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LAPLACIAN SPECTRUM OF FUZZY CYCLE

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Abstract

Laplacian spectrum plays a vivacious part in the field of sciences. Multiplicities of integer eigenvalues and spectral effects of various modifications for Laplacian spectrum of simple graphs has been conferred [1]. But, Laplacian spectrum of fuzzy graphs, because of its complexity in calculation, is a barrier in utilizing the concept in various fields. When the fuzzy graph is larger there arise a grim in calculation part. Minor error at any point will disturb the spectrum of Laplacian drastically. Our idea is to overcome the exertion and make it unpretentious by finding the Laplacian spectrum of any fuzzy graph directly from its edge membership value or vertex membership value. Thoroughly analyzing various types of fuzzy graphs for its characteristics, contour and comportment of its Laplacian Spectrum and explored some interesting outcomes. In this study, we focused on fuzzy cycle and vertex regular fuzzy cycle for its structure, pattern of labelling so as to deduce its least and greatest eigenvalues of its corresponding Laplacian matrix as well as their behavior by using only the edge membership value of the given graph. This study approaches the least and greatest eigen values of Laplacian matrix for a fuzzy cycle directly from its edge membership value itself.

Keywords: Laplacian Matrix, Laplacian Spectrum, Fuzzy Graph, Fuzzy Cycle, Vertex Regular Fuzzy Cycle.

1. INTRODUCTION

In 1973, Kaufmann defined fuzzy graph theory constructed on Zadeh's idea on fuzzy sets [3][4]. Later, in 1975, Azriel Rosenfeld elaborated on fuzzy graph theory [2]. In 2008, Nagoor Gani and Radha has established few results on regular fuzzy graphs [7].

We can find application of Laplacian spectrum of simple graphs in various fields but, application of Laplacian spectrum of fuzzy graphs has not been taken up because of its tremendous work in calculation. This invigorated us to work on it so as to develop an application which will effectively help in finding the Laplacian spectrum of given fuzzy graph only by considering either its vertex membership values or the edge membership values. We have already depicted an algorithm which computes Laplacian spectrum of Complete fuzzy graphs where without following the regular way by giving the vertex membership values of given fuzzy graph, we can directly find the Laplacian spectrum [10]. Then we extended our results on trace of Laplacian matrix of certain fuzzy graphs which will intern help us in finding its spectrum [11]. Gera, Ralucca and Stănică, Pante discussed Laplacian spectrum of generalized fuzzy graph in 2011[9]. Inspired with their thoughts we have also discussed the Laplacian spectrum of generalized fuzzy Petersen graphs [12]. Now, we have concentrated on fuzzy cycle and vertex regular fuzzy cycle and deduced its least and greatest eigen values of its corresponding Laplacian matrix without finding

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its Laplacian matrix. Also, we have discussed the nature of Laplacian spectrum for fuzzy cycle. Let us recoil some rudimentary definitions which will be helpful for understanding the essential fallouts.

Consider a simple connected graph G:(V, E) with n number of vertices and m number of edges where n and m are greater than one.

Then, a fuzzy graph G: (σ, μ) with n vertices and m edges where σ is a fuzzy subset from vertex set to [0,1] and μ is a fuzzy relation from V × V to [0,1] on σ such that μ (x, y) ≤ σ(x) Λ σ(y) ∀ x, y ∈ V, where Λ stands for minimum [2]. The adjacency matrix A(G) = $[a_{ij}]$ where a_{ij} = μ (u_i, u_j)[6]. Let $d_G(u) = \sum_{uv \epsilon E(G)} \mu(uv)$ and D(G) = $[d_{ij}]$ where d_{ij}= $\begin{bmatrix} d_G(v_i) & if \;\; i = j \\ 0 & otherwise \end{bmatrix}$ is the degree matrix where u is any vertex of a fuzzy graph [6]. The difference of adjacency matrix from the degree matrix is known as Laplacian matrix. If a cycle has more than one weakest arc then the cycle is called a fuzzy cycle (f – cycle) if it contains more than one weakest arc [8]. A fuzzy graph G:(σ, μ) is said to be regular fuzzy graph if every vertex has same degree [5][6][7]. With this understanding we will cross the threshold of the main results.

