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A DUAL-WAREHOUSE APPROACH FOR MANAGING PERISHABLE INVENTORY UNDER REALISTIC OPERATIONAL CONDITIONS

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Abstract

Inventory control under uncertain and time-varying demand is one of the most important issues in supply chain systems. In this research, a new two-warehouse inventory model with a realistic S-shaped demand function following the logistic growth pattern and a linear deterioration rate is introduced. In contrast to conventional models with constant or exponentially changing demand, the logistic curve employed here reflects the natural growth pattern of product demand—increasing slowly, accelerating at a middle point, and finally saturating. The structure of the model also represents real-world operations where demand is met by a rented warehouse initially and then shifts to an owned warehouse when the rented inventory is depleted. To properly model inventory behavior, the model is written in terms of differential equations that respond both to demand and degradation over time (time t). A complete numerical example is given, followed by sensitivity analysis that shows the effect of variations in demand intensity (parameter L) on total inventory cost. The findings show a strong negative correlation between rising demand and efficiency of cost, with the exponential function e^{kt_o} having a major effect on total expenses. This model adds to the literature by providing a more flexible inventory control framework that allows for strategic warehouse decisions and cost-efficient stock replenishment under uncertain demand scenarios.

Keywords: Exponential Function, Inventory Cost, Linear Deterioration, Supply Chain Systems, S-Shaped Demand.

1. INTRODUCTION

In supply chain and inventory management, effective management of stock is essential to optimize costs and satisfy customer demands. Most classical models of inventory are based on a single storage facility of infinite capacity and constant rates of demand. In the real world, conditions are often accompanied by constraints such as storage space limitation and changing patterns of demand. To manage these difficulties, companies use a two-warehouse system, an Own Warehouse (OW) with a finite capacity and a Rented Warehouse (RW) to hold spillover inventory. Handling deteriorating or perishing items introduces an added layer of complexity. Products such as foodstuffs, pharmaceuticals, and chemicals will decrease in value or usefulness over time and require models with deterioration rates. Knowing and predicting demand patterns is also crucial. Although most models use linear or static demand, real-world market performance is usually more complicated in form, like the S-shaped (sigmoidal) demand curve, which indicates the

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product life cycle phases: introduction, growth, maturity, and decline. The S-shaped pattern of demand has slow uptake at first, fast acceleration, and then a leveling off as the market becomes saturated. This is a typical behavior of demand in the adoption of new technology, fad products, and seasonal goods. By integrating this demand behavior into inventory models, it is possible to better forecast and manage the stock. At the same time, stored products over time become degraded, depending on such factors as storage conditions, nature of the products, and time. A linear rate of deterioration suggests a uniform decrease in product quality or quantity over time, which is a good approximation for most perishable products.

Research into inventory models that involve deterioration and sophisticated patterns of demand has developed significantly over the years. Initial models dealt with single-warehouse systems with linear or constant growing demand and accounted for numerous forms of deterioration. Researchers have worked extensively in inventory systems with deterministic and time-varying demand. Initial work started with [3], who presented an exact approach to computing optimal order quantity and timing. Follow-up work by [25] to [1] and others built on this, providing heuristic solutions to Donaldson's problem or formulating it for more realistic environments. Early models mostly dealt with a single warehouse with the assumption of infinite storage capacity. In real-world contexts, however, space within warehouses is generally limited. In order to avail bulk purchase discount or to handle huge quantities of inventory, companies typically overload their owned warehouse (OW) and thus use rented warehouses (RW), even though the holding cost is higher. This necessitates multi-warehouse systems.

It responded to that by investigating two-warehouse models for one OW with limited capacity and one RW with unlimited capacity in recent research. RW is often more expensive to store, so the items are sent out from it initially. The idea was first presented by [11], where a heuristic was utilized to find the optimum size of order. Building on this, [19] analyzed systems with time-varying polynomial demand and prepayments for non-instantaneous deteriorating items. [16] extended this to a three-warehouse framework involving one OW and two RWs, aiming to minimize costs with quadratic demand and time-dependent holding costs. [5] discussed lot sizing for inventory in the presence of trended demand and shortages, proposing an innovative replenishment policy that is responsive to varying system parameters. [12] formulated inventory models that are reactive to a period of rising demand followed by a period of level demand, emphasizing dynamic modeling in planning operations. Similarly, [26] examined quadratic demand in a two-warehouse setup, while [14] investigated systems with deteriorating goods. [9] suggested a linear programming technique for maximizing warehouse storage capacity with the emphasis on structural parameters for effective allocation of resources. [10] surveyed advanced warehouse planning methods, with special reference to the influence of design arrangements on system efficiency and performance. Further studies by [7] and [8] focused on supply chain and warehouse management, and [21], [22] evaluated EOQ models for perishables. Warehouse optimization was also explored in [6]. Models incorporating exponential demand and constant deterioration were discussed in [15], while [24] addressed general deterioration cases. Price-sensitive demand was studied in

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[17], and [18] incorporated LIFO and FIFO strategies under time-based demand. Additional contributions include [4] on ramp-type demand with perishables, [23] on inflation-adjusted, stock-dependent demand, [2] on delayed payments, [13] on fixed shelf-life products across OW and RW, and [20] on stock-dependent demand and deterioration in a two-warehouse context.

