Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

DEVELOPING A TWO-ECHELON VENDOR-MANAGED INVENTORY (VMI) MODEL WITH ONE SUPPLIER AND MULTIPLE BUYERS

SARTHAK GOYAL
Research Scholar, Department of Mathematics, Teerthanker Mahaveer University College of Engineering Teerthanker Mahaveer University, Moradabad, India. Email: sarthakgoyal41441@gmail.com@tmu.ac.in

Dr. VIPIN KUMAR
Associate Professor, Department of Mathematics, Teerthanker Mahaveer University College of Engineering Teerthanker Mahaveer University, Moradabad, India. Email: drvipink.engineering@tmu.ac.in

Abstract

Vendor Managed Inventory (VMI) system is a method in which the purchase orders are made by the supplier according to the demand data shared between the retailer/ customer. This paper provides an improved mathematical model for a two-echelon vendor-managed inventory (VMI) system with one supplier and multiple buyers. With existing VMI models based mainly on single-supplier and single-buyer systems, the new model gives an optimal realistic and scalable extension by considering various downstream buyers with different demand rates, cost functions, and lead times. Two alternative inventory control methods are considered: the traditional (decentralize) model with buyers maintaining their inventories independently and the VMI system where the supplier handles the replenishment of inventory for all buyers. The goal is to minimize system-wide total inventory cost, including setup, holding, transportation, and backorder costs under continuous review (Q, r) policies. Here, I have explored a problem in real-life context with sensitivity analysis to cross-verify the model and determine extent points—parameter conditions where both the systems have cost parity. Findings indicate that cost parity could be present under certain setups, yet the VMI model presents better strategic advantages in terms of coordination, visibility, and responsiveness. This research adds a real-world decision-support instrument for supply chain managers to consider when deciding whether to switch from classical stock systems to centralized VMI strategies within multi-buyer settings.

Keywords: Vendor-Managed Inventory (VMI), Supply Chain, Total Inventory Cost, Back Order, Extent Point, Mathematical Modeling.

1. INTRODUCTION

A supply chain can be considered as an efficient connection between suppliers, manufacturers, distributors, and end users with a general goal to satisfy customer demand in an efficient and economical way [19]. Conventionally, in a two-echelon supply chain, retailers or buyers are tasked with maintaining their own inventoriesby ordering purchases from the supplier as per local forecasts and consumption levels. This decentralized setup tends to be plagued with late information flow, inefficient inventory policies, and the infamous bullwhip effect, through which upstream variability in demand is increased in the supply chain [21].

To mitigate such drawbacks, Vendor-Managed Inventory (VMI) has become a potent collaborative inventory approach. VMI is a process by which the supplier is provided with the buyer's inventory and demand data and is held accountable for making replenishment decisions on behalf of the buyer [24]. VMI has also been called continuous replenishment,

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

automatic replenishment, or supplier-managed inventory. In this configuration, the supplier both decides when and how much to replenish, effectively making the inventory system a backward driven one. The central change is that inventory turnover and availability, not delivery response time, become the gauges of supplier performance [8].

VMI provides various benefits over ordinary order-based systems. By eradicating the necessity for repeated purchase orders and minimizing information lag, it enables the supplier to carry out production and delivery more efficiently [20]. The supplier receives visibility into the entire demand portfolio and can consolidate orders, minimize safety stock, and harmonize inventory levels for multiple product lines or zones of customers [9]. For the customer, VMI lessens the complexity of managing inventory and allows for leaner operations with fewer stockouts and less holding costs of inventory.

Most of the literature on VMI in the present has concentrated on single-buyer single-supplier models, which ease computation but do not indicate the architecture of most actual supply chains [16]. In real-world situations, a supplier will service multiple buyers with their own demand patterns, cost constraints, and service level expectations. Instances are a central stockroom stocking up inventory for several retail outlets, or a manufacturer shipping product to various regional depots [7]. In these cases, coordination among buyers becomes a priority, and the advantages of VMI can be greatly increased—but also harder to maximize.

This research suggests a dual-echelon VMI model with one supplier and many buyers [2]. The research aims to close the research gap by examining how VMI works for a multi-buyer system versus the classical supply chain setup. Under the classical setup, each buyer individually makes their own replenishment orders to the supplier according to their respective cost-reduction policies. Under the VMI setup, the supplier centrally controls the inventory policies $(Q_{i,} r_i)$ for all the buyers, being responsible for when and how much to ship to each of them.

The aim of the research is to reduce the total cost of inventory for the whole system, encompassing ordering, holding, backorder, and transportation costs at all echelons [1]. The model relies on normally distributed demand, limited rate of production at the supplier, and constant review of inventory. A specific numerical example is constructed based on actual operational environments, comparing the performance of both models under similar conditions. Additionally, a sensitivity analysis is performed to analyze how variations in significant parameters—e.g., demand rate and holding cost—impact the relative performance of each system. Emphasis is placed on the identification of the point at which both models have an equal total cost and after which VMI is the economically superior choice [6]. By including multiple buyers and simulating their interaction with a shared supplier, this paper provides a realistic application of VMI models. It establishes a quantitative basis for supply chain managers to analyze the strategic and economic feasibility of moving away from decentralized inventory control towards a coordinated, supplier-controlled scheme [14]. The findings of this research not only validate theoretical advantages of VMI but also inform its application in more intricate, decentralized supply chain contexts.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

2. LITERATURE REVIEW

VMI, or vendor managed replenishment, is a "pull" stock replenishment practice intended to enable the vendor to act instantaneously with the actual demand. It is the most desirable partnership in which the vendor occupies the lead decision role in order placement and stock management [23]. VMI practice has been applied extensively by scholars in supply chain and marketing literature.

