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POSSIBILITY INTERVAL-VALUED INTUITIONISTIC FUZZY SOFT SETS - TO PROTECT THE CRITICALLY ENDANGERED SPECIES FROM THE MUDUMALAI FOREST, NILGIRI'S, SOUTH INDIA

ARUNADEVI S

Department of Mathematics, Avinashilingam Institute of Home Science and Higher Education for Women, India. Email: 18phmap005@avinuty.ac.in

JAYANTHI D

Department of Mathematics, Avinashilingam Institute of Home Science and Higher Education for Women, India. Email: jeyanthi_mat@avinuty.ac.in

SARANYA M

Department of Mathematics, Sri Krishna Adithya College of Arts and Science, India.
Email: saranyaresearch94@gmail.com

Abstract

Preserving endangered species is crucial for maintaining ecological balance. This article presents a model designed to help ecologists identify species at high risk of extinction in the Mudumalai Forest of South India. Although population size is a key indicator, a comprehensive evaluation of various factors is necessary. The model utilizes Possibility Interval-Valued Intuitionistic Fuzzy Soft Sets (PIVIFSS) to address this complex decision-making process. A case study focusing on endangered species in Mudumalai validates the model, incorporating criteria from literature reviews and expert consultations. Six criteria and six alternatives are assessed using PIVIFSS through both top-down and bottom-up approaches. MATLAB programming showcases the model's effectiveness. The comparative analysis demonstrates that this methodology excels in complex decision-making, serving as a valuable tool for conservationists.

Keywords: Endangered Animals, Fuzzy Set, Intuitionistic Fuzzy Soft Sets, Interval-Valued Sets, Optimization, Possibility Sets.

1. INTRODUCTION

Protecting endangered species within forest ecosystems is crucial for several reasons. Forests support diverse species essential for ecological balance, and their extinction can disrupt this equilibrium, impacting the entire ecosystem. Conservation is vital for maintaining biodiversity, which supports ecosystem resilience, climate regulation, and essential resources like food, medicine, and clean water. Additionally, endangered species often hold cultural, scientific, and intrinsic value, making their preservation important from both ecological and ethical perspectives.

Ensuring their survival helps maintain healthy forests that benefit both the environment and human communities. Mudumalai Forest, located in the Nilgiri Hills of Tamil Nadu, India, is part of the Nilgiri Biosphere Reserve, known for its rich biodiversity. This forest is home to many endangered species, with threats including habitat degradation, poaching, human-wildlife conflict, and environmental changes affecting their

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survival.Vagueness is an inherent aspect of everyday life, prompting the need for mathematical tools to address such uncertainty.

Zadeh (1965) pioneered the fuzzy set concept to manage imprecision mathematically. Building on this foundation, Atanassov (1986) introduced intuitionistic fuzzy sets, followed by the introduction of interval-valued intuitionistic fuzzy sets by Atanassov and Gargov (1989). To further handle ambiguously defined objects and uncertainties, Molodtsov (1999) proposed soft set theory. Maji et al. (2003) highlighted the significance of soft set theory in resolving decision-making problems. Alkhazaleh et al. (2011) expanded this field by introducing the concept of soft multisets, along with the expert set and its applications, as presented by Alkhazaleh and Salleh (2011). And also the authors developed fuzzy parameterized interval-valued fuzzy soft sets and possibility fuzzy soft sets, exploring their applications in decision-making, medical diagnostics, engineering, and social sciences.

The concept of the possibility vague soft set was later defined by Alhazamyeh et al. (2012), while Maruah Bashir et al. (2012) introduced the notion of the possibility intuitionistic fuzzy soft set, further extending the possibility fuzzy soft set into this realm. Zhiming Zhang et al. (2014) proposed top-bottom and bottom-bottom algorithms for interval-valued intuitionistic fuzzy soft sets, albeit without considering possibility values.

1.1 Key Endangered Species:

1. **Asian Elephant (Elephasmaximus):** Mudumalai serves as a critical sanctuary for the Asian elephant, classified as endangered due to habitat fragmentation, human encroachment, and ivory poaching. This region has long been a key corridor for elephant migration, connecting multiple protected areas within the Nilgiri Biosphere Reserve.
2. **Indian Leopard (Pantherapardusfusca):** Leopards in Mudumalai face significant threats due to human-wildlife conflict, where predation on livestock often results in retaliatory killings. Additionally, habitat encroachment and the decline in prey species have exacerbated their vulnerability.
3. **Indian Wild Dog or Dhole (Cuonalpinus):** The Dhole, a highly social apex predator, has experienced a sharp decline in population within Mudumalai, primarily due to habitat degradation and a diminishing prey base. These factors have greatly contributed to their endangered status.
4. **Gaur (Bosgaurus):** The Indian bison, or Gaur, the largest wild cattle species, inhabits the Mudumalai region. Although not critically endangered, they are categorized as vulnerable due to ongoing threats from habitat loss and hunting pressures.
5. **Long-billed Vulture (Gyps indicus):** The critically endangered Long-billed Vulture, also known as the Indian Vulture, can be found in the Indian subcontinent, including Mudumalai Forest. The species has faced a drastic population decline, primarily

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due to the use of the veterinary drug diclofenac, which is lethal to vultures when they feed on the carcasses of treated livestock.