This research is focused on constructing a comprehensive inventory model that incorporates the dynamics of a two-warehouse system. The model considers an own warehouse (OW) with limited storage capacity and a rented warehouse (RW) used to accommodate excess inventory. It also includes an S-shaped pattern of demand, which is a good representation of actual real-world market adoption patterns, and a linear decay rate to model the progressive but steady reduction of the quality or quantity of items held over time. The major goals of this research are developing a mathematical model that characterizes inventory behavior in both warehouses under the specified demand and deterioration levels, obtaining total cost relationships that include holding, deterioration, and shortage costs, and determining optimal inventory policies that reduce these costs.

The model will also be subjected to sensitivity analysis to investigate the implications of altering different parameters on overall inventory performance and cost configuration. By integrating an S-shaped demand pattern with a linear rate of deterioration in a two-warehouse inventory model, the research presents a more real-world and pragmatic model of managing perishable items. The model facilitates more effective decision-making regarding inventory restocking schedules and distribution of storage between the own warehouse and the leased warehouse. It also helps save costs by reducing holding, deterioration, and shortage-based costs. Additionally, by facilitating improved inventory availability, the model will be able to improve customer satisfaction. The insights that can be obtained from this model are highly useful in product launch planning, particularly for products expected to exhibit a sigmoidal pattern of demand throughout its lifecycle.

2. NOTATIONS & ASSUMPTIONS

2.1. Notations

The notations used in this study are defined as follows:

$D(t) = Demand\ rate$

$\alpha = Deterioration\ rate\ in\ owned\ warehouse$

$\beta = Deterioration\ rate\ in\ rented\ warehouse$

$I_r = Inventory\ level\ in\ rented\ warehouse\ [0, t_r]$

$I_{o1} = Inventory\ level\ in\ owned\ warehouse\ [0, t_r]$

$I_{o2} = Inventory\ level\ in\ owned\ warehouse\ [t_r, T]$

$S_1 = Storage\ of\ rented\ warehouse$

$S_2 = Storage\ of\ owned\ warehouse$

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$C_H = Holding\ cost$

$C_o = Ordering\ cost$

$D_c = Deterioration\ cost$

$\sigma_1\ \&\ \sigma_2 = Holding\ cost\ of\ rented\ and\ owned\ warhouse\ respectively$

$\mu_1\ \&\ \mu_2 = Deterioration\ cost\ of\ rented\ and\ owned\ warhouse\ respectively$

$C_{Total} = Total\ inventory\ cost$

2.2. Assumptions

The model is formulated under the following set of assumptions:

- The inventory framework consists of a dual-warehouse setup, which includes:

A primary or own warehouse (OW) that has restricted storage space, and

An auxiliary or rented warehouse (RW) that provides adequate capacity to accommodate surplus inventory.

- The demand pattern is characterized by an S-shaped curve, which effectively models the various stages of a product's life cycle—starting with a slow uptake, followed by a rapid growth phase, and finally reaching a saturation point where demand levels off.

$$D(t) = \frac{L}{1 + e^{-k(t-t_o)}}$$

Where,

$L = saturation\ level\ of\ demand$

$k = Growth\ rate\ parameter$

$t_o = Inflection\ point$

- Products stored in both the own and rented warehouses experience linear deterioration, implying that the loss in quality or quantity progresses at a rate directly proportional to time.
- Deterioration begins as soon as the items are stocked, and it is assumed that items which deteriorate have no residual or salvage value.
- Inventory replenishment is considered instantaneous, meaning there is no lead time, and stockouts are not permitted within the system.
- Storage costs vary between the two warehouses; typically, the rented warehouse incurs higher holding expenses compared to the own warehouse.
- The model is designed for a limited time horizon, and analysis is conducted within a single replenishment period.

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3. MATHEMATICAL FORMULATION

To develop the proposed two-warehouse inventory model, we begin by formulating the mathematical structure that captures the inventory dynamics in both the rented and owned warehouses.

