shell-II	Shell-III	Shell-IV
0	297.7104534	300.5695264	294.9194262	297.7247264
0.1	352.0505412	355.4010821	348.7790047	352.0669892
0.2	421.2633477	425.1007184	417.5109368	421.2803876
0.3	476.006765	479.5057239	472.5789522	476.0114977
0.4	487.1311515	490.4878878	483.8469864	487.1315289
0.5	488.844461	492.1947535	485.5666817	488.8444135
0.6	487.1311515	490.4878878	483.8469864	487.1315289
0.7	476.006765	479.5057239	472.5789522	476.0114977
0.8	421.2633477	425.1007184	417.5109368	421.2803876
0.9	352.0505412	355.4010821	348.7790047	352.0669892
1	297.7247264	300.5695264	294.9194262	297.7247264

Table 4: Natural frequencies versus ring support's position

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at n=5 and head towards maximum frequency of the shell. While the curves of shell having length equal to 5,10 or 20, frequency curves have successively gradual increase after a small jump at initial circumferential waves. The trend of natural frequencies verses thickness of the shell is illustrated in figure 8. It is examined that as the thickness h increases frequencies get higher. Thickness has very minor or no influence for initial circumferential wave numbers. But for n¿2 variation becomes more prominent. Consequently, for h=0.1 frequency of shell- I gives its maximum value among the present data. The behavior of natural frequency (Hz) of shells for different radii is shown in figure 9. As the radius of the proposed bi-layered FGM cylindrical shell with ring supports at x=0, is increased less variation in natural frequencies is seen. Also, frequency decreases for higher radius means highest frequency is attain for the pair R=1, n=9 and least for R=10, n=9. From figure 10, it is analyzed those natural frequencies becomes higher according to the higher axial wave number. In figure 11 and 12, influence of power law exponents has been examined. Three cases $p < q$, $p > q$, $p = q$, for p the power law exponent of inner layer and q the power law exponent of the outer layer, are considered. And the evaluated results show that power law exponents do not have a specific impact on frequencies of bi-layered ring supported shells and the values in all considered cases chases each other.

4 Conclusion

The current study is about the analysis of natural frequency (Hz) of cylindrical shells having ring support along the longitudinal axis. Structurally shells are comprised of two perfectly bonded functionally graded layers and functionally graded material is composed of stainless steel and nickel. To inquire about the vibration behavior, the current problem framed into an eigenvalue problem by utilizing the thin shell theory of Love and numerically evaluated by Galerkin technique under the simply supported edge conditions.

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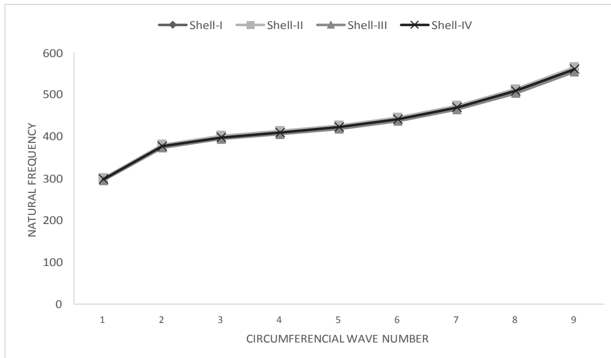

Figure 6: Frequency variation versus circumferential wave number; m=1, a=0, p=q=0.5, L=20m, R=1m, h=0.02m

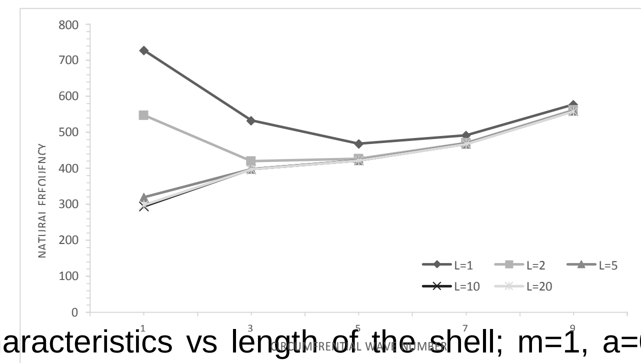

Figure 7: Frequency characteristics vs length of the shell; m=1, a=0, p=q=0.5, R=1m, h=0.02