6. **Bengal Tiger (Pantheratigristigris):** The Bengal tiger, an emblematic species of Mudumalai, has been endangered by poaching and habitat destruction. As part of a broader tiger conservation landscape in India, Mudumalai has been integrated into initiatives like Project Tiger to closely monitor and safeguard tiger populations.

1.2 Conservation Efforts:

Conservation in Mudumalai advanced significantly in the mid-20th century, starting with the creation of the Mudumalai Wildlife Sanctuary in 1940 due to rising concerns over declining wildlife. The area received additional protection in 2007 when it was designated a Tiger Reserve under Project Tiger, enhancing conservation efforts. Numerous initiatives have been implemented to protect endangered species in Mudumalai, including anti-poaching measures, habitat restoration, and community engagement to reduce human-wildlife conflicts. The forest department has also improved species monitoring using advanced technologies like camera traps and satellite tracking. Despite these efforts, challenges such as illegal poaching, deforestation, and climate change continue. Ongoing conservation strategies, supported by local and international organizations, aim to preserve Mudumalai's' biodiversity and safeguard its endangered species. This research applies a possibility interval-valued intuitionistic soft set approach, using both top-down and bottom-up strategies, to help ecologists prioritize the most critically endangered species in Mudumalai Forest, Tamil Nadu (South India).

This manuscript is organized as follows: Section 2 covers the preliminaries of interval-valued intuitionistic fuzzy sets, including the corresponding top-down and bottom-up approaches, as well as possibility intuitionistic fuzzy soft sets. In Section 3, we introduce a possibility interval-valued intuitionistic fuzzy soft set and describe the associated top-down and bottom-up approaches. Section 4 details the methodology, encompassing both top-down and bottom-up approaches using PIVIFSS, and includes a numerical example. Section 5 presents the results and discussion, based on MATLAB, while Section 6 offers the conclusion.

1.3 Motivation of the Study:

1. This innovative approach significantly advances Multi-Criteria Decision-Making (MCDM) by integrating both interval-valued and possibility parameters for membership and non-membership values, addressing a gap that previous methods did not cover.

2. PIVIFSS is particularly effective in addressing MCDM challenges, especially in scenarios with incomplete information, by introducing an additional layer to traditional fuzzy methodologies.

3. This extension significantly bolsters the robustness of decision-making in complex situations. However, while the added flexibility enhances the approach's

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adaptability, it also increases the complexity of the mathematical formulations and computational methods required.

4. MATLAB is utilized to assess and compare the outcomes of the proposed method with those of existing techniques.

2. PRELIMINARIES

Definition 2.1: [Yuncheng Jiang.et.al., 2010]

An Interval-valued Intuitionistic Fuzzy set on a universe X is an object of the form A = $\{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x): X \to Int([0,1])$ and$\gamma_A(x): X \to Int([0,1])$ stands for the set of all closed sub intervals of [0,1] satisfying the condition for all $x \in X$, $0 \le sup\mu_A(x) + sup\gamma_A(x) \le 1$.

Definition 2.2: [Zhiming Zhanget.al., 2014]

Let $C = \langle G, G_I \rangle$, be an IVIFSS over U, where $G_I \subseteq E$, E is the parameter set. Based on the IVIFSS over U for $C = \langle G, G_I \rangle$, define a top - bottom IVIFSS as,

1. $\mu_{\text{top - bottom}_C} = max_{x\in U}\mu_{\overline{G(E)}}(x) = \left[max_{x\in U}\mu_{\overline{G(E)L}}(x), max_{x\in U}\mu_{\overline{G(E)R}}(x)\right]$
2. $\gamma_{\text{top - bottom}_C} = min_{x\in U}\gamma_{\overline{G(E)}}(x) = \left[min_{x\in U}\gamma_{\overline{G(E)L}}(x), min_{x\in U}\gamma_{\overline{G(E)R}}(x)\right]$ for all $x \in A$.
3. We also define an IVIFSS over U, as Bottom - bottom - level IVIFSS as,
4. $\mu_{\text{Bottom - bottom}_C} = min_{x\in U}\gamma_{\overline{G(E)}}(x) = \left[min_{x\in U}\mu_{\overline{G(E)L}}(x), min_{x\in U}\mu_{\overline{G(E)R}}(x)\right]$
5. $\gamma_{\text{Bottom - bottom}_C} = min_{x\in U}\gamma_{\overline{G(E)}}(x) = \left[min_{x\in U}\gamma_{\overline{G(E)L}}(x), min_{x\in U}\gamma_{\overline{G(E)R}}(x)\right]$ for all $x \in A$.

Definition 2.3: [Alkhazaleh,S. and Salleh, A.R, 2012]

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, \ldots . x_n\}$ be the universal set of elements and $P = \{p_1, p_2, p_3, p_4, p_5, p_6, \ldots . p_m\}$ be the parameters set. The pair (U,P) is the soft universe set. Let $G: P \to IF_{(\text{U})}, \times IF_{(\text{U})}$ where $IF_{(\text{U})}, \times IF_{(\text{U})}$ is the collection of all IF subsets of U and *IF*(U)is the collection of all subsets of fuzzy set U. Let $p: P \to IF(\text{U})$ and let

$G_p: P \to (IF_{(\text{U})}, \times IF_{(\text{U})}) \times IF_{(\text{U})}$ be a function defined as,

