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A NOTE ON SUBSPACE HEXA GRAPH TOPOLOGICAL SPACE

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Abstract

In this note we discuss the subspace Hexa graph topological space using the Hexa graph topology ζ_H on a subgraph K of G .which will allow for further exploration of the Hexa graph theoretical approach to subspace Hexa graph topological space such as H-open graphs, H-closed graphs, H-complement of G .Further we join between Hexa graph topological space and minimal dominating sets of a graph G and some types of separation axiom on minimal dominating set of G are studied.

Keywords: Subspace Hexa Graph Topological Space, H-Open Graphs, H-Closed Graphs, H-Complement, HT_o -Space, HT_1 -Space, HT_2 -Space

1. INTRODUCTION AND PRELIMINARIES

The primary aim of this work is to visually illustrate the fundamental results of topology. By observing the graphical illustrations of topological spaces and properties on a two-dimensional surface, many topology results can become fascinating and easy to comprehend. Given a topological space (X,τ) and a subset W of X, the subspace topology on W is defined by $\tau_w = \{W \cap U / U \in \tau\}$. That is, a subset of W is open in the subspace topology if and only if it is the intersection of W with an open set in (X,τ). If W is equipped with the subspace topology then it is a topological space in its own right and is called a subspace of (X,τ).A set $S \subseteq V$ of vertices in graph $G=(V,E)$ is called dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S , S is called a minimal set if no proper subset $S^{"} \subseteq S$ is a dominating set [10].

Definition 1.1 (16). *Let $G=(V,E)$ be an undirected graph with six disjoint graph factorizations $J_h(s)$ where $h=1,2,3,4,5,\ 6$ of G. Consider $S_i, i=1,2,3,4,5,\ 6$ be any six disjoint subgraphs of G , then we define a lower hexa subgraph and upper hexa subgraph as*

$$H_{-}(S_i)=\bigcup_{s\in G}\{s: J_1(s)\subseteq S_1 \vee J_2(s)\subseteq S_2 \vee J_3(s)\subseteq S_3 \vee J_4(s)\subseteq S_4 \vee J_5(s)\subseteq S_5 \vee J_6(s)\subseteq S_6\},$$

$$H^{-}(S_i)=\bigcup_{s\in G}\{s: J_1(s)\cap S_1\neq\phi \wedge J_2(s)\cap S_2\neq\phi \wedge J_3(s)\cap S_3\neq\phi \wedge J_4(s)\cap S_4\neq\phi \wedge J_5(s)\cap S_5\neq\phi \wedge J_6(s)\cap S_6\neq\phi\}$$

The boundary hexa subgraph is defined as $B_N(S)=H^{-}(S_i)-H_{-}(S_i)$.

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Definition 1.2 (16). *Let G be the non-empty undirected graph and $J_h(s), h=1,2,3,4,5,\ 6$ be any six disjoint graph factorization of G. Then for any six disjoint subgraphs $S_i, i=1,2,3,4,5,\ 6$ of G and $\zeta_H(S_i)=\{G,\phi,H_-(S_i),H^-(S_i),B_N(S_i)\}$. Let $S_i \subseteq G, \zeta_H(S_i)$ satisfies the following axioms*

(1) $G, V_o \in \zeta_H(S_i)$, *where G = full graph with edge set, V_o = Null graph*

(2) *Any union of elements of $\zeta_H(S_i)$ is in $\zeta_H(S_i)$*

(3) *The finite intersection of the elements of $\zeta_H(S_i)$ is in $\zeta_H(S_i)$.*

Then a pair $\zeta_H(S_i)$ is known as Hexa graph topology on G with regard to a subgraph S_i of G. We call the pair $(G,\zeta_H(S_i))$ as the Hexa graph topological space on G. The elements of Hexa graph topological spaces are called Hexa open subgraph of G and the complement of the Hexa open subgraph of G is called an Hexa closed subgraph of G.