2. METHODOLOGY

Initially we started to check the Laplacian spectrum for smaller graphs. Then while dealing with larger graphs, we need to find the degree matrix, adjacency matrix and Laplacian matrix manually and then took assistance from MATLAB for finding its Laplacian spectrum. Using machine assistance is also not an easy task, since we have to enter the values of Laplacian matrix manually for getting its corresponding eigen values. We are trying to find the Laplacian spectrum of fuzzy graph from its membership values itself so that we can skip all these tiring procedures.

3. RESULTS AND DISCUSSIONS

3.1. Laplacian Spectrum of Fuzzy Cycle

Concentrating on fuzzy cycle and vertex regular fuzzy cycle, its structure, pattern of labelling we tried to deduce its least and greatest eigenvalues of its corresponding Laplacian matrix and also establish its behavior without following the usual method. We found that least and greatest Laplacian spectrum of fuzzy cycle can be deduced using the edge membership value of the given fuzzy cycle. On analyzing fuzzy cycle and examining the behavior of its Laplacian matrix, we extended the following theorems.

Theorem 3.1.1

For every fuzzy cycle, the least Laplacian spectrum is zero.

Proof:

Let G:(σ, μ) be the given fuzzy cycle with vertices n ≥ 3 and edges m ≥ 3. The Laplacian matrix of a fuzzy cycle is always symmetric in which sum of each row and each column

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is zero. Hence, the Laplacian matrix is a singular matrix. For any singular matrix, one of its eigen value will be zero. Also, eigen values of real symmetric matrix are real and positive only. Hence, the least Laplacian spectrum for any fuzzy cycle is zero.

Theorem 3.1.2

For any fuzzy cycle with n vertices, there are (n – 1) distinct Laplacian spectrum.

Proof:

By theorem 3.1.1, the least Laplacian spectrum is zero. Suppose, all the other eigenvalues of its corresponding Laplacian matrix are not distinct. Then, the eigen vectors corresponding to eigen values are linearly dependent. But, Laplacian matrix of fuzzy cycle is real and symmetric. We know that, real symmetric matrix has n linearly independent vectors which contradicts to our assumption. Hence, all the other (n – 1) Laplacian spectrum are distinct.

Note: The above theorem holds good even though the edge membership value gets repeated.

3.2. Laplacian Spectrum of Vertex Regular Fuzzy Cycle

A fuzzy cycle can be made regular by labelling its edge membership function in such a way that it satisfies the definition of vertex regular fuzzy graph. To convert an even fuzzy cycle into vertex regular even fuzzy cycle we can either label all the edges with same edge membership values or we can label alternate edges with same edge membership values. To convert an odd fuzzy cycle into vertex regular odd fuzzy cycle the only way to label its edges to make it vertex regular is to give same edge membership values to all edges [7]. We have extended these results so as to calculate its Laplacian spectrum.

Theorem 3.2.1

Let G:(σ, μ) be given vertex regular even fuzzy cycle with n vertices and $n \geq 4$. If $\mu = \mu_1 = \mu_2 = \cdots = \mu_n$ then the least and the greatest Laplacian spectrum is 0 and 4μ respectively. The remaining (n – 2) Laplacian spectrum will be in pair.

Proof:

Given G:(σ, μ) a vertex regular even fuzzy cycle with $n \geq 4$ vertices. Let us prove this theorem by induction on n.

Let n = 4. Since $\mu = \mu_1 = \mu_2 = \mu_3 = \mu_4$, the corresponding degree matrix and the adjacency matrix for the given vertex regular fuzzy cycle is,

$$D(G) = \begin{pmatrix} 2\mu & 0 & 0 & 0 \\ 0 & 2\mu & 0 & 0 \\ 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 2\mu \end{pmatrix} \qquad A(G) = \begin{pmatrix} 0 & \mu & 0 & \mu \\ \mu & 0 & \mu & 0 \\ 0 & \mu & 0 & \mu \\ \mu & 0 & \mu & 0 \end{pmatrix}$$