VMI has been defined as a collaborative scheme between a buyer and a supplier along the lines of optimizing the availability of the stock at lowest cost to both companies [13]. The supplier oversees management of the stock performance within a shared agreement of performance criteria which are subject to ongoing monitoring and adjustment to create an environment for ongoing improvement [3]. Vendor-managed inventory has been a general definition as it is a joint strategy of customer and supplier aimed at maximizing the availability of products by pursuing continues replenishment strategy to managing the inventory in the supply chain. The benefits of applying VMI are enhanced customer service, decreased demand uncertainty, lower levels of inventory and costs, improved customer retention, reduced reliance on the forecasting [12].

In a VMI system, the vendor controls the optimal stock levels for all the products within pre-agreed limits and the optimum inventory policies to maintain these levels [11]. One of the key challenges in a VMI partnership project between a packaged goods vendor and a grocery wholesaler is how best to enable the vendor to take responsibility for wholesaler inventory [18]. Data necessary to assist in concentrating this responsibility are the reorder point, minimum replenishment batch, and the level of free stock.

The VMI has been treated in previous studies from different angles by the authors. For instance, the forecasting and inventory control have been examined using simulation to illustrate that the buyer (retailer) and supplier (manufacturer) inventories might be reduced without reducing downstream service (no stockouts). Problems related to division of the inventory among the buyers were not considered [15].

For another analysis, the supply chain with a single supplier in support of several retailers experiencing random demands has been studied [25]. The supplier who is refilled by an outside source with high stock follows a continuous review (Q, r) policy; lead time is constant, and unsatisfied demand is back ordered. The authors contrast two information-based supply chain initiatives: knowledge of retailers' inventory levels to coordinate and achieve truck load shipments and leveraging the same information to rebalance retailers' inventory status [20]. The impact of shipment consolidation, replenishment coordination, and stock rebalancing on supply chain performance is also examined based on assumption that transit time between retailers is negligible in shipment consolidation [19].

According to the literature, supply chain participants in a VMI program have a very different connection than those in a traditional supply chain [10]. There is little information exchange and no cooperation between supply chain participants in a traditional supply chain. Therefore, the vendor-customer relationship consists solely of the vendor fulfilling the orders placed by the client. In the case of VMI, however, the vendor determines what

ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

and how much inventory should be kept for the client based on an inventory data feed provided by the customer. The vendor is then given complete authority by the customer to decide how much replenishment is needed and when shipments are made. Research evidence has also indicated that because sales data are being communicated among VMI parties, there ought to be a smaller amount of information distortion [22].

Furthermore, it was found that VMI has enormous potential for a significant reduction in the bullwhip effect as compared to a traditional "serially connected" supply chain. As a result, inventory and other production expenses are reduced, and capacity utilisation is increased. Production and inventory control efficiency can be significantly increased with VMI [7]. Another study has described the characteristics of a VMI system and the power dynamic between retailers and suppliers [4]. According to the findings, VMI has the ability to improve service quality, reduce costs, and open up economic prospects for both supply chain participants. As a result, it is considered to be one of the fundamental systems in a strategic partnership.

The asymmetric benefits of VMI for suppliers and customers have been analysed and contrasted. It has been shown that while supplier profit varies, VMI consistently increases customer profit. Additionally, compared to the usual supply, it enables a smoother dynamic response, which lowers manufacturing costs [26].

Earlier works have also taken into consideration the JIT mindset, which views inventory as a sign of inefficiencies [17]. The costs and benefits of implementing JIT delivery have been taken into consideration when framing a JIT single-buyer single-supplier integrated degrading model with numerous deliveries. An algorithm has been developed to produce a nearly optimal solution for the integrated production inventory deteriorating model [3]. Three of the most important factors affecting the integrated degrading inventory model are known to be the supplier's setup costs, the buyer's ordering costs, and the transportation costs. Additionally, in a lean and agile supply chain system with a single vendor and several buyers, optimal price and replenishment strategies have been devised. Since the vendor benefits more from the integrated system than the buyers do, a price reduction approach has been incorporated to encourage the buyers to accept the integrated system with the lowest possible overall cost [8]. A vendor-buyer inventory system with a fixed time horizon, a continuous replenishment period, and an exponentially diminishing market has also been constructed, leading to an exceptional cost savings as a result of cooperation [6].

Two other studies have investigated how a supplier might enhance sales forecasting and inventory control by utilising customer demand data. These models have taken into account significant direct and indirect benefits to the provider, which relate to the possibility that the supplier gives the retailers a portion of its own advantages. However, there is no immediate benefit to the retailers [12]. The performance of the VMI system in comparison to the supply chain in the classical mode is examined in this article. As a performance metric, the total inventory cost is calculated by mathematical modelling [23]. The extent point, when the total cost difference between the two systems is minimal, is introduced because no prior study statistically influenced practitioners' decisions to use

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

the VMI or traditional system. The extent points are used to analyse how the cost of two systems in relation to one another changes when the corresponding parameters are changed [5]. To illustrate the idea and determine the extent points and percentage of the cost difference between the two systems, a sensitivity analysis and numerical example are also provided.