$\boldsymbol{G_p(e)} = (G(p)(x), p(P)(x)), for\ all\ x \in U$ where

$$G(P)(x) = (\mu(x), \gamma(x)) for\ all\ x \in U$$

Then G_p is called Possibility Intuitionistic Fuzzy Soft Set (PIFSS) over the soft universe (U,P). For each parameter P_i ,$G_p(P_i) = (G(P_i)(x), p(P_i)(x))$ indicates not only the degree of belongingness of the elements of U in $G(P_i)$, but also the degree of possibility of belongingness of the elements of U in $G(P_i)$, which is represented by $p(P_i)$. So $Gp(P_{\boldsymbol{i}})$ =$\left\{\left(\frac{\boldsymbol{x_1}}{G(P_i)(\boldsymbol{x_1})}, p(P_i)(\boldsymbol{x_1})\right), \left(\frac{\boldsymbol{x_2}}{G(P_i)(\boldsymbol{x_2})}, p(P_i)(\boldsymbol{x_2})\right), \ldots, \left(\frac{\boldsymbol{x_n}}{G(P_i)(\boldsymbol{x_n})}, p(P_i)(\boldsymbol{x_n})\right)\right\}$

$\boldsymbol{G_p}$can be written as (G_p, P). If $A \subseteq P$ one can also have a PIFSS $(\boldsymbol{G_p}, A)$.

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3. THE TOP-BOTTOM LEVEL APPROACHUSINGPOSSIBILITY INTERVAL-VALUED INTUITIONISTIC FUZZY SOFT SETS

In this section, we present an adjustable approach to PIVIFSS using Top-Bottom-level approach for decision making problems. To illustrate the effectiveness of this method, we provide a practical example implemented in MATLAB.

Definition 3.1:

Let a set U be fixed. An PIVIFSS on a universe U with x is an object of the form,

$$G_{p}=\left\{\left(\frac{x_i}{G(P_i)(x_i)}, p(P_i)(x_i)\right)/x\in U\right\}$$

$G(P_i)(x_i) = \mu_{G_p(x)}\gamma_{G_p(x)}$ and $\mu_{G_p(x)}: X \to Int([0,1])$and $\gamma_{G_p(x)}: X \to Int([0,1])$ where $Int([0,1])$represents the set of all closed subintervals of [0,1] satisfying the condition $0 \le \mu_{G_p}(x) + \gamma_{G_p}(x) \le 1$.The notion of interval-valued intuitionistic fuzzy sets , which is characterized by a membership function and a non-membership function, whose values are intervals rather than exact numbers.

For an arbitrary set $G_p \subseteq [0,1]$, we define $\underline{G_p} = \inf(G_p)\ and\ \overline{G_p} = \sup(G_p)$

Definition 3.2:

Let $C = \langle G, G_p \rangle$, be an PIVIFSS over U, where $G_p \subseteq E$, E is the parameter set. Based on the PIVIFSS over U for $C = \langle G, G_p \rangle$,define a top - bottomPIVIFSS as,

- $\mu_{\text{top - bottom}_C} = max_{x\in U}\mu_{\overline{G(E)}}(x) = \left[max_{x\in U}\mu_{\overline{G(E)L}}(x), max_{x\in U}\mu_{\overline{G(E)R}}(x)\right]$
- $\gamma_{\text{top - bottom}_C} = min_{x\in U}\gamma_{\overline{G(E)}}(x) = \left[min_{x\in U}\gamma_{\overline{G(E)L}}(x), min_{x\in U}\gamma_{\overline{G(E)R}}(x)\right]$ for all $x \in A$.

We also define an PIVIFSSover U, asBottom - bottom - level PIVIFSS as,

- $\mu_{\text{Bottom - bottom}_C} = min_{x\in U}\gamma_{\overline{G(E)}}(x) = \left[min_{x\in U}\mu_{\overline{G(E)L}}(x), min_{x\in U}\mu_{\overline{G(E)R}}(x)\right]$
- $\gamma_{\text{Bottom - bottom}_C} = min_{x\in U}\gamma_{\overline{G(E)}}(x) = \left[min_{x\in U}\gamma_{\overline{G(E)L}}(x), min_{x\in U}\gamma_{\overline{G(E)R}}(x)\right]$ for all $x \in A$.

The Top - Bottom PIVIFSS is called the Top - bottom - threshold of the PIVIFSS$C = \langle G, G_p \rangle$ for the level soft set namely $L(C; top - bottom_C)$ (or) simply $L(C; top - bottom)$.

4. ILLUSTRATIVE EXAMPLE

An ecologist, Mr. Z, is committed to identifying and safeguarding the most endangered species in the Mudumalai forests of Tamil Nadu, South India, to avert their potential extinction. Although the road traversing the forest poses a substantial threat to numerous species, various other factors also contribute to their endangerment. In this study, we employ the proposed PIVIFSS framework to model a decision-making problem that will aid Mr. Z in selecting the species most at risk. The model is based on two approaches: the top-down level method and the bottom-up level method using

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PIVIFSS. Drawing from a comprehensive review of existing literature and insights provided by the Conservator of Mudumalai forests in the Nilgiri's (Coimbatore, Tamil Nadu) region, the critical parameters identified are:

1. Overhunting
2. Introduction of non-native species
3. Spread of diseases
4. Habitat degradation
5. Forest fires, and
6. Deforestation

Let the universal set U= {Asian Elephants, Indian Leopard, Indian Wild Dog(or)Dhole, Gaur, Long-billed Vulture and Bengal Tiger}be a sample set of sixendangered species from a non-homogeneous group of population can be represented as $U = \{s_{1,}\ \ s_{2,}\ \ s_{3,}\ \ s_{4,}\ \ s_{5,}\ \ s_6\}$ .Let the soft set $Z_P = \{\alpha_{1,}\ \ \alpha_{2,}\ \ \alpha_{3,}\ \ \alpha_{4,}\ \ \alpha_{5,}\ \ \alpha_6\}$ be the sixessential parameters in selection of the most endangered species from the forest.