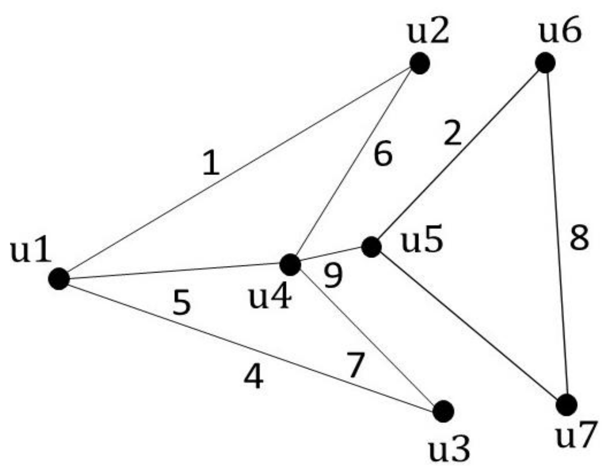
2. SUBSPACE HEXA GRAPH TOPOLOGICAL SPACE

Definition 2.1 *For any Hexa graph topological space (G,ζ_H), consider subgraph K of G, the collection of graphs obtained by taking the intersection of members of ζ_H is Hexa graph topology.*

The formal definition is as follows:

Let (G,ζ_H) be a Hexa graph topological space and let K be a subgraph of G. Then, the subspace Hexa graph topology defined as $\zeta_H^* = \{G_i \cap K : G_i \in \zeta_H\}$, where G_i is Hexa open subgraph in ζ_H is called subspace Hexa graph topological space . It is denoted by ζ_H^*

Example 2.2 *consider an undirected graph G with edges* $\{1,2,3,4,5,6,7,8,9\}$ *and 6 vertices*



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$S_i, i = 1,2,3,4,5,\ 6$	$J_h(s), h = 1,2,3,4,5,\ 6$	$H_-(S_i)$	$H^-(S_i)$
$\{1,3,4,8,5,6,7,9\}$	$\{\{1,4\},\{5\},\{6,7\},\{2,3,8\},\{9\}\}$	$\{1,4,5,6,7,9\}$	G
$\{1\}$	$\{\{1,2,3,4,9,8\},\{5,6,7\}\}$	φ	$\{1,2,3,4,9,8\}$
$\{4\}$	$\{\{1,2,3,4,5,8,6\},\{7,9\}\}$	ϕ	$\{1,2,3,4,5,8,6\}$
$\{2\}$	$\{\{1,2,3,4,5,8\},\{6,7,9\}\}$	ϕ	$\{1,2,3,4,5,8\}$
$\{3\}$	$\{\{1\},\{2,3,4,5,9,8\},\{6,7\}\}$	ϕ	$\{2,3,4,5,9,8\}$
$\{5\}$	$\{\{1\},\{2,3,5,8,9\},\{4,6,7\}\}$	ϕ	$\{2,3,5,9,8\}$

Hence the Lower Hexa subgraph and Upper Hexa subgraph and Boundary hexa subgraph is $H_-(S_i) = \{1,4,5,6,7,9\}$, $H^-(S_i) = \{2,3,8\}$, $B_N(S) = \phi$ then $\zeta_H(S_i) = \{G, \varnothing, \{2,3,8\}, \{1,4,5,6,7,9\}\}$ is a Hexa graph topology of G and the elements are Hexa open subgraphs of G .Take $K = \{1,2\}$ be a subgraph of G then its subspace Hexa graph topology is $\zeta_H^* = \{K, \phi, \{1\}, \{2\}\}$

Lemma 2.3 *Let K_1, K_2, K_3 with the graph operations union and intersection satisfies the following conditions:*

(1) $K1 \cup (K2 \cap K3) = (K1 \cup K2) \cap (K1 \cup K3)$

(2) $K1 \cap (K2 \cup K3) = (K1 \cap K2) \cup (K1 \cap K3)$

proof: Let $K1, K2, K3$ be a subgraphs of an undirected graph G .This graph is characterised by the edge set of G .