In the rented warehouse, inventory is depleted due to both demand and linear deterioration until time $t = t_r$, after which the stock is exhausted. In the owned warehouse, two separate inventory functions are considered: one (denoted as $I_{o1}(t)$) during the time when the rented warehouse is still fulfilling demand, and the other (denoted as $I_{o2}(t)$) after the system switches over to fulfill demand solely from the owned warehouse.

These inventory variations are modeled by first-order linear differential equations incorporating both logistic demand rate and time-dependent deterioration rate, allowing the behavior of the system to be analyzed over the entire planning horizon.

Inventory level in rented warehouse during $0 \le t \le t_r$ is given by the differential equation:

$$\frac{dI_r(t)}{dt} + (\beta + \beta t)I_{r1}(t) = -\frac{L}{1+e^{-k(t-t_o)}}; \qquad 0 \le t \le t_r \tag{1}$$

Inventory level in owned warehouse during $0 \le t \le t_r$ is given by the differential equation:

$$\frac{dI_{o1}(t)}{dt} + (\alpha + \alpha t)I_{o1}(t) =; \qquad 0 \le t \le t_r \tag{2}$$

Inventory level in rented warehouse during $t_r \le t \le T$ is given by the differential equation:

$$\frac{dI_{o2}(t)}{dt} + (\alpha + \alpha t)I_{o2}(t) = -\frac{L}{1+e^{-k(t-t_o)}}; \qquad t_r \le t \le t_o \tag{3}$$

Case I: When $t \gg t_o$ (Time much greater than the inflection point)

When time t is much larger than the inflection point t_o, the exponential term in the denominator becomes very small:

$$e^{-k(t-t_o)} \to 0$$

Thus, the demand function approximates to:

$$D(t) \approx \frac{L}{1+0} = L$$

Now solving the equations,

$$I_r(t) = \left(1 - \beta t - \frac{\beta t^2}{2}\right)\left[-L\left(t + \frac{\beta t^2}{2} + \frac{\beta t^3}{6}\right) + C_1\right] \tag{4}$$

$$I_{o1}(t) = C_2\left(1 - \alpha t - \frac{\alpha t^2}{2}\right) \tag{5}$$

$$I_{o2}(t) = \left(1 - \alpha t - \frac{\alpha t^2}{2}\right)\left[-L\left(t + \frac{\alpha t^2}{2} + \frac{\alpha t^3}{6}\right) + C_3\right] \tag{6}$$

$$I_r(t) = \left(1 - \beta t - \frac{\beta t^2}{2}\right)\left[-L\left(t + \frac{\beta t^2}{2} + \frac{\beta t^3}{6}\right) + S_1\right] \tag{7}$$

$$I_{o1}(t) = S_2 \frac{\left(1-\alpha t-\frac{\alpha t^2}{2}\right)}{\left(1-\alpha t_r-\frac{\alpha t_r^2}{2}\right)} \tag{8}$$

$$I_{o2}(t) = L\left(1-\alpha t-\frac{\alpha t^2}{2}\right)\left(T-t+\frac{\alpha}{2}(T^2-t^2)+\frac{\alpha}{6}(T^3-t^3)\right) \tag{9}$$

The total inventory cost will be given by:

1. Ordering cost
2. Holding cost

Holding cost in rented warehouse will be given by

$$= \sigma_1\left[\int_o^{t_r} I_r(t)dt\right]$$

$$= S_1 t_r + t_r^4\left[\frac{L\beta}{12}+\frac{L\beta^2}{8}\right] + t_r^2\left[-\frac{L}{2}-\frac{S_1\beta}{2}\right] + \frac{L\beta^2 t_r^5}{12} + t_r^3\left(\frac{L\beta}{6}-\frac{S_1\beta}{6}\right) + \frac{L\beta^2 t_r^6}{72} \tag{10}$$

Holding cost in owned warehouse will be given by

$$= \sigma_2\left[\left[\int_o^{t_r} I_{o1}(t)dt\right] + \left[\int_{t_r}^{T} I_{o2}(t)dt\right]\right]$$

$$= \left[\frac{S_2 t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{L(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2 t_r + 6T^3\alpha^2 + 3T^2\alpha^2 t_r^2 + 12T^2\alpha^2 t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2 t_r^3 + 12T\alpha^2 t_r^2 + 18T\alpha^2 t_r - 12T\alpha + \alpha^2 t_r^4 + 6\alpha^2 t_r^3 + 9\alpha^2 t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \tag{11}$$