Table 1: Membership and Non-membership values [Zhiming Zhanget.al., 2014]

U	α_1	α_2	α_3	α_4	α_5	α_6
s_1	[0.2,0.3], [0.5,0.6], 0.3	[0.6,0.7], [0.1,0.2], 0.3	[0.7,0.8], [0.2,0.2], 0.1	[0.6,0.8], [0.2,0.2]. 0.3	[0.2,0.3], [0.5,0.6], 0.4	[0.6,0.7], [0.2,0.3], 0.3
s_2	[0.2,0.4], [0.5,0.6], 0.2	[0.6,0.7], [0.2,0.3], 0.2	[0.7,0.8], [0.1,0.2], 0.4	[0.7,0.8], [0.2, 0.2] ,0.5	[0.2,0.4], [0.5,0.6], 0.2	[0.7,0.8], [0.1,0.2], 0.1
s_3	[0.7,0.8], [0.2,0.2],0.3	[0.3,0.5], [0.4,0.5], 0.3	[0.2,0.3], [0.5,0.6],0.1	[0.3,0.5], [0.4,0.5],0.3	[0.7,0.8], [0.2,0.2],0.4	[0.6,0.7], [0.2,0.3],0.3
s_4	[0.7,0.8], [0.1,0.2], 0.2	[0.4,0.5], [0.4,0.5], 0.2	[0.2,0.4], [0.5,0.6], 0.4	[0.4,0.5], [0.4,0.5], 0.5	[0.7,0.8], [0.1,0.2], 0.2	[0.7,0.8], [0.2,0.2], 0.2
s_5	[0.6,0.8], [0.1,0.2], 0.3	[0.6,0.8], [0.1,0.2], 0.3	[0.7,0.8], [0.1,0.2],0.2	⟨[0.8,0.8], [0.1,0.1], 0.3⟩	[0.4,0.8], [0.1,0.1], 0.4	[0.7,0.8], [0.1,0.2], 0.3
s_6	[0.8,0.9], [0.05,0.1], 0.2	[0.8,0.9], [0.05,0.1], 0.[illegible]	[0.8,0.9], [0.0,0.1], 0.4	[0.8,0.9], [0.05,0.0], 0.5	[0.5,0.8], [0.05,0.1], 0.2	[0.8,0.9], [0.1,0.05], 0.2

Hence Z(α_i) , i =1,2,3,4,5,6 is a set of all most endangered species from the forest that satisfy the respective parameter α_i, So the soft set (Z, Z_P) is a parameterized family$\{Z(\alpha_i)\}$ of subsets of U. The PIVIFSS$\langle Z, Z_p\rangle$describes the most endangered species from the forest to be selected with respect to the parameter over hunting(α_1) to the decision maker as follows,

$$Z_p(\alpha_1) = \left\{\langle\frac{s_1}{([0.2,0.3][0.5,0.6])}, 0.3\rangle, \langle\frac{s_2}{([0.2,0.4][0.5,0.6])}, 0.2\rangle, \ldots, \langle\frac{s_6}{([0.8,0.9][0.05,0.1])}, 0.2\rangle\right\}$$

Similarly Z(α_2), Z(α_3), Z(α_4), Z(α_5) and Z(α_6) can be definedPIVIFSS$\langle Z, Z_p\rangle$described with respect to the other parameters also. Based on

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the priority of the parameters the intuitionistic fuzzy values are defined with membership, non-membership and possible parameter values.

$Z_{P=}$ $\{\{\text{Introduction of non} - \text{native species}, \text{Spread of disease}, \text{Habit degradation}, \text{Forest Fire}, \text{Deforestation}\}\}$to the decision maker as follows,