Now, consider $K1 \cup (K2 \cap K3)$.

Then

(3) $E(K2 \cap K3) = E(K2) \cap E(K3)$

(4) $E(K1 \cup (K2 \cap K3)) = E(K1) \cup (E(K2) \cap E(K3))$

Since $E(Ki), i = 1,2,3$ are edges, the distribution properties of set are satisfied and hence equation (4) becomes:

(5) $E(K1) \cup (E(K2) \cap E(K3)) = (E(K1) \cup E(K2)) \cap (E(K1) \cup E(K3))$

from (4) and (5), we have

(6) $E(K1 \cup (K2 \cap K3)) = E(K1) \cup E(K2) \cap (E(K1) \cup E(K2)'$

Similarly, the Equation (2) can also be established.

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Theorem 2.4 *Let* (G,ζ_H) *be Hexa graph Topological space and* K *be a subgraph of* G *with* $\zeta_H^* = \{G_i \cap K : G_i \in \zeta_H\}$, *then the null graph* V_0 *with empty edge set and the graph* G *are Hexa open subgraph in* ζ_H^*

Proof: First we check whether the null graph V_0 *and subgraph* K *is in* ζ_H^* *.By the definition of* ζ_H^* *, the members of* ζ_H^* *are the graphs obtained by the intersection of members of* ζ_H *and* K*.Since* G *and* V_o *are in* ζ_H*, we have* $V_o \cap K = V_o \in \zeta_H^*$ *and* $G \cap K = K \in \zeta_H^*$*. Hence,* V_0 *and* K *are Hexa open in* ζ_H^* *.*

Theorem 2.5 *Let* $G_1, G_2,..., G_n$ *be Hexa open subgraphs in* ζ_H *and* K *be a subgraph of* G *with* $\zeta_H^* = \{G_i \cap K : G_i \in \zeta_H\}$ *,then* ζ_H^* *is Hexa closed subgraph under any intersection.*

Proof: Given $G_1, G_2,..., G_n$ *be Hexa open subgraphs in* ζ_H*. Since* ζ_H *is a Hexa graph topology on* G*.To prove that* ζ_H^* *is Hexa closed under any intersection. We have* $\zeta_H = \{G, V_0, G_1, G_2, G_3,..., G_n\}$*,the Hexa graph topology on* G*.Consider the collection,* $\zeta_H^* = \{G \cap K, V_0 \cap K, G_1 \cap K, G_2 \cap K, G_3 \cap K,..., G_n \cap K\}$*. Since* ζ_H *is a graph topology, it is Hexa closed under any intersection. That is for any Hexa open graphs* G_i *and* G_j *in* ζ_H *their intersection* $G_i \cap G_j$ *is Hexa open subgraph in* ζ_H*.*

Now consider the graphs $(G_i \cap K)$ *and* $(G_j \cap K)$ *in* ζ_H^* $(G_i \cap K) \cap (G_j \cap K) = G_i \cap K \cap G_j \cap K = (G_i \cap G_j) \cap K \in \zeta_H^*$*. Therefore,* $\{G_i \cap G_j\} \cap K \in \zeta_H^*$ *This can be extended to any intersection of graphs. Hence, we can say that* ζ_H^* *is Hexa closed under intersection.*

Theorem 2.6 *Let* $G_1, G_2,..., G_n$ *be Hexa open subgraphs in* ζ_H*.Since* ζ_H *is a Hexa graph topology on* G *,then* ζ_H^* *is Hexa closed under union.*

Proof: *Now, we need to show that* ζ_H^* *is Hexa closed under union. For this let us consider two Hexa open subgraphs* Gi *and* Gj *of* G*. Then,* $(Gi \cap K)$ *and* $(Gj \cap K)$ *belongs to* ζ_H^* *by the Definitions of subspace Hexa graph topological space . Now, we have to prove that* $(G_i \cap K) \cup (G_j \cap K) \in \zeta_H^*$ *we have* $(G_i \cap K) \cup (G_j \cap K) = (G_i \cup G_j) \cap K \in \zeta_H^*$*.By Theorem 2.4, we can say that any union of graphs are Hexa closed in* ζ_H^*