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Then the corresponding Laplacian matrix,

$$L(G) = D(G) - A(G) = \begin{pmatrix} 2\mu & -\mu & 0 & -\mu \\ -\mu & 2\mu & -\mu & 0 \\ 0 & -\mu & 2\mu & -\mu \\ -\mu & 0 & -\mu & 2\mu \end{pmatrix}$$

To find the Laplacian spectrum,

$$|L(G) - \lambda I| = \begin{vmatrix} 2\mu-\lambda & -\mu & 0 & -\mu \\ -\mu & 2\mu-\lambda & -\mu & 0 \\ 0 & -\mu & 2\mu-\lambda & -\mu \\ -\mu & 0 & -\mu & 2\mu-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (2\mu-\lambda)^4 - 4\mu^2(2\mu-\lambda)^2 = 0$$

$$\Rightarrow \quad (2\mu-\lambda)^2[(2\mu-\lambda)^2 - 4\mu^2] = 0$$

$$\Rightarrow \quad \lambda = 0,\ 2\mu,\ 2\mu,\ 4\mu.$$

Hence, the least Laplacian spectrum is 0 and the greatest Laplacian spectrum is 4μ

Let n = 6. Since $\mu = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6$,

Then the corresponding Laplacian matrix,

$$L(G) = D(G) - A(G) = \begin{pmatrix} 2\mu & -\mu & 0 & 0 & 0 & -\mu \\ -\mu & 2\mu & -\mu & 0 & 0 & 0 \\ 0 & -\mu & 2\mu & -\mu & 0 & 0 \\ 0 & 0 & -\mu & 2\mu & -\mu & 0 \\ 0 & 0 & 0 & -\mu & 2\mu & -\mu \\ -\mu & 0 & 0 & 0 & -\mu & 2\mu \end{pmatrix}$$

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To find the Laplacian spectrum,

$$|L(G)-\lambda I| = \begin{vmatrix} 2\mu-\lambda & -\mu & 0 & 0 & 0 & -\mu \\ -\mu & 2\mu-\lambda & -\mu & 0 & 0 & 0 \\ 0 & -\mu & 2\mu-\lambda & -\mu & 0 & 0 \\ 0 & 0 & -\mu & 2\mu-\lambda & -\mu & 0 \\ 0 & 0 & 0 & -\mu & 2\mu-\lambda & -\mu \\ -\mu & 0 & 0 & 0 & -\mu & 2\mu-\lambda \end{vmatrix}$$

$$|L(G)-\lambda I| = 0$$

$$\Rightarrow (\mu-\lambda)^2(3\mu-\lambda)^2(4\mu-\lambda)\lambda = 0$$

$$\Rightarrow \lambda = 0,\ \mu,\ \mu,\ 3\mu,\ 3\mu, 4\mu$$

Hence the least Laplacian spectrum is zero and the greatest Laplacian spectrum is 4μ. We observe that two rows and two columns are added as we increase the number of vertices by 2 which does not affect the least and greatest Laplacian spectrum and it remains the same.

Assume that the theorem is true for n – 2 vertices.

Hence for n – 2 vertices also the least Laplacian spectrum is zero and the greatest Laplacian spectrum is 4μ.

Let us consider for n vertices, that is, n = (n – 2) + 2, by our assumption the result is true for n – 2 vertices then after adding two more vertices also the least and greatest Laplacian spectrum does not change and it remains the same.

Also, the remaining (n – 2) Laplacian spectrum are in pair.

Hence the theorem.

Corollary 3.2.1

If all the diagonal elements of the Laplacian matrix are same then the greatest Laplacian spectrum is twice of its degree of vertex.

Corollary 3.2.2

Let G:(σ, μ) be given vertex regular even fuzzy cycle with $n = 4$ vertices. If $\mu = \mu_1 = \mu_2 = \mu_3 = \mu_4$ then the Laplacian spectrum of the given vertex regular fuzzy cycle is $0, 2\mu, 2\mu, 4\mu$.

Proof:

By theorem 3.2.1, the proof is complete.

