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A STUDY ON STRONGLY P-REGULAR TERNARY NEAR-RINGS

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ABSTRACT:

Here, we introduced the notion of strongly P-regular ternary near-ring. We defined ternary near-ring and discussed some of the theorems. The intent of this paper is to testify some concepts of strongly regular and strongly P-regular.

Keywords: *Ternary near-ring (TNR), Regular, P-regular, Strongly regular, Strongly P-regular, Zero-symmetric, Bi-ideal.*

INTRODUCTION:

Twenty-five years ago, tardy in 1968, the first conference on near-rings and near-field in the mathematische Forschungsinstitut oberwolfach in Germany. To deduced mathematical theorems from acongruous example. A right ternary near-ring is a generalization of a near-ring in ternary context. Algebraic structures and their properties in depth. The ternary algebraic system concept was first debut by Lehmer in 1932. Dutta and Kar debuted the notion of ternary semiring which is a generality of the ternary ring presented by Lister. Near-rings have a number of fascinating applications ranging from Geometry.

The topic of a conventional near-ring was acquainted in 1968 by J.C.Beidleman and later S.Leigh and H.E.Healtherly etc... S.J.Choi elongated P-regularity of a near-ring. Customary (von-Neumann customary) ring plays a consequential role in the structure theory of rings which was first introduced by Von-Newmann.

Definition: Let Ↄ be a non-empty set together with a binary operation addition and a ternary operation [ ] : Ↄ×Ↄ×Ↄ → Ↄ. Then (Ↄ,+,[ ]) is a right TNR.

1. (N,+) is a group.
2. $[[\zeta_1\zeta_2\zeta_3]\zeta_4\zeta_5]=[\zeta_1[\zeta_2\zeta_3\zeta_4]\zeta_5]=[\zeta_1\zeta_2[\zeta_3\zeta_4\zeta_5]]=[\zeta_1\zeta_2\zeta_3\zeta_4\zeta_5]$ for every $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5 \in$ Ↄ
3. $[(\zeta_1+\zeta_2)\zeta_3\zeta_4]=[\zeta_1\zeta_3\zeta_4]+[\zeta_2\zeta_3\zeta_4]$ for every $\zeta_1,\zeta_2,\zeta_3,\zeta_4 \in$ Ↄ .

Similarly left TNR and lateral TNR can be defined.

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Definition: Let Ɲ be a right TNR. Then Ɲ 0={𝓃 ϵ Ɲ / [𝓃00] =0} is the **zero-symmetric** part of Ɲ .

Definition: A right TNR is **regular**, if for every rβϵ Ɲ there exists ζ1,ζ2,ζ3 ϵ Ɲ such that rβ=rβ[ζ1ζ2ζ3]rβ=rβζ∗rβ. A regular element rβ ϵ Ɲ may equivalently be defined as rβ=[rβζ1rβζ2rβ] for some ζ1,ζ2 ϵ Ɲ . If Ɲ is regular then it is obvious that [Ɲ Ɲ Ɲ]=Ɲ .

Definition: An ideal P of Ɲ is **P-regular**, if rβϵ N there exists [ζ1ζ2ζ3=ζ∗] ϵ Ɲ such that rβ=rβζ∗rβ+p for some p ϵ P.

Definition: An ideal P of Ɲ is **Strongly P-regular**, if for every rβϵ Ɲ there exists [ζ1ζ2ζ3=ζ∗] ϵ Ɲ such that rβ=ζ∗rβ2+p for some p ϵ P.

Example: If module 6, Z6 is a strongly P-regular TNR.

Theorem 1: If Ɲ is a left strongly P-regular TNR. Then rβ=ζ∗rβ2+p for some rβϵ Ɲ and [ζ1ζ2ζ3=ζ∗]ϵ Ɲ where P is an arbitrary ideal .It is also a P-regular as well as regular.

Proof:

Since Ɲ be a left strongly P-regular right TNR.

We know that left strongly P-regular is rβ =[ζ1ζ2ζ3]rβ2+p=ζ∗rβ2+p .

Then by using couples of lemma in near-ring, we have (rβ−(ζ∗rβ2+p))=00, rβ(rβ −(ζ∗rβ2+p))=rβ00, rβζ∗rβ(rβ −(ζ∗rβ2+p))=rβζ∗rβ00.

Therefore (rβ−(ζ∗rβ2+p))2=(rβ−(ζ∗rβ2+p))(rβ −(ζ∗rβ2+p)) =rβ00−rβζ∗rβ00+p
= (rβ00−rβζ∗)rβ00+p.

Now (ζ∗rβ2+p))3=(rβ −(ζ∗rβ2+p))(rβ−(ζ∗rβ2+p))2=(rβ−(ζ∗rβ2+p))(rβ00−rβζ∗)rβ00+p=(rβ00−rβζ∗)rβ00rβ+p(rβ)−(rβ00−rβζ∗)rβ00(ζ∗rβ2+p)−p(ζ∗rβ2+p)=(rβ00−rβζ∗)rβ00+p(rβ)−(rβ00−rβ00)(ζ∗rβ2+p)−p(ζ∗rβ2+p)
=(rβ00−rβζ∗)rβ00+p.

Similarly (rβ−(ζ∗rβ2+p))2=(rβ−(ζ∗rβ2+p).

Hence we have 0=(rβ−(ζ∗rβ2+p)rβ =(rβ−(ζ∗rβ2+p))2 rβ =(rβ00−rβζ∗)r100+p =(rβ−(ζ∗rβ2+p))2 =(rβ−(ζ∗rβ2+p).