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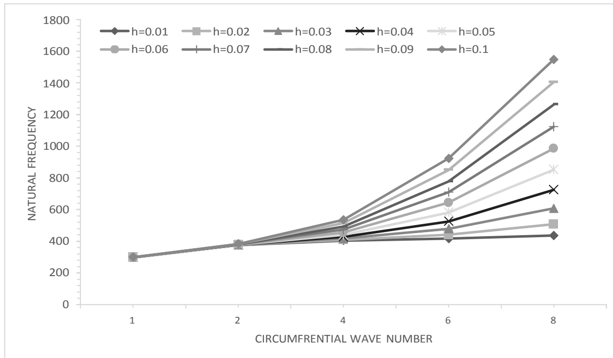

Figure 8: Variation of frequencies versus shell's thickness; m=1, a=0, p=q=0.5, R=1m, L=20m

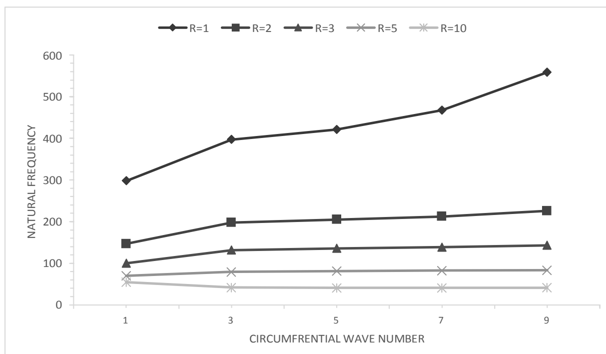

Figure 9: Variation of frequencies versus radius of the shell; m=1, a=0, p=q=0.5, h=0.02m, L=20m

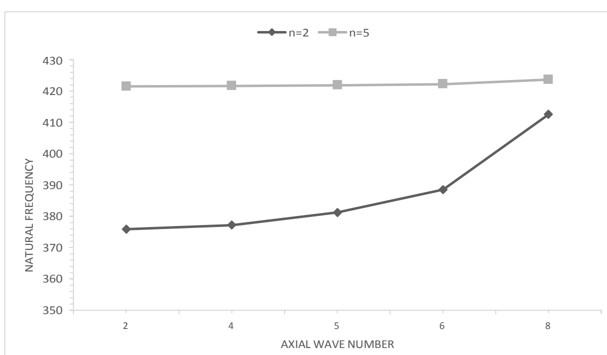

Figure 10: Variation of frequencies versus axial wave number; a=0, p=q=0.5, h=0.02m, L=20m, R=1m

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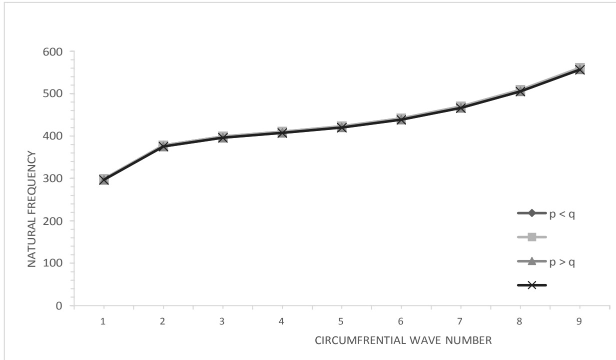

Figure 11: Variation of frequencies for unequal power law exponents p,q; a=0, h=0.02m, L=20m, R=1m, m=1

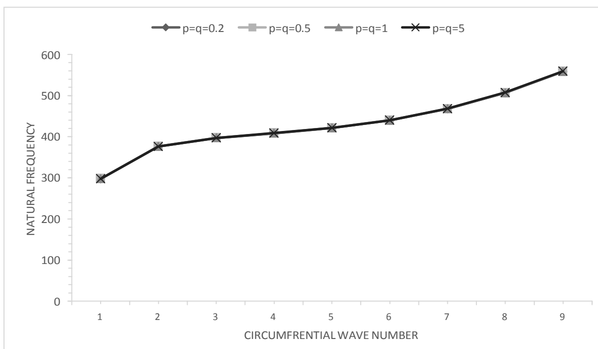

Figure 12: Variation of frequencies for equal power law exponents p, q; a=0, h=0.02m, L=20m, R=1m, m=1