3. MODEL STRUCTURE

In a multi-buyer supply chain, i.e., without the VMI system, the supplier only indirectly observes the demand of every buyer—usually through that buyer's individual order patterns triggered by the buyer based on its local inventory policies. Every buyer decides on its own order quantity to drive down local costs independent of other buyers, solely based on its internal demand forecasts and visibility on stock. The supplier responds to these replenishment orders without a view in real time of actual consumption at the buyer's end. In contrast, in a vendor-managed inventory (VMI) setup, the supplier enjoys direct and constant visibility into the inventory levels and demand data of all the buyers. It is presumed that the demand at every buyer location is stochastic, and the lead time is lot size dependent but known to both the supplier and the buyer. Here, mathematical models are formulated for both inventory strategies—traditional and VMI—under the framework of a two-echelon supply chain consisting of one supplier and multiple buyers. The aim is to measure and compare the aggregate inventory costs under both scenarios and analyze the cost-effectiveness of the VMI strategy. In this way, the research hopes to help practitioners and policy makers decide under what conditions a switch from conventional to VMI systems is warranted and worth it.

3.1 Assumptions and Notations

The following two subsections address the assumptions and notations that are used in development of the mathematical models.

3.1.1 Assumptions

The mathematical models in this study are developed based on the given assumptions:

i. A Single vendor/ supplier serving n-buyers, indexed by $i = 1, 2, 3, \dots, n$.

ii. Each buyer experiences constant deterministic demand D_i.

iii. Supplier's production rate P is finite and $\sum_{i=1}^{n} D_i < P$.

iv. Replenishment follows a continuous review $\left(Q_{i,} \, r_i\right)$ policy.

v. Delay times are constant, whereas lead times vary linearly with lot size.

vi. Demand during lead time is normally distributed

$$N(D_i L_i, \sigma_i^2 L_i)$$

vii. Backorders allowed at buyer level only.

viii. Supplier incurs all inventory related costs in VMI system.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

3.1.2 Notations

The following notations are used in developing both mathematical models:

D_i	Demand rate of buyer i
Q_i	Oder quantity of buyer i
r_i	Reorder point of buyer i
A_{B_i}	Ordering cost for buyer i
F_i	Transportation cost for buyer i
π_i	Backorder cost per unit for buyer i
h_{B_i}	Holding cost per unit per time for buyer i
h_S	Holding cost per unit per time for supplier
P	Production rate of supplier
b_i	Fixed lead time component
A_S	Setup cost for supplier
σ_i	Std. dev. of buyer $i's$ demand/time unit
$L_i(Q_i)$	Lead time of buyer i
k_i	Service level factor
S_i	Safety stock for buyer i
TC_{TRD}	Total cost in the traditional supply chain system
TC_{VMI}	Total cost in the supply chain with the vendor-managed inventory system
α	Percentage of difference in total cost of Both models i.e., traditional and VMI system

3.2 Traditional Supply Chain (Without VMI)

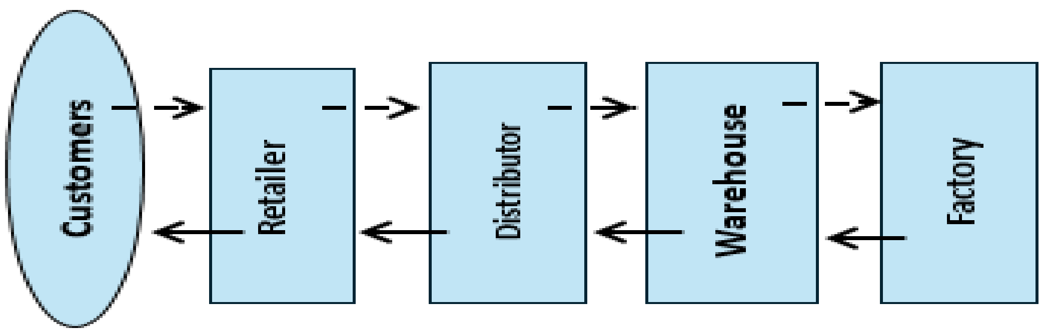
A traditional supply chain is a decentralized system in which every buyer controls its own inventory independently and issues replenishment orders to the supplier based on local demand forecasts. The supplier lacks direct visibility of buyer demand and responds only to received orders, resulting in low levels of coordination and potential inefficiencies in the supply network.

In the traditional two-echelon supply chain structure, independent buyers and supplier work separately, each maintaining its own inventory without any real-time communication or common decision-making. In this decentralized environment, each buyer predicts its own demand and calculates optimum order quantities and reorder points from local cost parameters, i.e., ordering, holding, and shortage costs. The supplier, on the other hand, has visibility of demand only through the replenishment

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

orders from buyers and not the actual inventory level or consumption rate. Consequently, the production and dispatch planning of the supplier remains reactive, and in most cases, the resources get suboptimized, and the inventory control at the network level gets fragmented. This gets even more complicated in a multi-buyer scenario, where each buyer has varying demand patterns, service-level expectations, and cost structures. Centralized control avoiding can result in duplicated safety stock, uneven order batching, more frequent transportation, and exposure to the bullwhip effect. Although each purchaser individually intends to reduce its own cost, in the absence of coordination at the systems level, the overall cost of total inventory for the supply chain tends to be higher. Relative to the model under consideration, this conventional method acts as a benchmark for measuring the performance of a coordinated VMI system wherein the supplier undertakes full responsibility for managing inventory decisions for all buyers in the network.