Hence $\langle Z, Z_p\rangle$ =

$$\left\{\begin{array}{c}
\text{Over hunting} = \left\{\langle\frac{s_1}{([0.2,0.3][0.5,0.6])}, 0.3\rangle, \langle\frac{s_2}{([0.2,0.4][0.5,0.6])}, 0.2\rangle, \ldots, \langle\frac{s_6}{([0.8,0.9][0.05,0.1])}, 0.2\rangle\right\} \\
\text{Introduction of non} - \text{native species} = \left\{\langle\frac{s_1}{([0.6,0.7][0.1,0.2])}, 0.3\rangle \langle\frac{s_2}{([0.6,0.7][0.2,0.3])}, 0.2\rangle, \langle\frac{s_3}{([0.2,0.3][0.5,0.6])}, 0.3\rangle, \ldots, \langle\frac{s_6}{([0.8,0.9][0.05,0.1])}, 0.2\rangle\right\} \\
\text{Spread of diseases} = \langle\frac{s_1}{([0.7,0.8][0.2,0.2])}, 0.1\rangle \langle\frac{s_2}{([0.7,0.8][0.1,0.2])}, 0.4\rangle, \langle\frac{s_3}{([0.6,0.7][0.1,0.2])}, 0.2\rangle, \ldots, \langle\frac{s_6}{([0.8,0.9][0.0,0.1])}, 0.4\rangle \\
\text{Habit degradation} = \left\{\langle\frac{s_1}{([0.6,0.8][0.2,0.2])}, 0.3\rangle \langle\frac{s_2}{([0.7,0.8][0.2,0.2])}, 0.5\rangle, \langle\frac{s_3}{([0.2,0.4][0.5,0.6])}, 0.3\rangle, \ldots, \langle\frac{s_6}{([0.8,0.9][0.05,0.0])}, 0.5\rangle\right\} \\
\text{Forest Fire} = \left\{\left\{\langle\frac{s_1}{([0.2,0.3][0.5,0.6])}, 0.4\rangle \langle\frac{s_2}{([0.2,0.4][0.5,0.6])}, 0.2\rangle, \langle\frac{s_3}{([0.3,0.5][0.4,0.5])}, 0.2\rangle, \ldots, \langle\frac{s_6}{([0.5,0.8][0.05,0.1])}, 0.2\rangle\right\}\right\} \\
\text{Deforestation} = \left\{\langle\frac{s_1}{([0.6,0.7][0.1,0.2])}, 0.3\rangle \langle\frac{s_2}{([0.7,0.8][0.1,0.2])}, 0.3\rangle \langle\frac{s_3}{([0.8,0.9][0.0,0.1])}, 0.2\rangle, \ldots, \langle\frac{s_6}{([0.7,0.7][0.1,0.2])}, 0.1\rangle\right\}
\end{array}\right\}$$

4.1 New Illustration - Proposed Method using Top-bottom PIVIFSS

Algorithm:

i. Define interval-valued intuitionistic fuzzy soft sets with possibility values on a linguistic scale (refer to Table 1).

ii. For the parameterα_1, select the highest membership value and the lowest non-membership value across all species (refer to Table 1)..

iii. Similarly, identify the maximum and minimum values for each parameter and apply the Top-Bottom algorithm to PIVIFSS (as per Definition 2.2).

iv. Assign a choice value of 1 to the species corresponding to the selected parameters, while all other species receive a choice value of 0 (Zhiming Zhang et.al., 2014).

v. The species with the highest choice value is identified as the most endangered.

Methodology

The threshold value for the possibility value is p=0.4.So the parameters with the possibility value less than 0.4 can be neglected. Hence above calculation for PIVIFSS becomes as Hence (PIVIFSS)$\langle Z, Z_p\rangle$ is a parameterized family$\{\overline{Z_p}(\alpha_i), i = 1,2,3,4,5,6\}$ of (PIVIFSS) on U Hence every(PIVIFSS) can be viewed as an (IVIFSS)which can be represented as in Table 4 above for $\langle Z, Z_p\rangle$for any i=1,2,3,4,5,6 and j= 1,2,3,…6

Let us reconsider the (PIVIFSS) for $C = \langle Z, Z_p\rangle$ with Table 1.

The top - bottom - threshold of $C = \langle Z, Z_p\rangle$ for an (PIVIFSS)$C = \langle Z, Z_p\rangle$ can be calculated as follows:

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$$Top - bottom_{\langle Z,Z_p\rangle} = \left\{\begin{matrix} \langle \alpha_3, [0.8,0.9], [0.0,0.1], 0.4\rangle, \\ \langle \alpha_4, [0.8,0.9], [0.05,0.0], 0.5\rangle \end{matrix}\right\}$$

The top - bottom - level soft sets of $C = \langle Z, Z_p\rangle$ is a soft set $L\left((Z, Z_p)\,;\, top - bottom\right)$ and it can be calculated as follows:

$$\tilde{Z}_{top-bottom_C(\alpha_3)} = L\big(\tilde{Z}(\alpha_3); [0.8,0.9], [0.0,0.1], 0.4\big)$$

$$\tilde{Z}_{top-bottom_C(\alpha_4)} = L\big(\tilde{Z}(\alpha_4); [0.8,0.9], [0.05,0.0], 0.5\big) = \{s_6\}$$

The Table2 gives the tabular representation of the top - bottom level soft set

$L\left((Z, Z_p)\,;\, top - bottom\right)$. If$C_i \in \tilde{Z}_{top-bottom_C(\alpha_j)}$, then$C_{ij} = 1$, otherwise $C_{ij} = 0$.

Table 2: [Choice value calculation] [Zhiming Zhanget.al., 2014]

U	α_3	α_4
s_1	0	0
s2	0	0
s_3	0	0
s_4	0	0
s_5	0	0
s_6	1	1

Hence the choice values from the above table can be calculated asthe table representation of the top-bottom-level soft set $L\left((\tilde{Z}, Z_p); top - bottom\right)$

Table 3: [Optimal choice value] [Zhiming Zhanget.al., 2014]

U	α_3	α_4	Choice value (C_i)
s_1	0	0	$Cv_1 = 0$
s2	0	0	$Cv_2 = 0$
s_3	0	0	$Cv_3 = 0$
s_4	0	0	$Cv_4 = 0$
s_5	0	0	$Cv_5 = 0$
s_6	1	1	$Cv_6 = 2$

From above, $\max_{1\le i\le 6}\{Cv_i\} = \{S_6\}$is the optimal choice value.