The adopted methodology is verified by doing a comparative study of the present approach with those available in the literature. Ring support has dominant effectiveness on vibration characteristics of the shells. The frequency curve for the ring supports position is symmetric about the center of the shell. By altering the FGM constituents used for layer, 4-types of double layered cylindrical shells are designed. Among these four types, shell-III, whose inner and outer layers are constructed as N - SS and SS – N respectively, gives comparatively minimum values of natural frequency (Hz). In all comparisons for shell parameters, it is examined that frequency curves of shells chase each other. Length, radius, and width of the shell have a clear impact on the shell's natural frequencies. This work could be extended for more layered shells with different FGM constituents or with a different methodology. One could also consider the problem for multi-ring supported shells.

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APPENDIX I

$$L_{11} = A_{11}\frac{dU^2}{dx^2} - \frac{n^2 A_{66}}{R^2}U(x)$$

$$L_{12} = n\left(\frac{(A_{12}+A_{66})}{R} + \frac{(B_{12}+2B_{66})}{R^2}\right)\frac{dV}{dx}$$

$$L_{13} = \left(\frac{A_{12}}{R} + \frac{n^2(B_{12}+2B_{66})}{R^2}\right)\left(W(x) + (x-a)\frac{dW}{dx}\right) - B_{11}\left(3\frac{d^2W}{dx^2} + (x-a)\frac{d^3W}{dx^3}\right)$$

$$L_{21} = -n\left(\frac{(A_{12}+A_{66})}{R} + \frac{(B_{12}+B_{66})}{R^2}\right)\frac{dU}{dx}$$

$$L_{22} = \left(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2}\right)\frac{d^2V}{dx^2} - \frac{n^2}{R^2}\left(A_{22} + \frac{2B_{22}}{R} + \frac{D_{22}}{R^2}\right)V(x)$$

$$L_{23} = \frac{n}{R}\left(B_{12} + 2B_{66} + \frac{D_{12}+2D_{66}}{R}\right)\left(2\frac{dW}{dx} + (x-a)\frac{d^2W}{dx^2}\right) - \left(\frac{n}{R^2}\left(A_{22} + \frac{B_{22}}{R}\right) + \frac{n^3}{R^3}\left(B_{22} + \frac{D_{22}}{R}\right)\right)(x-a)W(x)$$

$$L_{31} = B_{11}\frac{d^3U}{dx^3} - \left(\frac{A_{12}}{R} + \frac{n^2(B_{12}+2B_{66})}{R^2}\right)\frac{dU}{dx}$$

$$L_{32} = \frac{n}{R}\left(B_{12} + 2B_{66} + \frac{D_{12}+4D_{66}}{R}\right)\frac{d^2V}{dx^2} - \left(\frac{n}{R^2}\left(A_{22} + \frac{B_{22}}{R}\right) + \frac{n^3}{R^3}\left(B_{22} + \frac{D_{22}}{R}\right)\right)V(x)$$

$$L_{33} = \left(\frac{2B_{12}}{R} + \frac{n^2(2D_{12}+4D_{66})}{R^2}\right)\left(2\frac{dW}{dx} + (x-a)\frac{d^2W}{dx^2}\right) - \frac{1}{R^2}\left(A_{22} + \frac{2n^2B_{22}}{R} + \frac{n^4D_{22}}{R^2}\right)(x-a)W(x) - D_{11}\left(4\frac{d^3W}{dx^3} + (x-a)\frac{d^4W}{dx^4}\right)$$