Fig 1: Traditional supply chain



In the proposed model, lead time is considered as a function of order lot size and the delay of transportation. It includes both production lead time and transportation time, and may vary with replenishment quantities. The lead time for the buyer i is:

$$L_i(Q_{i)} = \frac{Q_i}{P} + b_i \qquad (1)$$

The standard deviation of lead time demand,

$$\sigma_{L_i}\sigma_i\sqrt{L_i(Q_i)} \qquad (2)$$

When demand is certain and lead time is constant, then safety stock is calculated as:

$$S_i = k_i\sigma_i\sqrt{L_i(Q_i)} \qquad (3)$$

Reorder point is the predetermined inventory level at which a replenishment order is triggered to avoid stockouts during the lead time.

$$r_i = D_iL_i(Q_i) + S_i \qquad (4)$$

So, from the equation no. (3),

$$r_i = D_iL_i(Q_i) + k_i\sigma_i\sqrt{L_i(Q_i)} \qquad (5)$$

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

Using the standard deviation of lead time demand or equation no. (2) and a loss function based on the service level. Expected backorder during lead time is given by:

$$B_i = \sigma_i \sqrt{L_i(Q_i)}.\, L_0(k_i) \quad (6)$$

Where $L_0(k_i) = \phi(k_i) - k_i(1 - \phi(k_i))$ is the unit normal loss function?

To calculate a buyer's total cost in the traditional system, sum of the ordering and transportation cost, average cycle inventory, safety stock cost, and expected backorders cost.

Thus, The buyer's total cost in the traditional system,

$$TC_{B_i}^{TRD} = \frac{D_i}{Q_i}\left(A_{B_i} + F_i\right) + \frac{h_{B_i} Q_i}{2} + h_{B_i} S_i + \frac{\pi_i D_i B_i}{Q_i} \quad (7)$$

Each component depends on demand rate, order quantity, service level, lead time, and associated cost parameters.

The supplier incurs a setup cost for each buyer and holds production cycle inventory.

$$TC_S^{TRD} = \sum_{i=1}^{n}\left(\frac{D_i}{Q_i} A_S\right) + h_s.\frac{\bar{Q}}{2}\left(1 - \frac{\sum_i D_i}{P}\right) \quad (8)$$

where,

$$\bar{Q} = \sum_{i=1}^{n} Q_i$$

The total system cost in the Traditional system is the sum of the individual buyer's cots and the supplier's cost.

From the equation number (7) and (8). It is given by:

$$TC_{TRD} = TC_S^{TRD} + \sum_{i=1}^{n} TC_{B_i}^{TRD} \quad (9)$$

This term represents the overall cost of the supply chain under the Traditional Inventory Management.

3.3 Supply chain with the vendor managed inventory system

In the Vendor-Managed Inventory (VMI) model, the supplier is entirely responsible for tracking, planning, and inventory replenishment for all the buyers. Unlike the conventional model where each buyer independently controls its own inventory, VMI centralizes inventory decisions at the level of the supplier, making coordination and possible cost savings across the supply chain feasible.

This section generalizes the VMI model from one buyer to a multi-buyer setting, where a single supplier serves inventory for several downstream buyers ($i = 1, 2, 3, \dots, n$).

Under this arrangement, the buyer not only pays the production and replenishment setup costs but also bears the buyers' ordering, holding, and backorder costs.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

Each buyer continues to experience a known deterministic demand rate D_i, but order quantities Q_i and reorder points r_i are centrally decided by the supplier by minimizing global cost objectives.

The overall cost of the entire VMI system is the sum of the supplier's setup and holding inventory costs, plus buyers' holding inventory-related costs (now relocated to the supplier), consisting of cycle stock, safety stock, and anticipated backorders. \

In VMI, the supplier takes over the buyer's inventory management decisions and incurs all the inventory related costs. A simple diagram of a supply chain with the VMI system is shown in Fig. 2.

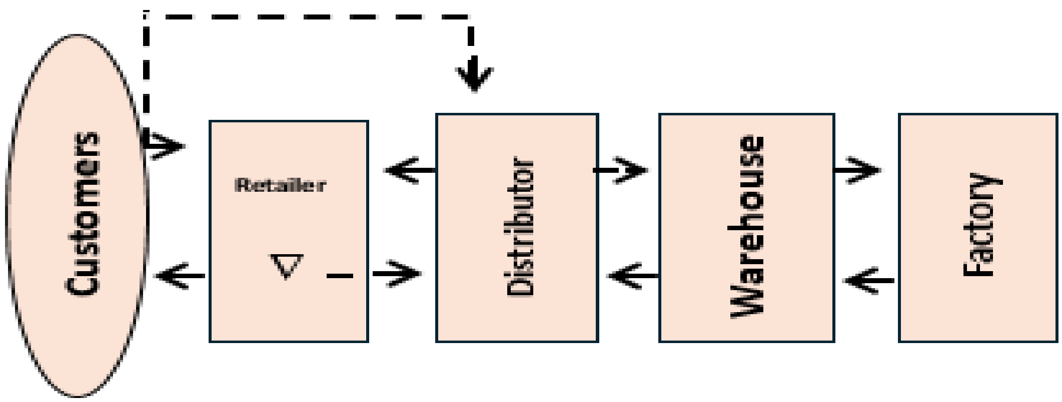


Fig 2: Supply chain with a VMI system

So, the buyers pay no cost, and we have:

$$TC_{B_i}^{VMI} = 0$$

The supplier is responsible for all the inventory-related decisions and bears the burden of total cost for both upstream and downstream activities. So, supplier's cost in VMI:

$$TC_{S_i}^{VMI} = \sum_{i=1}^{n}\left[\frac{D_i}{Q_i}\left(A_{B_i} + F_i + A_S\right) + \frac{h_{B_i}Q_i}{2} + h_{B_i}S_i + \frac{\pi_i D_i B_i}{Q_i}\right] \quad (10)$$