4.2 New Illustration - Proposed Method using Bottom-bottom PIVIFSS

Algorithm:

1. Define interval-valued intuitionistic fuzzy soft sets with possibility values (refer to Table 1).
2. For the parameter α_1, select the lowest membership value and the lowest non-membership value across all species.

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3. Similarly, identify the maximum and minimum values for each parameter and apply the Bottom-Bottom algorithm to PIVIFSS (as per Definition 2.2).
4. Assign a choice value of 1 to the species corresponding to the selected parameters, while all other species receive a choice value of 0 (Zhiming Zhang et.al., 2014).
5. The species with the highest choice value is identified as the most endangered.

Methodology

The PIVIFSS Bottom - bottom is called the Bottom - bottom - threshold of the PIVIFSS $\complement = \langle G, G_p \rangle$ for the level soft set namely $L(\complement\,; Bottom - bottom_{\complement})$ (or) simply $L(\complement\,; Bottom - bottom)$. The Bottom - bottom - level soft sets of $\complement = \langle Z, Z_p \rangle$ is a soft set. The threshold value for the possibility value is p=0.4. So the parameters with the possibility value less than 0.4 can be neglected. Hence above calculation from Table 6 becomes as$\left((Z, Z_p)\,;\ Bottom - bottom\right)$ and it can be calculated as follows:

1. $\tilde{Z}_{Bottom-bottom_{\complement}(\alpha_3)} = L\left(\tilde{Z}(\alpha_3); [0.2, 0.4], [0.0, 0.1], 0.4\right) = \{s_6\}$,
2. $\tilde{Z}_{Bottom-bottom_{\complement}(\alpha_4)} = L\left(\tilde{Z}(\alpha_4); [0.2, 0.4], [0.0, 0.05], 0.5\right) = \{s_6\}$,

The table4 gives the tabular representation of the Bottom - bottom level soft set choice value

$L\left((Z, Z_p)\,;\ Bottom - bottom\right)$. If $C_i \in \tilde{Z}_{Bottom-bottom_{\complement}(\alpha_j)}$, then$C_{ij} = 1$, otherwise $C_{ij} = 0$

Table 4: [Choice value calculation] [Zhiming Zhanget.al., 2014]

U	α_3	α_4
s_1	0	0
s_2	0	0
s_3	0	0
s_4	0	0
s_5	0	0
s_6	1	1

Hence the choice values from the above table can be calculated as the table representation of the Bottom-bottom-level soft set $L\left((\tilde{Z}, Z_p);\ Bottom - bottom\right)$

Table 5: [optimal choice value] [Zhiming Zhanget.al., 2014]

U	α_3	α_4	Choice value (C_i)
s_1	0	0	$Cv_1 = 0$
s_2	0	0	$Cv_2 = 0$
s_3	0	0	$Cv_3 = 0$
s_4	0	0	$Cv_4 = 0$
s_5	0	0	$Cv_5 = 0$
s_6	1	1	$Cv_6 = 2$

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From above, $\max_{1\le i\le 6}\{Cv_i\}=\{C_6\}$ is the optimal choice value.

4.3 Sensitivity Analysis

In our comparative analysis of the proposed Possibility Interval-Valued Intuitionistic Fuzzy Soft Set (PIVIFSS) algorithm against the existing Interval-Valued Intuitionistic Fuzzy Soft Set (IVIFSS) algorithm, focusing on both top-bottom and bottom-bottom approaches, we have derived the following results.

ALGORITHM/RESULTS	Actual Parameters	Actual endangered species	Parameters and species considered after applying the algorithm	Optimized decision (Most endangered species)
IVIFSS (Top-Bottom)	6	6	6,6	S_6
IVIFSS (Bottom-Bottom)	6	6	6,6	S_6
PIVIFSS (Top-Bottom)	6	6	2,6	S_6
PIVIFSS (Bottom-Bottom)	6	6	2,6	S_6

Hence from section (4.1), (4.2) and (4.3) we had compared the optimal solution obtained using the existing method(IVIFSS Bottom-Bottom) and our proposed method(PIVIFSS Bottom-Bottom). It is observed that our proposed method optimize the results of the existing method by considering less parameters.