Total holding cost will be given by:

$$C_H = \sigma_1\left[\int_o^{t_r} I_r(t)dt\right] + \sigma_2\left[\left[\int_o^{t_r} I_{o1}(t)dt\right] + \left[\int_{t_r}^{T} I_{o2}(t)dt\right]\right]$$

$$C_H = \sigma_1\left[S_1 t_r + t_r^4\left[\frac{L\beta}{12}+\frac{L\beta^2}{8}\right] + t_r^2\left[-\frac{L}{2}-\frac{S_1\beta}{2}\right] + \frac{L\beta^2 t_r^5}{12} + t_r^3\left(\frac{L\beta}{6}-\frac{S_1\beta}{6}\right) + \frac{L\beta^2 t_r^6}{72}\right] + \sigma_2\left[\frac{S_2 t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{L(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2 t_r + 6T^3\alpha^2 + 3T^2\alpha^2 t_r^2 + 12T^2\alpha^2 t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2 t_r^3 + 12T\alpha^2 t_r^2 + 18T\alpha^2 t_r - 12T\alpha + \alpha^2 t_r^4 + 6\alpha^2 t_r^3 + 9\alpha^2 t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \tag{12}$$

3. Deterioration cost

Deterioration costs in both warehouses will be given by

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$$D_c = \mu_1\left[\int_o^{t_r} I_r(t)dt\right] + \mu_2\left[\left[\int_o^{t_r} I_{o1}(t)dt\right] + \left[\int_{t_r}^{T} I_{o2}(t)dt\right]\right]$$

Cost of deterioration in rented warehouse will be given by

$$= \mu_1\left[S_1t_r + t_r^4\left[\frac{L\beta}{12} + \frac{L\beta^2}{8}\right] + t_r^2\left[-\frac{L}{2} - \frac{S_1\beta}{2}\right] + \frac{L\beta^2t_r^5}{12} + t_r^3\left(\frac{L\beta}{6} - \frac{S_1\beta}{6}\right) + \frac{L\beta^2t_r^6}{72}\right] \quad (13)$$

Cost of deterioration in owned warehouse will be given by

$$= \mu_2\left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{L(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \quad (14)$$

Total deterioration will be given by

$$D_c = \mu_1\left[S_1t_r + t_r^4\left[\frac{L\beta}{12} + \frac{L\beta^2}{8}\right] + t_r^2\left[-\frac{L}{2} - \frac{S_1\beta}{2}\right] + \frac{L\beta^2t_r^5}{12} + t_r^3\left(\frac{L\beta}{6} - \frac{S_1\beta}{6}\right) + \frac{L\beta^2t_r^6}{72}\right] + \mu_2\left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{L(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \quad (15)$$

Total Relevant cost will be given by:

$$C_{total} = \frac{1}{T}[Ordering\ cost + Holding\ cost + Deterioration\ cost]$$

$$C_{total} = \frac{1}{T}\left[C_o + (\sigma_1+\mu_1)\left[S_1t_r + t_r^4\left[\frac{L\beta}{12} + \frac{L\beta^2}{8}\right] + t_r^2\left[-\frac{L}{2} - \frac{S_1\beta}{2}\right] + \frac{L\beta^2t_r^5}{12} + t_r^3\left(\frac{L\beta}{6} - \frac{S_1\beta}{6}\right) + \frac{L\beta^2t_r^6}{72}\right] + (\sigma_2+\mu_2)\left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{L(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right]\right] \quad (16)$$

Case II: When $t \ll t_o$ (Time much less than the inflection point)

In the early stages, when $t \ll t_o$, the exponential term becomes very large:

So, $D(t) \approx \frac{L}{e^{-kt_o}} = Le^{kt_o}$

Now solving the differential equations

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$$\frac{dI_r(t)}{dt} + (\beta + \beta t)I_{r1}(t) = -Le^{kt_o}$$

$$I_r(t) = \left(1 - \beta t - \frac{\beta t^2}{2}\right)\left[-Le^{kt_o}\left(t + \frac{\beta t^2}{2} + \frac{\beta t^3}{6}\right) + C_4\right] \quad (17)$$

$$I_{o1}(t) = C_5\left(1 - \alpha t - \frac{\alpha t^2}{2}\right) \quad (18)$$

$$I_{o2}(t) = \left(1 - \alpha t - \frac{\alpha t^2}{2}\right)\left[-Le^{kt_o}\left(t + \frac{\alpha t^2}{2} + \frac{\alpha t^3}{6}\right) + C_6\right] \quad (19)$$

$$I_r(t) = \left(1 - \beta t - \frac{\beta t^2}{2}\right)\left[-Le^{kt_o}\left(t + \frac{\beta t^2}{2} + \frac{\beta t^3}{6}\right) + S_1\right] \quad (20)$$

$$I_{o1}(t) = S_2\frac{\left(1-\alpha t-\frac{\alpha t^2}{2}\right)}{\left(1-\alpha t_r-\frac{\alpha t_r^2}{2}\right)} \quad (21)$$

$$I_{o2}(t) = Le^{kt_o}\left(1 - \alpha t - \frac{\alpha t^2}{2}\right)\left(T - t + \frac{\alpha}{2}(T^2 - t^2) + \frac{\alpha}{6}(T^3 - t^3)\right) \quad (22)$$