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APPENDIX II

$$c_{11} = A_{11} I_1 - \frac{n^2 A_{66}}{R^2} I_2$$

$$c_{12} = n \left(\frac{(A_{12}+A_{66})}{R} + \frac{(B_{12}+2B_{66})}{R^2} \right) I_3$$

$$c_{13} = \left(\frac{A_{12}}{R} + \frac{n^2 (B_{12} + 2B_{66})}{R^2} \right) (I_4 + I_5) - B_{11} (3I_4 + I_6)$$

$$c_{21} = -n \left(\frac{(A_{12}+A_{66})}{R} + \frac{(B_{12}+B_{66})}{R^2} \right) I_7$$

$$c_{22} = \left(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2} \right) I_1 - \frac{n^2}{R^2} \left(A_{22} + \frac{2B_{22}}{R} + \frac{D_{22}}{R^2} \right) I_2$$

$$c_{23} = -\left(\frac{n}{R^2} \left(A_{22} + \frac{B_{22}}{R} \right) + \frac{n^3}{R^3} \left(B_{22} + \frac{D_{22}}{R} \right) \right) I_8 + \frac{n}{R} \left(B_{12} + 2B_{66} + \frac{D_{12}+2D_{66}}{R} \right) (2I_4 + I_9)$$

$$c_{31} = B_{11} I_{10} - \left(\frac{A_{12}}{R} + \frac{n^2 (B_{12}+2B_{66})}{R^2} \right) I_{11}$$

$$c_{32} = \frac{n}{R} \left(B_{12} + 2B_{66} + \frac{D_{12}+4D_{66}}{R} \right) I_9 - \left(\frac{n}{R^2} \left(A_{22} + \frac{B_{22}}{R} \right) + \frac{n^3}{R^3} \left(B_{22} + \frac{D_{22}}{R} \right) \right) I_8$$

$$c_{33} = \left(\frac{2B_{12}}{R} + \frac{n^2 (2D_{12}+4D_{66})}{R^2} \right) (2I_4 + I_{13}) - \frac{1}{R^2} \left(A_{22} + \frac{2n^2 B_{22}}{R} + \frac{n^4 D_{22}}{R^2} \right) I_{12} - D_{11} (4I_4 + I_{14})$$

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APPENDIX III

$$I_1 = \int_0^L U(x)\frac{d^2U}{dx^2}\,dx = \int_0^L V(x)\frac{d^2V}{dx^2}\,dx$$

$$I_2 = \int_0^L U^2(x)\ \ dx = \int_0^L V^2(x)\ \ dx$$

$$I_3 = \int_0^L U(x)\frac{dV}{dx}\,dx$$

$$I_4 = \int_0^L U(x)\,W(x)\ dx\ = \int_0^L U(x)\ \frac{d^2W}{dx^2}\,dx$$
$$= \int_0^L (x-a)\,W(x)\frac{dW}{dx}\,dx = \int_0^L (x-a)\,W(x)\ \frac{d^3W}{dx^3}\ \ dx$$

$$I_5 = \int_0^L (x-a)\,U(x)\frac{dW}{dx}\,dx$$

$$I_6 = \int_0^L (x-a)\,U(x)\frac{dW}{dx}\,dx\ = \int_0^L (x-a)\,U(x)\ \frac{d^3W}{dx^3}\,dx$$

$$I_7 = \int_0^L V(x)\frac{dU}{dx}\,dx$$

$$I_8 = \int_0^L (x-a)\,V(x)\,W(x)\ dx$$
$$I_9 = \int_0^L (x-a)\,V(x)\ \frac{d^2W}{dx^2}\,dx = \int_0^L (x-a)\,W(x)\ \frac{d^2V}{dx^2}\,dx$$

$$I_{10} = \int_0^L (x-a)\,W(x)\ \frac{d^3U}{dx^3}\,dx$$

$$I_{11} = \int_0^L (x-a)\,W(x)\frac{dU}{dx}\,dx$$

$$I_{12} = \int_0^L (x-a)^2\,W^2(x)\ dx$$

$$I_{13} = \int_0^L (x-a)^2\,W(x)\ \frac{d^2W}{dx^2}\,dx$$

$$I_{14} = \int_0^L (x-a)^2\,W(x)\ \frac{d^4W}{dx^4}\,dx$$