5. RESULTS AND DISCUSSION

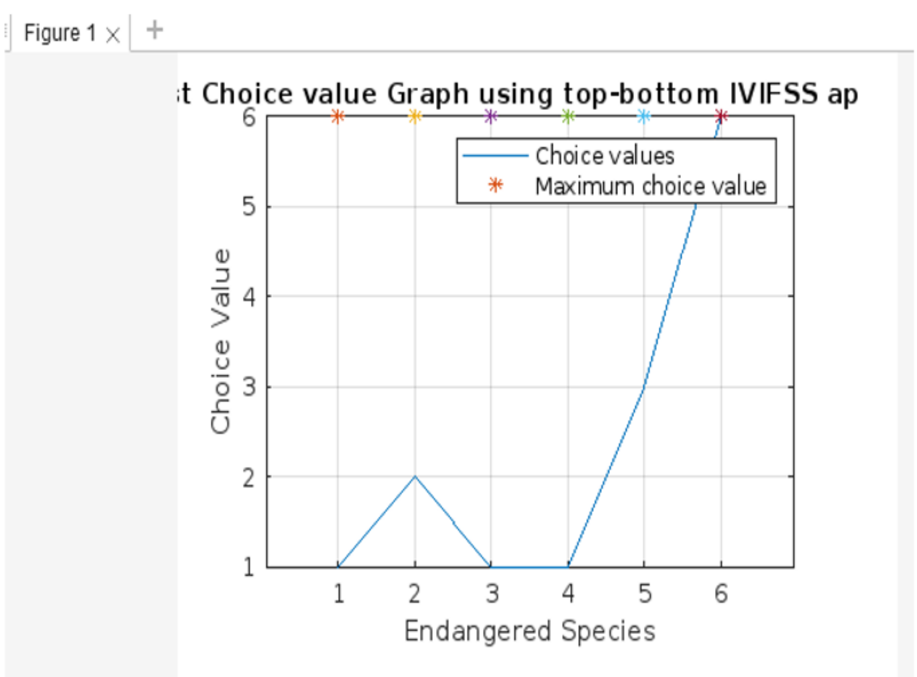

Fig 5.1: (Choice value graph using Top-Bottom IVIFSS)

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5.1 Results Obtained Using PIVIFSS (Top-Bottom Approach)

Forest Top bottom PIVIFSS

Forest membership (a_1, b_1) and non-membership (a2, b2) values

c1 =

0.3000	0.2000	0.6000	0.5000	0.3000
0.6000	0.7000	0.1000	0.2000	0.3000
0.7000	0.8000	0.3000	0.2000	0.2000
0.6000	0.8000	0.3000	0.2000	0.3000
0.3000	0.2000	0.6000	0.5000	0.4000
0.6000	0.8000	0.3000	0.2000	0.3000

c5 =

0.6000	0.8000	0.1000	0.2000	0.3000
0.6000	0.8000	0.1000	0.2000	0.3000
0.7000	0.8000	0.2000	0.1000	0.1000
0.8000	0.8000	0.1000	0.1000	0.3000
0.4000	0.8000	0.1000	0.1000	0.4000
0.7000	0.8000	0.1000	0.2000	0.3000

c6 =

0.8000	0.9000	0.0500	0.1000	0.2000
0.8000	0.9000	0.0500	0.1000	0.1000
0.9000	0.8000	0	0.1000	0.4000
0.8000	0.9000	0.0500	0	0.5000
0.5000	0.8000	0.0500	0.1000	0.1000
0.8000	0.9000	0.1000	0.2000	0.2000

ch =

0	0	0	0	0	1
0	0	1	0	0	1
0	0	0	0	0	1
0	0	0	0	0	1
0	0	1	0	1	1
1	1	1	1	1	1

Choice table for the Forest by Top-Bottom Approach is
Maximum choice value =

ch value = 6

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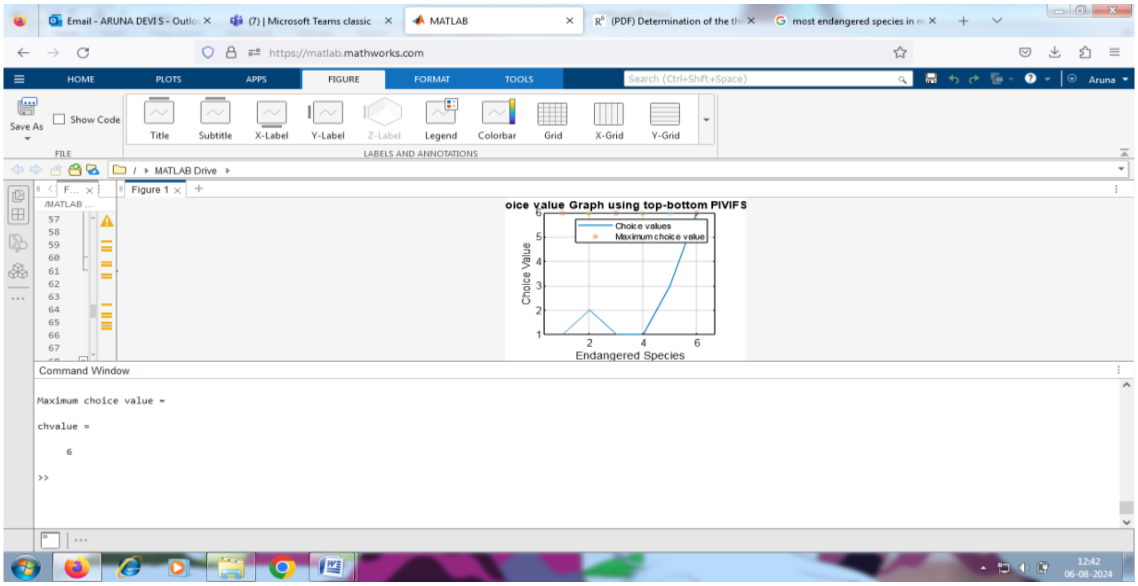

Fig 5.2: (Choice value graph using Bottom-Bottom PIVIFSS)