The total inventory cost will be given by:

1. Ordering cost
2. Holding cost

Holding cost in rented warehouse will be given by

$$= \sigma_1\left[\int_o^{t_r} I_r(t)dt\right]$$

$$= S_1t_r - t_r^2\left[\frac{S_1\beta}{2} + Le^{kt_o}\right] - \frac{\beta t_r^3(S_1 - Le^{kt_o})}{6} + \frac{L\beta^2 t_r^5 e^{kt_o}}{12} + \frac{L\beta^2 t_r^6 e^{kt_o}}{72} + \frac{L\beta t_r^4 e^{kt_o}(3\beta+2)}{24} \quad (23)$$

Holding cost in owned warehouse will be given by

$$= \sigma_2\left[\left[\int_o^{t_r} I_{o1}(t)dt\right] + \left[\int_{t_r}^{T} I_{o2}(t)dt\right]\right]$$

$$= \left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{Le^{kt_o}(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \quad (24)$$

Total holding cost will be given by:

$$C_H = \sigma_1\left[\int_o^{t_r} I_r(t)dt\right] + \sigma_2\left[\left[\int_o^{t_r} I_{o1}(t)dt\right] + \left[\int_{t_r}^{T} I_{o2}(t)dt\right]\right]$$

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$$C_H = \sigma_1\left[S_1t_r - t_r^2\left[\frac{S_1\beta}{2} + Le^{kt_o}\right] - \frac{\beta t_r^3(S_1 - Le^{kt_o})}{6} + \frac{L\beta^2t_r^5e^{kt_o}}{12} + \frac{L\beta^2t_r^6e^{kt_o}}{72} + \frac{L\beta t_r^4e^{kt_o}(3\beta+2)}{24}\right] + \sigma_2\left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{Le^{kt_o}(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \quad (25)$$

3. Deterioration cost

Deterioration costs in both warehouses will be given by

$$D_c = \mu_1\left[\int_o^{t_r} I_r(t)dt\right] + \mu_2\left[\left[\int_o^{t_r} I_{o1}(t)dt\right] + \left[\int_{t_r}^{T} I_{o2}(t)dt\right]\right]$$

Cost of deterioration in rented warehouse will be given by

$$= \mu_1\left[S_1t_r - t_r^2\left[\frac{S_1\beta}{2} + Le^{kt_o}\right] - \frac{\beta t_r^3(S_1 - Le^{kt_o})}{6} + \frac{L\beta^2t_r^5e^{kt_o}}{12} + \frac{L\beta^2t_r^6e^{kt_o}}{72} + \frac{L\beta t_r^4e^{kt_o}(3\beta+2)}{24}\right] \quad (26)$$

Cost of deterioration in owned warehouse will be given by

$$= \mu_2\left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{Le^{kt_o}(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \quad (27)$$

Total deterioration will be given by

$$D_c = \mu_1\left[S_1t_r - t_r^2\left[\frac{S_1\beta}{2} + Le^{kt_o}\right] - \frac{\beta t_r^3(S_1 - Le^{kt_o})}{6} + \frac{L\beta^2t_r^5e^{kt_o}}{12} + \frac{L\beta^2t_r^6e^{kt_o}}{72} + \frac{L\beta t_r^4e^{kt_o}(3\beta+2)}{24}\right] + \mu_2\left[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{Le^{kt_o}(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 + 3T^2\alpha^2t_r^2 + 12T^2\alpha^2t_r + 9T^2\alpha^2 - 6T^2\alpha + 2T\alpha^2t_r^3 + 12T\alpha^2t_r^2 + 18T\alpha^2t_r - 12T\alpha + \alpha^2t_r^4 + 6\alpha^2t_r^3 + 9\alpha^2t_r^2 + 6\alpha t_r^2 + 12\alpha t_r - 36]\right] \quad (28)$$

Total Relevant cost will be given by:

$$C_{total} = \frac{1}{T}[Ordering\ cost + Holding\ cost + Deterioration\ cost]$$

$$C_{total} = \frac{1}{T}\Bigg[C_o + (\sigma_1 + \mu_1)\left[S_1t_r - t_r^2\left[\frac{S_1\beta}{2} + Le^{kt_o}\right] - \frac{\beta t_r^3(S_1 - Le^{kt_o})}{6} + \frac{L\beta^2t_r^5e^{kt_o}}{12} + \frac{L\beta^2t_r^6e^{kt_o}}{72} + \frac{L\beta t_r^4e^{kt_o}(3\beta+2)}{24}\right] + (\sigma_2 + \mu_2)\Bigg[\frac{S_2t_r(\alpha t_r^2+3\alpha t_r-6)}{3(\alpha t_r^2+2\alpha t_r-2)} - \frac{Le^{kt_o}(T-t_r)^2}{72}[T^4\alpha^2 + 2T^3\alpha^2t_r + 6T^3\alpha^2 +$$