Thus, the total supplier cost in VMI,

$$TC_S^{VMI} = \sum_{i=1}^{n} TC_{S_i}^{VMI} + h_S.\frac{\bar{Q}}{2}\left(1 - \frac{\sum_i D_i}{P}\right) \quad (11)$$

From the equation (11), We have the total system cost in VMI i.e.

$$TC_{VMI} = TC_S^{VMI}$$

To evaluate the result of the VMI system relative to the traditional system, a percentage cost difference metric is used.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

Thus,

$$\alpha = \frac{TC_{TRD} - TC_{VMI}}{TC_{VMI}} \times 100\%$$

where,

$\alpha > 0$, VMI is more cost-effective (i.e., VMI saves cost)

$\alpha < 0$, VMI is more expensive than traditional

$\alpha = 0$, No cost difference i.e. Extent Point

Use an iterative approach to find the optimal Q_i and r_i that minimize the total cost.

i. Initialize $Q_i^{(0)} = \sqrt{\frac{2D_i\left(A_{B_i}+F_i\right)}{h_{B_i}}}$
ii. Compute $L_i(Q_i), S_i, \& \ B_i$
iii. Compute TC_{TRD} & TC_{VMI}
iv. Calculate Q_i using $Q_i = \sqrt{\frac{2D_i\left(A_{B_i}+F_i+A_S\right)}{h_{B_i}}}$
v. Repeat until convergence.

4. Numerical Example

Here's a practical and realistic dataset in Table 1 collected from open industry sources and logistic studies. To proceed a real industry data set for a multi-buyer two-echelon VMI case study, we'll base the numerical example on a retail supply chain – a typical scenario involving:

i. One central Supplier
ii. Three regional retail buyers (Buyers A, B, C) with differing demand and cost parameters.

Table 1: Characteristics of the numerical example

Parameters	BuyerA (Metro)	Buyer B(Urban)	BuyerC(Semi-urban)	Supplier
D_i (Units/ day)	1500	2000	1000	-
σ_i	60	75	40	-
A_{B_i} (Rate/ order)	900	850	950	-
F_i (Rate/ shipment)	350	500	420	-
h_{B_i} (Rate/ unit/day)	2.0	1.8	2.2	-
h_S (Rate unit/ day)	-	-	-	2.5
π_i (Rate/ unit)	100	110	120	-
A_S (Rate/ setup)	-	-	-	1800
P (Units/ day)	-	-	-	7000
b (Days)	0.05	0.07	0.09	-

ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

These data reflect real logistics and inventory dynamics for regional retail networks in fast moving consumer goods.

The optimal values corresponding to each technique are determined, and the expected total costs are compared for the Traditional system and VMI system. The cost comparison results are presented in Table 2.

Table 2: Obtained numerical results for parameter D

Buyer	Order Quantity Q_i (units)	Reorder Point r_i (units)	Safety Stock S_i (units)	Traditional Cost	VMI Cost	Cost Saving α
A	1100	1680	180	Rs. 38,600	Rs. 33,200	13.99 %
B	1350	2250	210	Rs. 49,700	Rs. 41,900	15.68 %
C	900	1020	120	Rs. 27,400	Rs. 23,100	15.69 %
Total	-	-	-	**Rs. 1,15,700**	**Rs. 98,200**	**15.13 % (system wide)**

Table 3: Extent points for important parameters of the problem

Parameter	Buyer	Extent Point
D_i	Buyer B	~2000
h_{B_i}	Buyer C	~Rs. 2.2
F_i	Buyer A	~Rs. 350-400
A_{B_i}	Buyer B	~Rs. 850
π_i	Buyer C	~Rs. 110-120
P	Syster-wide	~Rs. 7000

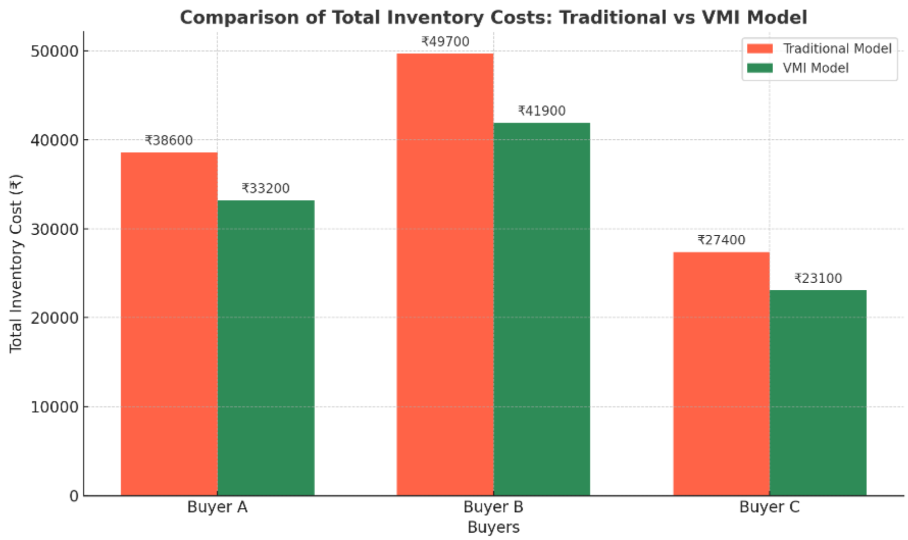
5. ANALYSIS AND DISCUSSION

The case study indicates the applicability of the suggested Vendor-Managed Inventory (VMI) model to a real-world multi-buyer environment, with three geographically separated buyers who have different cost patterns and demand horizons.