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Theorem 2.7 *Let $K1, K2, K3, ..., Kj, ...$ be any graphs. Then, for a fixed integer $i \in \{1, 2, ..., \}$ Consider a Hexa graph topological space (G, ζ_H). Let K be a subgraph of G and $\zeta_H^* = \{G_i \cap K : G_i \in \zeta_H\}$. Then ζ_H^* is Hexa graph topology on K.*

Proof: Now, we shall prove that the collection Hexa graph topology on the subgraph K of G. $\zeta_H^* = \{G_i \cap K : G_i \in \zeta_H\}$ is a Hexa graph topology on the subgraph K of G. From Theorem 2.3,2.4,2.5 we prove that the collection ζ_H^* is Hexa graph topology on K

Lemma 2.8 *Let (K, ζ_H^*) be a subspace graph topological space of (G, ζ_H). If P is Hexa open subgraph in ζ_H^* and K is an Hexa open subgraph of ζ_H, then P is an Hexa open subgraph of ζ_H.*

Proof: Given that K is a subgraph of G and (K, ζ_H^*) a subspace graph topological space of (G, ζ_H). Let P be a Hexa open subgraph of K which implies that $P \in \zeta_H^*$ and let K be a Hexa open subgraph of ζ_H, that is $K \in \zeta_H$. We need to show that P is a Hexa open subgraph of G. Since P is a Hexa open subgraph of K, $P = G_i \cap K$ for some $G_i \in \zeta_H$. Since G_i and K are Hexa open subgraph, by the third axiom of Hexa graph topology, their intersection is Hexa closed in ζ_H. That is $P = G_i \cap K \in \zeta_H$. Hence, P is Hexa open in G.

Lemma 2.9 *Let (G, ζ_H) be a Hexa graph topological space and (K, ζ_H^*) be the subspace Hexa graph topological space of (G, ζ_H). Let P be a subgraph of K. Then, the subspace Hexa graph topology on P inherited from the Hexa graph topology (K, ζ_H^*) is the same as the subspace Hexa graph topology on P inherited from the Hexa graph topology (G, ζ_H).*

Proof: Let (G, ζ_H) be a Hexa graph topological space and (K, ζ_H^*) be a subspace Hexa graph topological space of G. Let P be a subgraph of K. consider the subspace Hexa graph topological space of P inherited from K that is, $\zeta_H^*(p) = \{K_i \cap P : K_i \in \zeta_H^*\}$. Since $K_i \in \zeta_H^*$, by Definition , we have $K_i = G_i \cap K$ for some $i \in I$. Therefore,

$\zeta_H^*(p) = \{(G_i \cap K) \cap P : G_i \in \zeta_H\} = \{G_i \cap (K \cap P) : G_i \in \zeta_H\} = \{G_i \cap P : G_i \in \zeta_H\}$. Hence, we can say that the subspace Hexa topology on P obtained from the subspace Hexa graph topological space on K is the same as the subspace Hexa graph topology on P inherited from the Hexa graph topology on G.

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3 . H-CLOSED GRAPHS IN SUBSPACE HEXA GRAPH TOPOLOGICAL SPCAE

In graph topology, a graph is either d-closed or is neighborhood closed in a topological space if the decomposition complement or the neighborhood complement is open. But in Hexa graph topology, a graph is H-closed if the H-complement is Hexa open graphs. In this section, we discuss the H-closed graphs in subspace Hexa graph topology.