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Corollary 3.2.3

Let G:(σ, μ) be given vertex regular even fuzzy cycle with $n = 6$ vertices. If $\mu = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6$ then the Laplacian spectrum of the given vertex regular fuzzy cycle is $0,\ \mu,\ \mu,\ 3\mu,\ 3\mu, 4\mu$.

Proof:

By theorem 3.2.1, the proof is complete.

Theorem 3.2.2

Let G:(σ, μ) be the given regular even fuzzy cycle with n ≥ 4 vertices. If $\mu_1 = \mu_3 = \ldots = \mu_{n-1}$ and $\mu_2 = \mu_4 = \ldots = \mu_n$ then the greatest Laplacian spectrum is 2(μ_1 + μ_2).

Proof:

Given G:(σ, μ) a vertex regular even fuzzy cycle with $n \geq 4$ vertices. Let $\mu_1 = \mu_3 = \ldots = \mu_{n-1}$ and $\mu_2 = \mu_4 = \ldots = \mu_n$. Then, Laplacian matrix for the given vertex regular even fuzzy cycle is,

$$L(G) = D(G) - A(G) = \begin{pmatrix} \mu_1 + \mu_2 & -\mu_1 & 0 & & \ldots & -\mu_2 \\ -\mu_1 & \mu_1 + \mu_2 & -\mu_2 & & \ldots & 0 \\ 0 & -\mu_1 & \mu_1 + \mu_2 & & \ldots & 0 \\ & & \ldots & & . & \\ & & \ldots & & & . \\ & & \ldots & & . & \\ -\mu_2 & 0 & 0 & -\mu_1 & \ldots & \mu_1 + \mu_2 \end{pmatrix}$$

By theorem 3.1.1 and corollary 3.2.1, we see that the least value of Laplacian spectrum is zero and the greatest value of Laplacian spectrum is 2(μ_1 + μ_2).

Proposition 3.2.1

Let G:(σ, μ) be given vertex regular even fuzzy cycle with $n = 4$k where k = 1, 2, ... vertices. If $\mu_1 = \mu_3 = \ldots = \mu_{n-1}$ and $\mu_2 = \mu_4 = \ldots = \mu_n$ then except for the least and the greatest Laplacian spectrum, two Laplacian spectrum will be distinct and all the other (n – 4) Laplacian spectrum will be in pair.

For example, Consider a vertex regular even fuzzy cycle with n = 16 vertices. Let $\mu_1 = \mu_3 = \ldots = \mu_{15}$= 0.71 and $\mu_2 = \mu_4 = \ldots = \mu_{16}$ = 1. Then the Laplacian spectrum,

$$S(G) = \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{matrix} 0.1263 & 0.4836 & 1.0029 \\ 2 & 2 & 2 \end{matrix} \begin{pmatrix} 1.42 & 2 \\ 1 & 1 \end{pmatrix} \begin{matrix} 2.4171 & 2.9364 & 3.2937 \\ 2 & 2 & 2 \end{matrix} \begin{pmatrix} 3.42 \\ 1 \end{pmatrix} \right)$$

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Proposition 3.2.2

Let G:(σ, μ) be given vertex regular even fuzzy cycle with $n = 4k + 2$ where k = 1, 2, ... vertices. If $\mu_1 = \mu_3 = \ldots = \mu_{n-1}$ and $\mu_2 = \mu_4 = \ldots = \mu_n$ then except for the least and the greatest Laplacian spectrum, the other (n – 2) Laplacian spectrum will be in pair.

For example, Consider a vertex regular even fuzzy cycle with n = 18 vertices. Let $\mu_1 = \mu_3 = \ldots = \mu_{17} = 0.2$ and $\mu_2 = \mu_4 = \ldots = \mu_{18} = 0.4$.

Then the Laplacian spectrum,

$$S(G) = \left(\begin{array}{cccccccccc} 0 & 0.0321 & 0.1227 & 0.2536 & 0.3772 & 0.8228 & 0.9464 & 1.0773 & 1.1679 & 1.2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \end{array} \right)$$

Theorem 3.2.4

Let G:(σ, μ) be the given regular odd fuzzy cycle with n ≥ 5 vertices. Then the least value of Laplacian spectrum is zero and the remaining values of Laplacian spectrum will be in pair.