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$$3T^2\alpha^2t_r^2+12T^2\alpha^2t_r+9T^2\alpha^2-6T^2\alpha+2T\alpha^2t_r^3+12T\alpha^2t_r^2+18T\alpha^2t_r-12T\alpha+\alpha^2t_r^4+6\alpha^2t_r^3+9\alpha^2t_r^2+6\alpha t_r^2+12\alpha t_r-36]\Big]\Bigg] \quad (29)$$

4. SOLUTION PROCEDURE

The primary objective is to minimize the total inventory cost C_{Total}, where t_r (time of depletion of the rented warehouse) and T (The total cycle time) serve as the key decision variable. The optimal values of t_r and T are determined by solving the equations that yield the lowest possible value of the total cost function. To address this optimization problem, the following systematic approach can be adopted. The necessary conditions for minimizing the cost function $C_{Total}(t_r, T)$, are given by:

$$\frac{\delta C_{Total}(t_r,T)}{\delta t_r}=0$$

$$\frac{\delta C_{Total}(t_r,T)}{\delta t_r}=0$$

And,

$$\frac{\delta^2 C_{Total}(t_r,T)}{\delta t_r^2}>0$$

$$\frac{\delta^2 C_{Total}(t_r,T)}{\delta T^2}>0$$

The curvature of the total cost function is analyzed using the well-established Hessian matrix approach. In this context, the Hessian matrix corresponding to the total cost function is given by:

$$\begin{vmatrix} \frac{\delta^2 C_{Total}(t_r,T)}{\delta t_r^2} & \frac{\delta^2 C_{Total}(t_r,T)}{\delta t_r \delta T} \\ \frac{\delta^2 C_{Total}(t_r,T)}{\delta T \delta t_r} & \frac{\delta^2 C_{Total}(t_r,T)}{\delta T^2} \end{vmatrix}$$

The leading principal minors of the Hessian matrix $H(t_r, T)$ are denoted as H_1 and H_2 both of which are found to be greater than zero. Since all principal minors are positive, the total cost function is confirmed to be convex.

Or

$$\left\{\left(\frac{\delta^2 C_{Total}(t_r,T)}{\delta t_r^2}\right)*\left(\frac{\delta^2 C_{Total}(t_r,T)}{\delta T^2}\right)-\left(\frac{\delta^2 C_{Total}(t_r,T)}{\delta t_r \delta T}\right)^2\right\}>0$$

Which is also known as $H_2 > 0$.

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This confirms that the stationary point corresponds to a minimum. Using the derived equations, the optimal value of t_r and T, along with the minimum total cost, can be computed with the help of MATLAB software.

5. NUMERICAL ILLUSTRATION & SENSITIVITY ANALYSIS

5.1. Numerical Illustration

A numerical illustration is presented to validate and demonstrate the behavior of the proposed two-warehouse inventory model incorporating an S-shaped demand rate and a linear deterioration rate.

The numerical values of relevant parameters are chosen to simulate practical inventory dynamics.

This analysis highlights the effect of demand progression over time, particularly how the logistic (S-shaped) demand influences the timing of inventory depletion in both the rented and owned warehouses.

$$L = 750, \quad k = 0.6, \quad t_o = 5, \quad t = 0 \; to \; 10 \; days$$

Now using,

$$D(t) = \frac{L}{1 + e^{-k(t-t_o)}}$$

We get,

Table 1: Demand showing a logistic growth trend

t (days)	Demand *D(t)*
0	22.7
1	45.3
2	84.4
3	150.4
4	261.1
5	375
6	488.8
7	599.6
8	665.6
9	704.7
10	727.3

The logistic demand curve with carrying capacity $L = 750$ and inflection point at $t = 5 \; days$ shows a typical s-shaped pattern.

Demand starts slowly in the early days, grows rapidly around the midpoint, and saturates as it approaches the maximum capacity.

This behavior is crucial in warehouse inventory modelling, where anticipating rapid demand increases helps optimize resource allocation and warehouse switching strategies. Below is the graph showing behavior of the demand function.