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Therefore we get rβ=ζ∗rβ2+p =[ζ1ζ2ζ3]rβ2+p=ζ∗rβ2+p .Since, P is an arbitrary Ideal {0}. Hence rβ= rβζ∗ rβ.

Theorem:2: If ℷ be a right strongly P-regular TNR.Then rβ=rβ2ζ∗+p for some rβϵ ℷ and [ζ1ζ2ζ3=ζ∗]ϵ ℷ̂ where P is an ideal. It is also a P-regular.

Proof:

Similar proof of the above theorem.

Proposition: If ℷ is a TNR with unital element i and (ϱϱi)=ϱ then ϱ is an idempotent.

Proof:

Let ϱ=ϱϱi for ϱ ϵ ℷ and i is an unital element.

Then ϱ=ϱϱi=ϱ(ϱϱi)i=ϱϱ(ϱii)=ϱϱϱ=ϱ3.

Similarly we get iϱϱ=ϱiϱ=ϱ then ϱ3= ϱ.

Theorem:3: If any idempotent ϱ and any ζ∗ in ℷ , ϱ=ϱ ζ∗ϱ+p where P be a arbitrary ideal.

Proof:

Let ϱ2=ϱ and [ζ1ζ2ζ3=ζ∗]ϵ ℷ , then clearly ϱ=ζ∗ϱ2 +p.

We have (ϱ−(ϱ ζ∗ϱ+p))=00

And ϱ(ϱ−(ϱ ζ∗ϱ+p))=ϱ00, ϱ ζ∗ϱ(ϱ−(ϱ ζ∗ϱ+p))=ϱζ∗ϱ00.

Using same way for above theorem, We have (ϱ−(ϱ ζ∗ϱ+p))2=(ϱ00−ϱζ∗)ϱ00+p.

Therefore (ϱ−(ϱ ζ∗ϱ+p))3=(ϱ−(ϱ ζ∗ϱ+p))2,

similarly (ϱ−(ϱ ζ∗ϱ+p))2=(ϱ−(ϱ ζ∗ϱ+p)).

Consequently, We get

0=(ϱ −(ϱ ζ∗ϱ+p))

=(ϱ −(ϱ ζ∗ϱ+p))ϱ =(ϱ −(ϱ ζ∗ϱ+p))2 ϱ =ϱ −ϱ ζ∗ϱ+p.

Hence ϱ =ϱ ζ∗ϱ+p.

Theorem:4: A ternary sub near-ring B1of a regular TNR is a bi-ideal of TNR. iff B1=B1ℷ B1.

Proof:

If B1=B1ℷ B1,then we have B1 is a bi-ideal of ℷ .Only if, Let take B1 is a bi-ideal of a regular TNR. Let bα ϵ B1,there exist ζ∗ ϵ ℷ such that bα=bαζ∗ bα.

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This implies that bα ϵ B1ℷ B1and hence B1=B1ℷ B1.

Again B1ℷ B1⊆B1ℷ B1ℷˆB1⊆B1(ℷ B1ℷ)B1⊆B1B1B1⊆B1.

Hence B1=B1ℷ B1.

Theorem:5: A ternary sub near-ring B1 of a strongly TNR of ℷ is a Bi-ideal of ℷ iff B1=B12 ℷ .

Proof:

If we know that B1=B1ℷ B1=B1(B1ℷ)=B12ℷ .

Only if, take B1 is a Bi-ideal of a regular TNR of ℷ .

Theorem:6: A ternary sub near-ring B1 of a strongly P-regular near-ring ℷ is a bi-ideal of ℷ iff it is strongly P-regular TNR of ℷ .

Proof:

We know that, If ℷ is strongly P-regular TNR. (i.e) bα =bα 2ζ∗+p where bα ϵ ℷ ,ζ∗ ϵ ℷ and p ϵ P.

By using above theorem, Let p is a bi-ideal(B) of TNR, such that bα =bαζ bα .

Then bα =bα 2 ζ∗+bα ζbα =bα 2ζ∗+bα⊆b1 2ζ∗+p.

Again

bα 2ζ∗+p=bα 2ζ∗+bα ζbα =bα 2ζ∗+bα (bα n)=bα 2(ζ∗+ζ)=bα 2ℷ =bα ℷ bα ⊆bα .

Therefore bα =bα 2ζ∗+p.

Theorem:7: If right TNR is a zero symmetric. Then ϱ an idempotent bi-ideal of ℷ .

Proof:

Let ϱ be an idempotent bi-ideal B1 of ℷ , for some bα ϵ B1 ,ζ ϵ ℷ ,ϱ ϵ B1 .

Then (i) [ϱϱϱ]=ϱ ,(ii) [ϱ(bαϱbα)ϱ]⊆ϱ,(iii) [bα(ζϱζ)bα]⊆bα.

Now[ϱ[ζϱζ]ϱ]=[ϱϱϱ][ζϱζ][ϱϱϱ]⊆[ϱ[ϱ[ϱζϱ]ϱ]ϱ]⊆[ϱϱϱ]=ϱ.

Hence ϱ is a Bi-ideal of ℷ .

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CONCLUSION:

Here this paper bi-ideal, regular, P-regular and strongly P-regular ternary near-rings were defined and discussed some of the theorems. In a zero-symmetric ternary near-ring concept utilizing some of the theorems. Then bi-ideals in a strongly regular ternary near-ring have been realized as right ternary sub near-ring as well as strongly P-regular ternary near-ring. For future work a homogeneous approach can be explored for different kinds of the ideal utilizing their ideal (P) concept.

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