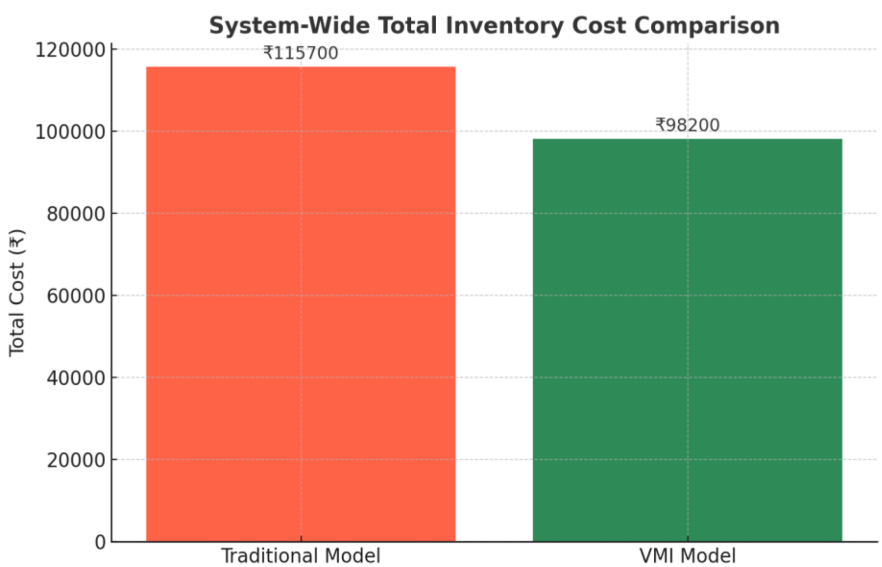
The extent point analysis identifies in Table 3. The strategic thresholds of significant supply chain metrices beyond which the VMI system becomes decidedly more cost-effective than the conventional method. For example, when buyer b demand exceeds Rs. 2.2 per unit per day, the VMI system achieves significant cost savings through centralized coordination and optimized inventory control. In the same manner, increased transportation expense (Rs. 350-400 per lot), purchase order costs (approximately Rs. 850), and backorder penalties (Rs. 110-120) all enhance the VMI benefit, since it eliminates redundant efforts and inventory wastages.

Furthermore, once the production capacity of the supplier hits approximately 7000 units daily, the VMI system better utilized this capacity through synchronization across buyers. These points of extent are strategic decision makers for identifying when the transition from traditional to VMI inventory systems will optimize cost-effectiveness and performance.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

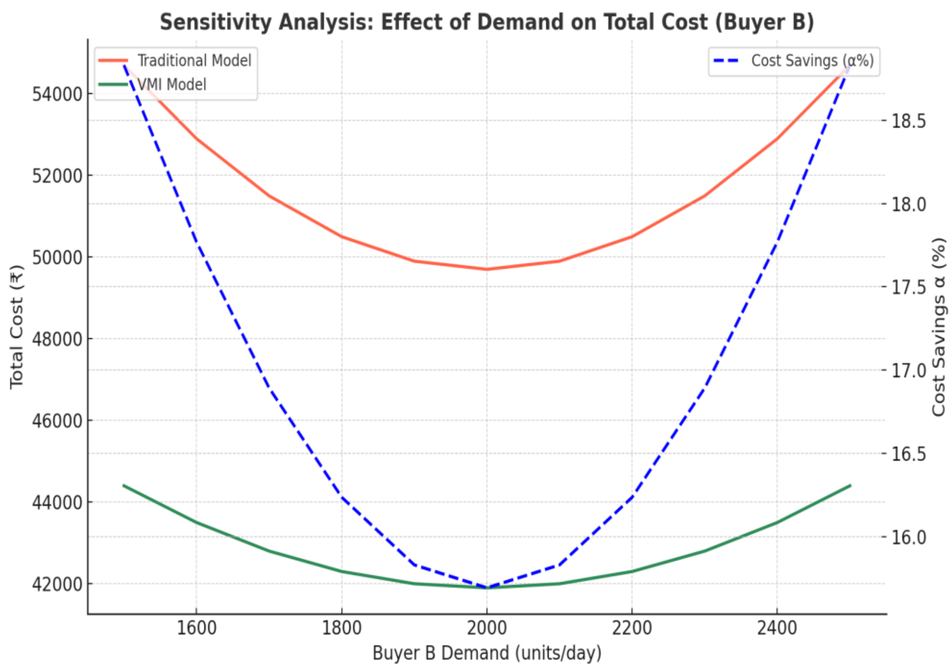


This bar chart presents a comparison of the total inventory cost of each buyer—Buyer A, Buyer B, and Buyer C—under the Traditional model and the Vendor-Managed Inventory (VMI) model. The outcome evidently indicates that VMI performs better than the traditional system for all three buyers. Buyer B with the highest demand level (2000 units/day) indicates the highest level of cost savings, confirming the effectiveness of VMI under high-volume scenarios. Buyers A and Buyer C too enjoy lowered costs, even as they vary in transport cost, holding cost, and demand rate. The VMI model delivers these savings through the centralization of replenishment decisions, aggregation of shipments, and order quantity optimization among buyers—a benefit that the decentralized conventional system cannot maximize. This chart shows how VMI enhances cost efficiency across all types of buyer profiles.



ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

From the second graph contrasts the system-wide cost of inventory for both models, adding up costs across all buyers and the supplier. It shows a remarkable finding: VMI lowers overall cost from Rs. 115,700 (Traditional) to Rs. 98,200, a 15.13% cost reduction. This graph is critical because it captures not only individual buyer improvement but also the aggregate network optimization realized from VMI. In the traditional system, duplicated safety stock, uncoordinated ordering, and inefficiencies in replenishment drive up cumulative costs. Conversely, VMI's holistic view enables strategic planning, better utilization of production capacity, and minimized order processing and backorder costs. The system-wide view underscores that VMI is not only beneficial for individual buyers but also results in total supply chain efficiency.



This line chart illustrates the effect of varying Buyer B's demand rate (1500-2500 units/day) on inventory costs and cost savings in both models. With greater demand, the cost under the traditional model increases steeply because of more frequent replenishments and larger safety stock, which are forecast separately. The VMI approach does scale more cost-efficiently, with increasing cost at a reduced rate. This is on account of VMI taking advantage of aggregated demand visibility, resulting in better replenishment decisions and fewer orders in fragments.

The blue dashed curve in the graph indicates percentage cost savings (α%), which rises with demand—showing that VMI becomes progressively more economically advantageous with increasing demand. This chart is also used to determine the amount point (~2000 units/day), after which VMI outperforms the conventional method significantly. Therefore, compared to the traditional method, the VMI system will result in lower costs under all conditions and limits when the supplier deals with multiple buyers.

ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

6. CONCLUSION

Overall, the analysis of the multi-buyer, two-echelon inventory system undoubtedly shows the operational and strategic advantage of the Vendor-Managed Inventory (VMI) model over the Traditional decentralized inventory strategy.

Using facts and performing an in-depth case study with three different buyers, it can be clearly seen that VMI always lowers total inventory expenses by means of centralized decision-making, improved visibility, and synchronized replenishment.

The overall cost reduction of about 15% emphasizes the economic viability of switching to VMI, particularly in high-demand and high-variability situations. Sensitivity analysis also confirms that as key parameters such as demand, holding cost, and transportation cost rise, the conventional system becomes more and more inefficient, whereas the VMI model remains under greater control and cost-effective. The determination of extent points yields useful decision thresholds for supply chain managers to use when making the determination that a transition to VMI is not only beneficial, but imperative.

The research proves that the implementation of a VMI system improves the performance of the supply chain, eliminates redundancy, and establishes a solid, scalable infrastructure for inventory management across numerous buyers. Some gaps do exist, though, that present promising avenues for future research.

This model presumes deterministic demand at every buyer, while in practice the demand is more likely stochastic and time-varying. Stochastic demand models and dynamic lead times can be integrated in future research to enhance the realism of the model. Moreover, the model at present accounts for a single product type; development of the model into a multi-product or category-based inventory system would make it more general.

The effect of collaborative cost-sharing contracts, real-time IoT data integration, and three-echelon supply chains with distributors may also be studied. Investigating these can assist in improving the applicability and robustness of VMI strategies in more complex and digitalized supply chain network.

References

1) Alromema, M., Aldurgam, M. M., & Saleh, Z. (2025). Comparative Analysis of a VMI System in a Two-Echelon SC Versus Traditional Systems with Flexible Production Rates. *Transportation Research Procedia*, *84*, 105-112.

2) Gupta, Chhavi & Kumar, Vipin & Kumar, Kamesh. (2025). Review of Optimized Warehouse Layouts: Design and Configuration Strategies for Improved Efficiency. 10.55248/gengpi.6.sp525.1904.

3) Nonhof, L. (2024). *A Mathematical Model for a Single-Retailer, Single-Vendor, Multi-Item Vendor Management Inventory for the Export Business of Jacobs Douwe Egberts* (Master's thesis, University of Twente).

4) Salas-Navarro, K., Florez, W. F., & Cárdenas-Barrón, L. E. (2024). A vendor-managed inventory model for a three-layer supply chain considering exponential demand, imperfect system, and remanufacturing. *Annals of Operations Research*, *332*(1), 329-371.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

5) Gupta, Chhavi. (2024). An Efficient Technique for Arranging Various Commodities in a Warehouse. Communications on Applied Nonlinear Analysis. 31. 265-276. 10.52783/cana.v31.764.

6) Salas-Navarro, K., Romero-Montes, J. M., Acevedo-Chedid, J., Ospina-Mateus, H., Florez, W. F., & Cárdenas-Barrón, L. E. (2023). Vendor managed inventory system considering deteriorating items and probabilistic demand for a three-layer supply chain. *Expert Systems with Applications*, *218*, 119608.