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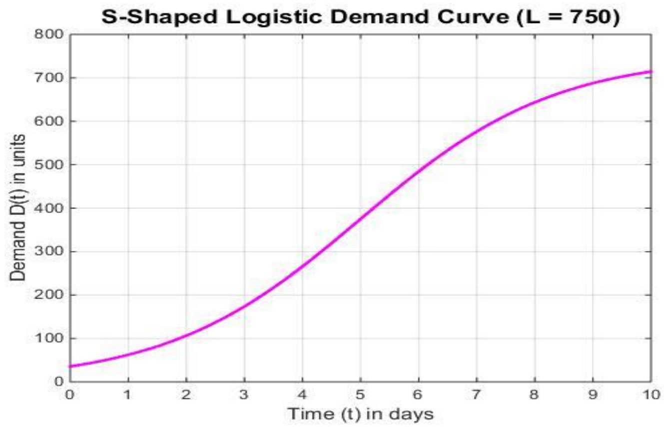

Fig 1: Graph showing a logistic growth trend with respect to demand

Now, to demonstrate the applicability of the proposed inventory model, a numerical example is presented using parameter values. The model is being solved by using the MATLAB software.

For Case I,

$$\alpha = 0.1 per\ unit,\ \ \beta = 0.15 per\ unit,\ \ S_1 = 200 units,\ \ S_2 = 150 units$$

$$\sigma_1 = 2.5 per\ unit,\ \ \sigma_2 = 2 per\ unit,\ \ \mu_1 = 1.5 per\ unit,\ \ \mu_2 = 2 per\ unit,$$

$$L = 250 units,\ \ C_o = 500,\ \ T^* = 2.5453\ days,\ \ t_r^* = 1.8902\ days,$$

$$C_{Total}^* = ₹785.6358$$

The results indicate that the rented warehouse stock depletes after approximately 1.89 days, after which the central warehouse begins fulfilling the remaining demand. The total cost of ₹785.6358 suggests the system operates efficiently under the given parameters.

For Case II,

$$\alpha = 0.1 per\ unit,\ \ \beta = 0.15 per\ unit,\ \ S_1 = 200 units,\ \ S_2 = 150 units$$

$$\sigma_1 = 2.5 per\ unit,\ \ \sigma_2 = 2 per\ unit,\ \ \mu_1 = 1.5 per\ unit,\ \ \mu_2 = 2 per\ unit,\ \ k = 0.05$$

$$L = 250 units,\ \ C_o = 500,\ \ T^* = 6.6\ days,\ \ t_r^* = 3.45\ days,\ \ t_o = 3\ days$$

$$C_{Total}^* = ₹854.298$$

The results indicate that the rented warehouse stock depletes after approximately 3.45 days, after which the central warehouse begins fulfilling the remaining demand. The total cost of ₹854.298 suggests the system operates efficiently under the given parameters.

5.2. Sensitivity Analysis

To understand the influence of key input parameters on the total inventory cost C_{Total}, a sensitivity analysis is carried out by varying the value of the market saturation level L, which governs the upper bound of the S-shaped demand function.

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Table 2: Sensitivity Analysis of C_{Total} with respect to demand level L

Parameter	Change in Parameter (L)	Change in Total Cost (C_{Total}) (Case I)	Change in Total Cost (C_{Total}) (Case II)
L	225	832.0063	1124.55
	230	822.7322	1070.50
	235	813.4581	1016.45
	240	804.1840	962.40
	245	794.9099	908.35
	250	785.6358	854.30
	255	776.3616	800.25
	260	767.0875	746.20
	265	757.8134	692.15
	270	748.5393	638.10
	275	739.2652	584.05

The sensitivity analysis, conducted by varying the demand level parameter L from its base value of 250, reveals a clear inverse relationship between demand and total inventory cost. In Case I, as L increases from 225 to 275, the total cost steadily decreases from around ₹832.00 to ₹739.27. This indicates that higher demand levels lead to greater cost efficiency within the model. The likely reasons for this trend include better inventory turnover, which helps lower holding and deterioration costs, and economies of scale, where larger replenishment quantities result in reduced per-unit ordering and storage costs. In Case II, as L increases from 225 to 275, the total cost steadily decreases from around ₹1124.55 to ₹584.05. k (growth rate of demand) & t_o (inflection point of S-shaped demand) significantly affect the total inventory cost through their combined exponential impact e^{kt_o}. This term amplifies multiple cost components, particularly those related to deterioration and holding. Even modest increases in k or t_o can cause a nonlinear surge in cost, making parameter optimization critical.