Proof:

Let us prove this theorem by induction on n. Let n = 5. Since $\mu = \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$, the corresponding degree matrix and the adjacency matrix for the given vertex regular fuzzy cycle is,

$$D(G) = \begin{pmatrix} 2\mu & 0 & 0 & 0 & 0 \\ 0 & 2\mu & 0 & 0 & 0 \\ 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 2\mu \end{pmatrix} A(G) = \begin{pmatrix} 0 & \mu & 0 & 0 & \mu \\ \mu & 0 & \mu & 0 & 0 \\ 0 & \mu & 0 & \mu & 0 \\ 0 & 0 & \mu & 0 & \mu \\ \mu & 0 & 0 & \mu & 0 \end{pmatrix}$$

Then the corresponding Laplacian matrix,

$$L(G) = D(G) - A(G) = \begin{pmatrix} 2\mu & -\mu & 0 & 0 & -\mu \\ -\mu & 2\mu & -\mu & 0 & 0 \\ 0 & -\mu & 2\mu & -\mu & 0 \\ 0 & 0 & -\mu & 2\mu & -\mu \\ -\mu & 0 & 0 & -\mu & 2\mu \end{pmatrix}$$

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To find the Laplacian spectrum,

$$|L(G)-\lambda I| = \begin{vmatrix} 2\mu-\lambda & -\mu & 0 & 0 & -\mu \\ -\mu & 2\mu-\lambda & -\mu & 0 & 0 \\ 0 & -\mu & 2\mu-\lambda & -\mu & 0 \\ 0 & 0 & -\mu & 2\mu-\lambda & -\mu \\ -\mu & 0 & 0 & -\mu & 2\mu-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (2\mu-\lambda)^5 - 2\mu^2(2\mu-\lambda)^3 - (2\mu-\lambda)\mu^4 = 0$$

$$\Rightarrow \quad (2\mu-\lambda)[(2\mu-\lambda)^4 - 2\mu^2(2\mu-\lambda)^2 - \mu^4] = 0$$

$$\Rightarrow \quad (2\mu-\lambda)[(2\mu-\lambda)^2 - \mu^2]^2 = 0$$

$$\Rightarrow \quad (2\mu-\lambda)[(2\mu-\lambda)^2 - \mu^2][(2\mu-\lambda)^2 - \mu^2] = 0$$

$$\Rightarrow \quad \lambda = 0,\ \mu,\ \mu, 2\mu, 2\mu$$

Hence, the least Laplacian spectrum is zero and the remaining values of Laplacian spectrum will be in pair.

Whenever two vertices are added it will contribute two eigen values which will be same. Assume that the theorem is true for n – 2 vertices.

Hence for n – 2 vertices also the least Laplacian spectrum is zero and the remaining values of Laplacian spectrum will be in pair. Let us consider for n vertices.

Consider n = (n – 2) + 2, by our assumption the result is true for n – 2 vertices then by adding two more vertices, one more pair of eigen values will be added then also Laplacian spectrum will be in pair.

Corollary 3.2.4

Let G:(σ, μ) be the given regular odd fuzzy cycle with $n = 3$ vertices. Then the Laplacian spectrum is 0, $3\mu, 3\mu$.

Proof:

Let G:(σ, μ) be the given regular odd fuzzy cycle with $n = 3$ vertices.

By theorem 3.1.1, the least value of Laplacian spectrum is zero.

Also, the only possibility to label an odd fuzzy cycle so as to make it regular is $\mu = \mu_1 = \mu_2 = \mu_3$.

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Then, $|L(G) - \lambda I| = \begin{vmatrix} 2\mu - \lambda & -\mu & -\mu \\ -\mu & 2\mu - \lambda & -\mu \\ -\mu & -\mu & 2\mu - \lambda \end{vmatrix} = 0$

$$\Rightarrow \quad 9\mu^2\lambda - \lambda^3 - 6\mu\lambda^2 = 0$$

$$\Rightarrow \quad \lambda(9\mu^2 - \lambda^2 - 6\mu\lambda) = 0$$

$$\Rightarrow \quad (3\mu - \lambda)^2\lambda = 0$$

$$\Rightarrow \quad \lambda = 0, 3\mu, 3\mu.$$

Hence, the least Laplacian spectrum is 0 and the greatest Laplacian spectrum is 3μ.