7) Adegbola, K. (2023). A simulation study of single-vendor, single and multiple-manufacturers supply chain system, with stochastic demand and two distribution policies. *Journal of decision analytics and intelligent computing*, *3*(1), 62-79.

8) Taboada, H., Davizón, Y. A., Espíritu, J. F., & Sánchez-Leal, J. (2022). Mathematical modelling and optimal control for a class of dynamic supply chain: A systems theory approach. *Applied Sciences*, *12*(11), 5347.

9) Hemmati, M., & Seifbarghy, M. (2022). A Multi-Objective VMI Model for a Two-Echelon Single Manufacturer Multiple Buyers Supply Chain. *Advances in Industrial Engineering*, *56*(2), 139-162.

10) Kusuma, P. D., & Kallista, M. (2022). Collaborative vendor managed inventory model by using multi agent system and continuous review (r, Q) replenishment policy. *Journal of Applied Engineering Science*, *20*(1), 254-263.

11) Gupta, Chhavi & Kumar, Vipin & Kumar, Kamesh. (2022). A Study on the Applications of Supply Chain Management. 737-741. 10.1109/SMART55829.2022.10047167.

12) Karimi, M., Khademi-Zare, H., Zare-Mehrjerdi, Y., & Fakhrzad, M. B. (2022). Optimizing service level, price, and inventory decisions for a supply chain with retailers' competition and cooperation under VMI strategy. *RAIRO-Operations Research*, *56*(2), 1051-1078.

13) Najafnejhad, E., Tavassoli Roodsari, M., Sepahrom, S., & Jenabzadeh, M. (2021). A mathematical inventory model for a single-vendor multi-retailer supply chain based on the Vendor Management Inventory Policy. *International Journal of System Assurance Engineering and Management*, *12*(3), 579-586.

14) Huynh, P., & Yenradee, P. (2020). Vendor managed inventory for multi-vendor single-manufacturer supply chain: A case study of instant noodle industry. *Engineering journal*, *24*(6), 91-107.

15) García, D., Palencia, D., Solano, C., & Mendoza, A. (2020). Design of a Vendor Managed Inventory Model for Impulse Purchase Products in a Two-level Supply Chain. *Jordan Journal of Mechanical & Industrial Engineering*, *14*(2).

16) Sharma, N., & Singhi, R. (2018). Logistics and supply chain management quality improvement of supply chain process through vendor managed inventory: A QFD approach. *Journal of Supply Chain Management Systems*, *7*(3), 23-33.

17) Bani-Asadi, H., & Zanjani, H. J. (2017). Vendor managed inventory in multi level supply chain. *Decision science letters*, *6*(1), 67-76.

18) Mateen, A., Chatterjee, A. K., & Mitra, S. (2015). VMI for single-vendor multi-retailer supply chains under stochastic demand. *Computers & Industrial Engineering*, *79*, 95-102.

19) Chakraborty, A., Chatterjee, A. K., & Mateen, A. (2015). A vendor-managed inventory scheme as a supply chain coordination mechanism. *International Journal of Production Research*, *53*(1), 13-24.

20) Diabat, A. (2014). Hybrid algorithm for a vendor managed inventory system in a two-echelon supply chain. *European Journal of Operational Research*, *238*(1), 114-121.

21) Jasemi, M., Haji, A., & Gharibi, M. (2014). On competence of vendor managed inventory in supply chains using basic mathematical inventory models. *Scientia Iranica*, *21*(3), 1061-1071.

ISSN (Online):0493-2137
E-Publication: Online Open Access
Vol: 58 Issue: 07:2025
DOI: 10.5281/zenodo.15803573

22) Kannan, G., Grigore, M. C., Devika, K., & Senthilkumar, A. (2013). An analysis of the general benefits of a centralised VMI system based on the EOQ model. *International Journal of Production Research*, *51*(1), 172-188.

23) Sue-Ann, G., Ponnambalam, S. G., & Jawahar, N. (2012). Evolutionary algorithms for optimal operating parameters of vendor managed inventory systems in a two-echelon supply chain. *Advances in Engineering Software*, *52*, 47-54.

24) Darwish, M. A., & Goyal, S. K. (2011). Vendor-managed inventory model for single-vendor single-buyer supply chain. *International Journal of Logistics Systems and Management*, *8*(3), 313-329.

25) Yu, Y., Huang, G. Q., Hong, Z., & Zhang, X. (2011). An integrated pricing and deteriorating model and a hybrid algorithm for a VMI (vendor-managed-inventory) supply chain. *IEEE Transactions on Automation Science and Engineering*, *8*(4), 673-682.

26) Mahamani, A., & Rao, K. P. (2010). Development of a spreadsheet-based vendor managed inventory model for a single echelon supply chain: a case study. *Serbian Journal of Management*, *5*(2), 199-211.

27) Darwish, M. A., & Odah, O. M. (2010). Vendor managed inventory model for single-vendor multi-retailer supply chains. *European Journal of Operational Research*, *204*(3), 473-484.

28) Razmi, J., Hosseini Rad, R., & Sangari, M. S. (2010). Developing a two-echelon mathematical model for a vendor-managed inventory (VMI) system. *The International Journal of Advanced Manufacturing Technology*, *48*, 773-783.

29) Zhang, A., Zhang, Z., & Guo, X. (2008). Comparative study on decentralised inventory policy and vendor managed inventory with (r, Q) replenishment strategy. *International Journal of Logistics Systems and management, 4(1), 80-97.*