Definition 3.1 *Let* G *be a graph and let* $K=(V_K,E_K)$ *be a subgraph of the graph* $G=(V,E)$*. The complement of the subgraph* K *with respect to the graph* G *is the graph* $K^*=(V^*,E^*)$ *induced by the edge set* $E^*=E-E_K$ *is called the H-complement of* K*.*

Definition 3.2 *A subgraph* K *in a Hexa graph topological space is H-closed if its H - complement* K^* *is Hexa open in the Hexa graph topological space.*

Theorem 3.3 *Let* (K,ζ_H^*) *be a subspace Hexa graph topological space of a Hexa graph topological space* (G,ζ_H) *for a subgraph K. Suppose that a subgraph* M *is H-closed in* (K,ζ_h^*)*. Then, the edge set of* M *is,* $E(M)=E(K)-E(G_i\cap K)$ *where* $G_i\in\zeta_H$*.*

Proof: Let (G,ζ_H) be a Hexa graph topology and (K,ζ_H^*) be a subspace Hexa graph topology for a subgraph K of G. Let M be an H-closed subgraph in (K,ζ_H^*). Then, by Definition of H-closed the decomposition complement M^* of M is Hexa open in (K,ζ_h^*). Since M^* is Hexa open in ζ_H, then $M^*=K_i\cap K$ where $K_i\in\zeta_H$.

$$E(M^{*-})=E(K)-E(M)E(Gi\cap K)=E(K)-E(M)E(M)=E(K)-E(G_i\cap K)$$

When $|Gi\cap K|<|E(K)|$, and $|G_i\cap K|=\phi$ then, $E(K^*)$ will be a proper subset of $E(K)$ and hence the subgraph with this edge set will be a proper subgraph of K.

Theorem 3.4 *In a subspace Hexa graph topological space, the graph* K *and the null graph* V_0 *are H-closed.*

Proof. Let (K,ζ_H^*) *be a subspace Hexa graph topological space of a Hexa graph topological space* (G,ζ_H) *for a subgraph* K*. We need to prove that the graph* K *and the null graph* V_o *are H-closed. By Theorem 3.3, for any H-closed graph* M *of subspace Hexa graph topological space,* $E(M)=E(K)-E(K_i\cap K)$ *where* $K_i\in\zeta_H$*. Suppose* $K_i\cap K=K$ *then,* $E(M^{*-})=E(K)-E(K)=\phi$*. Then, the subgraph induced by empty edge set becomes* V_o*. Hence,* M *will be* V_o*. Now, suppose that* $E(K_i\cap K)=\phi$ *then,* $E(M^{*-})=E(K)-\phi=E(K)$ *and the subgraph induced by the edge set* $E(M)$ *will be K. Hence, K is Hexa closed*.

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Theorem 3.5 *Let (G,ζ_H) be a Hexa graph topological space and (K,ζ_H^*) be a subspace Hexa graph topological space. Let K_i be a H-closed subgraph in ζ_H, then $K_i \cap K$ is a H-closed subgraph in the subspace Hexa graph topological space (K,ζ_H^*).*

Proof: Let (G,ζ_H) be a Hexa graph topological space and K_i be a H-closed graph in the subspace Hexa graph topological space. Then, by Definition of H-closed, the H-complement of K_i is Hexa open in ζ_H and

$$E(Ki^*) = E(G) - E(Ki)$$

The subgraph induced the edge set $E(Ki^*)$ is Hexa open in ζ_H.

$$E(Ki^*) = E(G) - E(Ki)$$

Since the K is H-closed

$E(K) = E(K^*)$ *and*

$E(K_i) \cap E(K) = E(Ki \cap K)$, *we have*

$E(Ki^*) \cap E(K^*) = E(K) - E(Ki \cap K)'$

$E(Ki^* \cap K) = E(K) - E(Ki \cap K)$. *By Definition of subspace Hexa graph topological space, we have $Ki^* \cap K \in \zeta_H^*$, and hence $Ki \cap K$ is Hexa closed in ζ_H^*.*