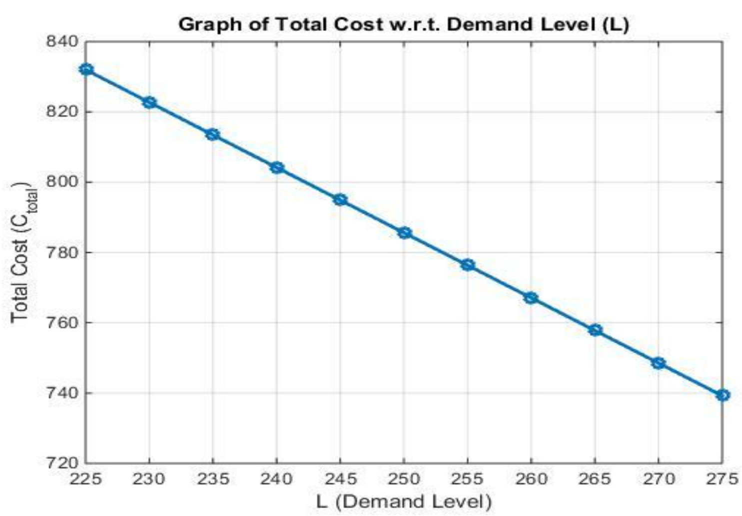

Fig 2: Graph showing a relationship between total cost and demand level for Case I

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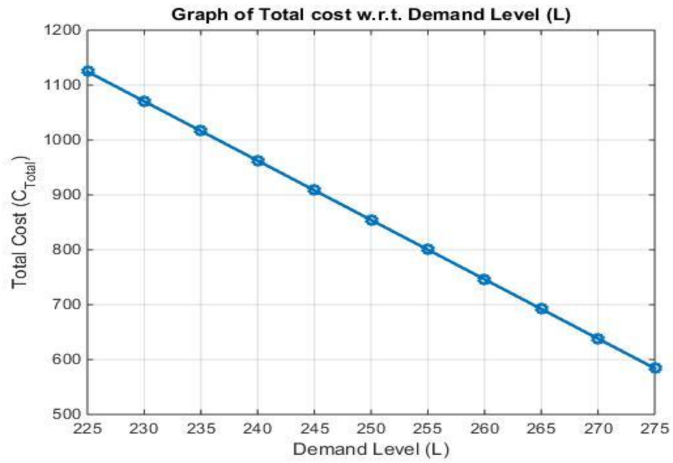

Fig 3: Graph showing a relationship between total cost and demand level for Case II

6. RESULT AND DISCUSSION

The logistic demand curve with carrying capacity $L = 750$ and an inflection point at $t = 5\,days$ shows a typical s-shape pattern-slow growth at the beginning, followed by rapid acceleration around the middle point, and finally ending in saturation. Such a pattern is of significant importance in inventory modeling, especially in planning rental-own transitions between warehouses.

The demand curve drawn from the plot shows how to better allocate stock and lower holding costs by forecasting mid-horizon demand bursts. Sensitivity analysis, where L is varied from 225 to 275, demonstrates a definite inverse correlation between demand and overall inventory cost. In Case I, cost falls from ₹832.00 to ₹739.27 as L increases, suggesting greater turnover and lower deterioration-related costs with higher demand.

In Case II, cost falls more dramatically from ₹1124.55 to ₹584.05, exhibiting higher sensitivity because of perhaps higher transfer or holding expenses. In addition, the exponential term e^{kt_o} is critical in cost behavior; slight increases in k (growth rate) or t_o (inflection point) lead to nonlinear cost growth. Therefore, optimizing these parameters becomes necessary to ensure cost minimization with responsiveness to demand patterns. The analysis verifies that logistic demand modeling is well-suited to provide cost-effective decisions in two-warehouse systems.

7. CONCLUSION

This study proposes a complete two-warehouse inventory model that integrates an S-shaped (logistic) demand pattern and a linear deterioration rate. The model represents real market dynamics where demand starts slowly, builds up close to an inflection point, and eventually saturates. This modeling strategy allows for more realistic depiction of product lifecycles and customer adoption patterns.

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The model contains differential equations that reflect inventory behavior in both leased and owned warehouses, with deterioration and time-varying demand. By numerical example and sensitivity analysis, the research exhibits how quantities like carrying capacity L, growth rate of demand k, and inflection point t_o play a substantial role in impacting the overall inventory cost.

The findings indicate that increasing levels of demand result in lower total costs through enhanced turnover and lesser holding and deterioration costs. Additionally, the exponential impact of e^{kt_o} on cost elements emphasizes the significance of optimizing demand intensity and timing.

Generally, the model is beneficial for inventory decision-making in settings of uncertain demand and demand change over time. It accommodates strategic warehouse switching and cost reduction, which makes it an effective supply chain planning tool. Extensions to this model in future work can include shortages, inflationary effects, or multi-item inventory systems.

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