5.2 Results Obtained Using IVIFSS (Bottom-Bottom Approach)

Forest

Forest membership (a1, b1) and non-membership (a2, b2) values

p1 =

0.6000	0.8000	0.1000	0.2000	0.3000
0.6000	0.8000	0.1000	0.2000	0.3000
0.7000	0.8000	0.2000	0.1000	0.1000
0.8000	0.8000	0.1000	0.1000	0.3000
0.4000	0.8000	0.1000	0.1000	0.4000
0.7000	0.8000	0.1000	0.2000	0.3000

p6 =

0.8000	0.9000	0.0500	0.1000	0.2000
0.8000	0.9000	0.0500	0.1000	0.1000
0.9000	0.8000	0	0.1000	0.4000
0.8000	0.9000	0.0500	0	0.5000
0.5000	0.8000	0.0500	0.1000	0.1000
0.8000	0.9000	0.1000	0.2000	0.2000

p2 =

0.2000	0.4000	0.8000	0.5000	0.2000
0.6000	0.8000	0.3000	0.2000	0.1000

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0.7000	0.8000	0.1000	0.1000	0.4000
0.7000	0.8000	0.2000	0.2000	0.5000
0.2000	0.4000	0.8000	0.5000	0.2000
0.7000	0.8000	0.3000	0.2000	0.2000

ch =

0	0	1	0	1	1
0	0	1	0	0	1
0	0	0	0	0	0
0	0	0	0	0	1
0	0	0	0	0	1
1	1	1	1	1	1

Choice table for the Forest by Bottom-Bottom Approach is Maximum choice value =
ch value = 6

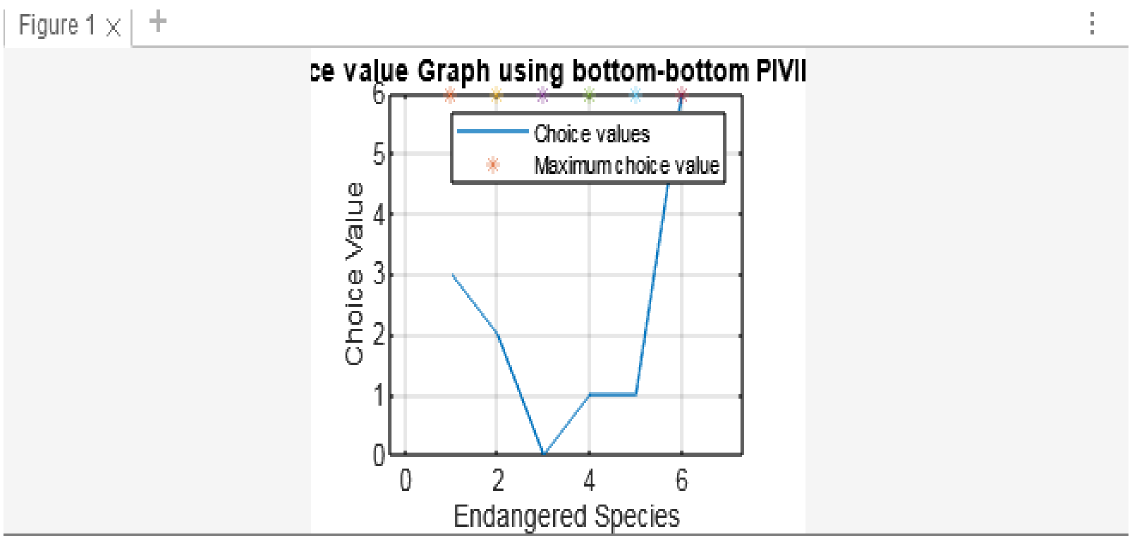

Fig 5.3: (Choice value graph using Bottom-Bottom PIVIFSS)

6. CONCLUSION

To conclude, the integration of membership values with possibility values, as demonstrated in the comparative analysis of the PIVIFSS Top-Bottom and PIVIFSS Bottom-Up algorithms, proves to be highly advantageous for decision-making under uncertainty. By ensuring that parameters with exceptionally low possibility values are not overly influential, this approach leads to more balanced and accurate outcomes in complex scenarios. For instance, when applied to the conservation prioritization of species such as the critically endangered Bengal Tiger in the Mudumalai forest, the ecologist is able to make informed and practical decisions. This prioritization exemplifies the real-world efficacy of the PIVIFSS algorithms, which guide strategic interventions in ecologically sensitive situations where uncertainties are prevalent.

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Furthermore, the robustness of the PIVIFSS methods in handling ambiguity and uncertainty underscores their utility in broader applications. Both the Top-Bottom and Bottom-Up approaches demonstrate consistent performance, providing clear, prioritized results despite the inherent complexities of environmental and ecological data. As such, these algorithms not only facilitate better decision-making but also contribute significantly to the development of more nuanced conservation strategies.

In future research, expanding the applicability of these algorithms to handle larger values of **nnn** could further enhance their utility in more expansive datasets and more intricate decision-making environments. Additionally, there is considerable potential for extending the PIVIFSS methodology to other mathematical frameworks, such as interval-valued rough sets and Fermatean rough sets. These extensions would broaden the scope of the approach, making it even more adaptable to various decision-making contexts, especially those involving multi-dimensional uncertainty. The evolution of this methodology promises to further refine decision processes across disciplines, solidifying the role of PIVIFSS in uncertain, data-intensive fields.

Data Availability Statement

The data is taken from Zhiming Zhang et.al., 2014.

Conflicts of Interest

The authors declare no conflict of interest.

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