4. SOME TYPES OF SEPARATION AXIOMS IN HEXA GRAPH TOPOLOGICAL SPACE

Throughout this section, we take any two minimal edge dominating sets among family of all minimal dominating sets of a graph G. For that introduced some new definitions such as HT_0-graph, HT_1-graph, HT_2-graph. Also use minimum edge dominating sets to satisfy hausdorff axiom of undirected graph G

Definition 4.1 *A graph G with Hexa graph topological space is said to be HT_o-space if h_1, h_2 be a edges of G there exist an minimal edge dominating sets $D \subseteq G$ such that either $h_1 \in D, h_2 \notin D$ (or) $h_2 \in D, h_1 \notin D$*

Definition 4.2 *A graph G with Hexa graph topological space is said to be HT_1-space if h_1, h_2 be a edges of G there exist an minimal edge dominating sets $D_1, D_2 \subseteq G$ such that $h_1 \in D_1$, $h_2 \notin D_1$ and $h_2 \in D_2$, $h_1 \notin D_2$*

Definition 4.3 *A graph G with Hexa graph topological space is said to be HT_2-space if $h_1, h_2 \in G$ there exist an disjoint minimal edge dominating sets $D_1, D_2 \subseteq G$ such that $h_1 \in D_1$ and $h_2 \in D_2$ also $D_1 \cap D_2 = \phi$*

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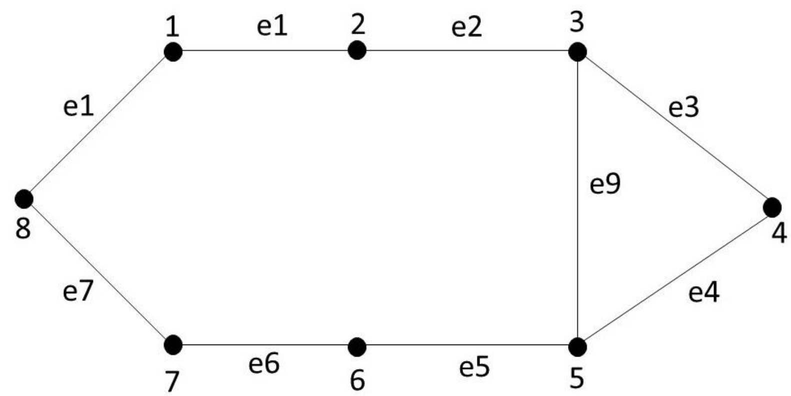
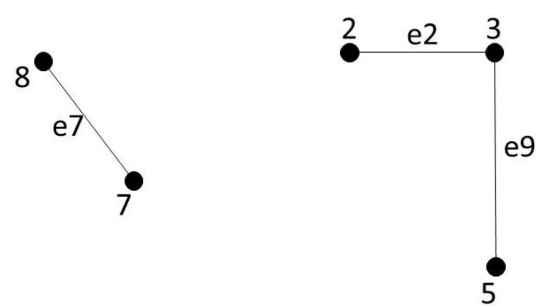
Example 4.4 *Let* G *be an undirected graph with 8 vertices* $\{1,2,3,4,5\ ,6,7,8\}$ *and 9 edges* $\{e1, e2, e3, e4, e5, e6, e7, e8\}$

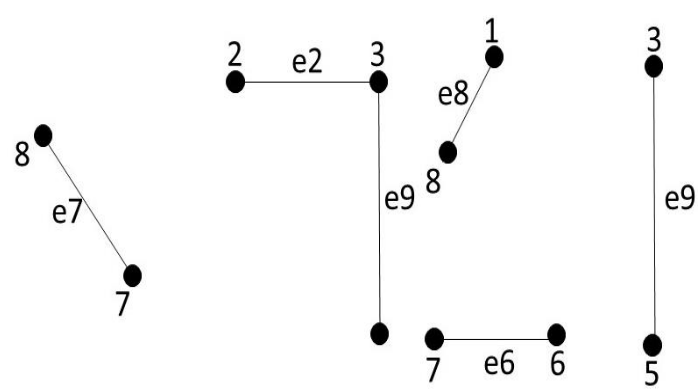


(1) Let $h_1 = e_7$, $h_2 = e_8$ be the distinct edges of G then the Minimal edge dominating set D is



Here $D \subseteq G$ such that $h_1 = e_7 \in D$, $h_2 = e_8 \notin D$ then G is HT_o -space.