Theorem 3.2.5

Let G:(σ, μ) be given vertex regular even fuzzy cycle with n vertices and $n \geq 3$. If $\mu = \mu_1 = \mu_2 = \cdots = \mu_n$ then when n is odd and for $k = \frac{n-1}{2}$ the Laplacian spectrum is given as the sequence $[0, x_2, x_4, \dots, x_{2k}]$ where $x_i = 2\mu - 2\mu\cos\frac{i\pi}{2k+1}$, $i = 1, .., k$ and when n is even and $k = \frac{n-2}{2}$ the Laplacian spectrum is given as the sequence $[0, x_1, x_2, \dots, x_k]$ where $x_i = 2\mu - 2\mu\cos\frac{i\pi}{k+1}$, $i = 1, .., k$.

Proof.

Let G:(σ, μ) be given vertex regular even fuzzy cycle with n vertices and $n \geq 3$. Let $\mu = \mu_1 = \mu_2 = \cdots = \mu_n$.

Case (i): Let n be odd. The least Laplacian spectrum is zero and remining values are in pair. Hence the values to be generated is $k = \frac{n-1}{2}$. By using cosine transformation, define $x_i = A - A\cos\theta_i$, where A = 2 μ . The total number of internal values is 2k+1, since only the start value is considered and skip the first value at $\theta = 0$, since it would once again give 0, the increment starts from i=2. Thus, increasing values of Laplacian spectrum from 2 up to 2k is calculated by using $\theta_i = \frac{i\pi}{2k+1}$ $i = 2,4, \dots, 2k$.

Case (ii): Let n be even. The least Laplacian spectrum is zero and the greatest Laplacian spectrum is 4 μ and the intermediate values are in pair. Hence the values to be generated is $k = \frac{n-2}{2}$. By using cosine transformation, define $x_i = A - A\cos\theta_i$, where A = 2 $\mu \epsilon [0, 4\mu]$. The total number of internal values is k+1, since the start and end values are to be considered and skip the first value at $\theta = 0$, since it would once again give 0, the increment starts from i=1. Thus, increasing values of Laplacian spectrum from 0 upto 4 μ is calculated by using $\theta_i = \frac{i\pi}{k+1}$ $i = 1,2, \dots, k$.

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3.2.1 Algorithm

Step 1 Input the value of n and μ

Step 2 If n is odd then go to Step 3 else go to Step 4

Step 3 k = (n-1)/2

for (i=2; i≤ $2k$; ++2i)

$x_i = 2\mu - 2\mu\cos\frac{i\pi}{2k+1}$

print x_i

Step 4 k = (n-2)/2

for (i=1; i≤ k; ++i)

$x_i = 2\mu - 2\mu\cos\frac{i\pi}{k+1}$

print x_i, x_i

print 4 μ

Step 5 end

Proof of Correction

The Laplacian spectrum of a fuzzy cycle with the constraint $\mu = \mu_1 = \mu_2 = \cdots = \mu_n$ can be enumerated in polylog time $\mathcal{O}(n)$.

Proof. Let G (σ, μ) be a fuzzy cycle. Input the value of n and μ. According the value of n, whether it is odd or even the 'for loop' will be executed and the exact values of Laplacian spectrum will be printed accordingly in polylog time $\mathcal{O}(n)$.

4. CONCLUSION

Our aim is to develop an application to calculate the Laplacian spectrum for given fuzzy graph straightforwardly by skipping the usual procedure which will be helpful in overcoming all limitations. More exertion is in thru to extend this paper in various types of fuzzy graphs and finding Laplacian spectrum of the corresponding Laplacian matrix without following the usual steps. This study will undeniably pave a way for unrivaled practice of eigen values of Laplacian matrix in several grounds.

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