(2) Let $h_1 = e_7$, $h_2 = e_6$ be the distinct edges of G then the Minimal edge dominating sets D_1, D_2 as follows :



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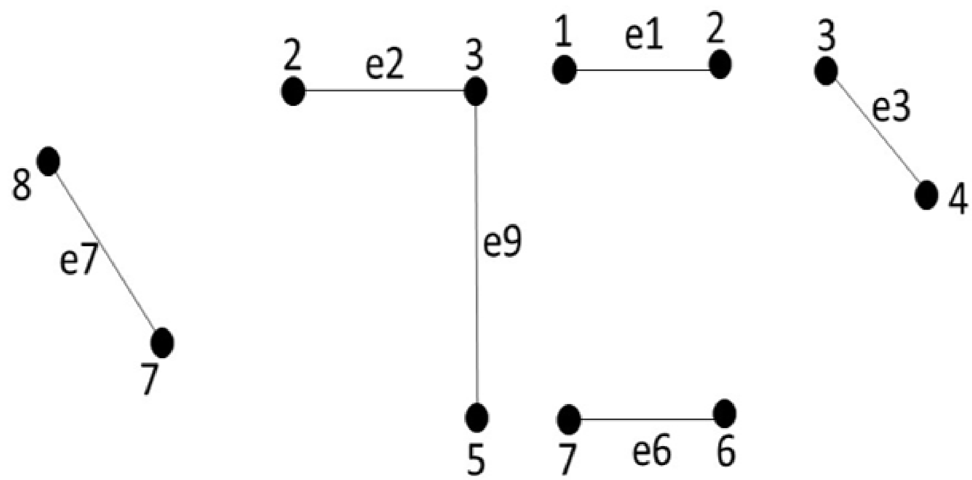
Take $h_1 = e_7 \in D_1$, $h_2 = e_6 \notin D_1$ and $h_2 = e_6 \in D_2$, $h_1 = e_7 \notin D_2$ then G is HT_1-space and D_1, D_2 may not be disjoint.

(3) Let $h_1 = e_7$, $h_2 = e_6$ be the distinct edges of G then the Minimal edge dominating sets D_1, D_2 as follows :



Take $h_1 = e_7 \in D_1$, $h_2 = e_6 \in D_2$ and $D_1 \cap D_2 = \phi$ then G is HT_2-space.

Theorem 4.5 *If a graph G is HT_2-space then it is HT_1 space*

Proof: Let G is HT_2 space then for any two distinct edges e_1, e_2 there are two disjoint minimal edge dominating sets D_1, D_1 such that $e_1 \in D_1$ and $e_2 \in D_2$. To prove G is HT_1-space. It is clearly that for all two edges e_1, e_2 there are any two minimal edge dominating sets D_1, D_2 such that $e_1 \in D_1$, $e_2 \notin D_1$ and $e_2 \in D_2$, $e_1 \notin D_2$. Hence G is HT_1- space.(by the definition of HT_1 space)

Theorem 4.6 *If a graph G is HT_1-space then it is H T_0-space.*

Proof: Let G is HT_1-space .To prove G is HT_0-space for all two edges e_1, e_2 there are two minimal edge dominating sets D_1, D_2 such that $e_1 \in D_1$, $e_2 \notin D_1$ thus G is HT_o-space.

5. CONCLUSION

The main purpose of the present paper is the relationship between hexa graph topology and subspace hexa graph topology were studied. And some types of seperation axioms in Hexa graph topological space were discussed. Further studies in this area are yet to be observed.

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